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Stabilizing Controllers for Plants with Perturbations

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Abstract This paper gives a convenient parametrization for the class of all stabilizing controllers in the presence of plant perturbation. The perturbations are constrained in such class as plants are stabilizable by a nominal controller. By using the controller stabilizing a given plant with perturbation, we can obtain a parametrization of all stable closed-loop transfer functions, which are affine in the free parameter of the controller. It is easy to extend the controller to the case of a two-degree-of-freedom controller.

1. Introduction

The Kucera-Yula parametrization gives a convenient representation for the class of all stabilizing controllers in terms of an arbitrary proper stable transfer function. The parametrization is powerful for robust controller design and other control problems [1,2]. By using the Kucera-Yula parametrization, the stability of designed control system can be guaranteed for a nominal plant while the optimization proceeds on the parameters. Moreover, since closed-loop transfer functions are affine in the free parameter of the controller, optimization for performance and robustness of the controller within an H_2 or H^{∞} framework is attractive. However, in the case that there exists perturbation in plant dynamics, we cannot ensure any longer the stability of control system. Obinata and Moore [3] have extended the Kucéra-Yula parametrization to the case of stabilizing controllers for two or more plants. Tay et al. [4-7] have given a similar extension to the case that there exists perturbations in plant dynamics. Moreover, the idea of enhancing the performance of a fixed robust controller when applied to plants other than the nominal one is also explored using adaptive (on-line) techniques. In this paper, we treat the similar parametrization for the class of all stabilizing controllers for plants with perturbations based on the former results. The perturbations are constrained in such class as the plants are stabilizable by the nominal controller. Such class of plants can be parametrized in terms of the stable rational proper transfer functions based on the parametrization theory. The parametrization for the case of perturbed plants can be also extended to a two-degree-of-freedom controller. The derived representations for stabilizing controllers are convenient for robust controller design and adaptive control problems.

In the following, R_P denotes the class of rational proper transfer functions, and RH^{∞} denotes the class of stable transfer functions in R_P.

2. Preliminaries

In this section, we first review the parametrization theory for the class of all stabilizing controllers for a nominal plant, and the dual class of plants stabilized by a nominal controller. Using convenient parametrizations of plants and controllers, we investigate closed-loop stability conditions for plants with perturbations parametrized in terms of a transfer function R, and feedback controllers parametrized in terms of a transfer function Q.

Consider a stabilizable and detectable nominal plant $P_0 \in R_P$, and proper stabilizing controllers for P_0 as $C \in R_P$, as in Fig.1, where the closed-loop system is well posed. That is

$$\begin{bmatrix} I & -C \\ -P_0 & I \end{bmatrix}^{-1} \text{ exists and belongs to } RH^{\infty} \qquad (1)$$

Consider also coprime factorizations for the plant P_0 and some stabilizing controller C_0 for P_0 as

$$P_0 = N_0 M_0^{-1} = \tilde{M}_0^{-1} \tilde{N}_0 ; \quad N_0, M_0, \tilde{N}_0, \tilde{M}_0 \in RH^{\infty} \qquad (2)$$

$$C_0 = U_0 V_0^{-1} = \tilde{V}_0^{-1} \tilde{U}_0 ; \quad U_0, V_0, \tilde{U}_0, \tilde{V}_0 \in RH^{\infty} \qquad (3)$$

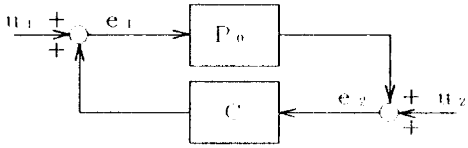


Fig.1 Feedback control system.

which satisfy the double Bezout identity as

$$\begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N}_0 & \tilde{M}_0 \end{bmatrix} \begin{bmatrix} M_0 & U_0 \\ N_0 & V_0 \end{bmatrix} = \begin{bmatrix} M_0 & U_0 \\ N_0 & V_0 \end{bmatrix} \begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N}_0 & \tilde{M}_0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{4}$$

The class of all stabilizing controllers for the nominal plant P_0 can be characterized in terms of an arbitrary $Q \in RH^\infty$ as (see [1,2])

$$\begin{aligned} C(Q) &= UV^{-1}, \quad U = U_0 + M_0 Q, \quad V = V_0 + N_0 Q \\ &= \tilde{V}^{-1}\tilde{U}, \quad \tilde{U} = \tilde{U}_0 + Q\tilde{M}_0, \quad \tilde{V} = \tilde{V}_0 + Q\tilde{N}_0 \end{aligned} \tag{5}$$

The controller $C(Q)$ of (5) can be also written via (4) as

$$C(Q) = C_0 + \tilde{V}_0^{-1} Q (I + V_0^{-1} N_0 Q)^{-1} V_0^{-1} \tag{6}$$

which can be reorganized as in Fig.2 with

$$J_0 = \begin{bmatrix} C_0 & \tilde{V}_0^{-1} \\ V_0^{-1} & -V_0^{-1} N_0 \end{bmatrix} \in R_P \tag{7.a}$$

$$T_0 = \begin{bmatrix} \begin{bmatrix} I & -C_0 \\ -P_0 & I \end{bmatrix}^{-1} & \begin{bmatrix} M_0 \\ N_0 \end{bmatrix} \\ \begin{bmatrix} \tilde{N}_0 & \tilde{M}_0 \end{bmatrix} & 0 \end{bmatrix} \in RH^\infty \tag{7.b}$$

It is known that the associated four closed-loop transfer functions between the u_i and e_i of Fig.2 are affine in Q as follows: (see [4])

$$\begin{bmatrix} I & -C(Q) \\ -P_0 & I \end{bmatrix}^{-1} = \begin{bmatrix} I & -C_0 \\ -P_0 & I \end{bmatrix}^{-1} + \begin{bmatrix} M_0 \\ N_0 \end{bmatrix} Q \begin{bmatrix} \tilde{N}_0 & \tilde{M}_0 \end{bmatrix} \in RH^\infty \tag{8}$$

The dual result is immediate on interchanging the role of controller and plant in the parametrization theory for the class of all stabilizing controllers. That is, under the above definitions, the class of all proper plants stabilizable by C_0 is characterized by an arbitrary $R \in RH^\infty$ as

$$\begin{aligned} P(R) &= NM^{-1}, \quad N = N_0 + V_0 R, \quad M = M_0 + U_0 R \\ &= \tilde{M}^{-1}\tilde{N}, \quad \tilde{N} = \tilde{N}_0 + R\tilde{V}_0, \quad \tilde{M} = \tilde{M}_0 + R\tilde{U}_0 \end{aligned} \tag{9}$$

Then, the closed-loop transfer function matrix of $(P(R), C_0)$ is given by

$$\begin{bmatrix} I & -C_0 \\ -P(R) & I \end{bmatrix}^{-1} = \begin{bmatrix} I & -C_0 \\ -P_0 & I \end{bmatrix}^{-1} + \begin{bmatrix} U_0 \\ V_0 \end{bmatrix} R \begin{bmatrix} \tilde{V}_0 & \tilde{U}_0 \end{bmatrix} \in RH^\infty \tag{10}$$

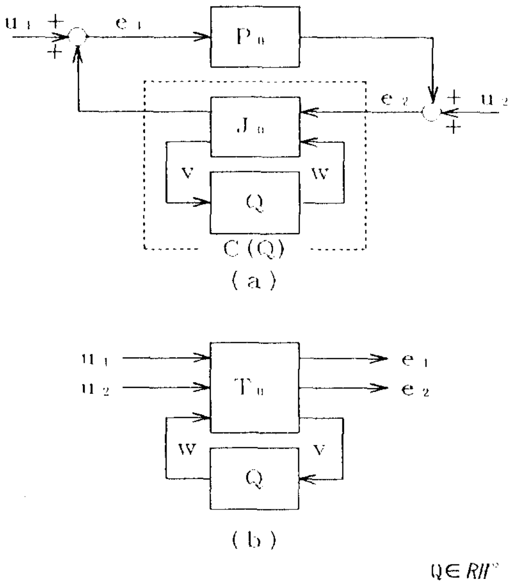


Fig.2 Class of all stabilizing controllers.

Concerning robust stabilization for plant $P(R)$ with feedback controller $C(Q)$, we can obtain the following theorem.

Theorem 1(Tay et al.[7]).

Let (P_0, C_0) be a stabilizing nominal plant-controller pair. Let $C(Q)$ be the class of controllers parametrized by Q as in (5) and $P(R)$ be the class of plants parametrized by R as in (9). (Here there is no requirement that $Q, R \in RH^\infty$.) Then, $(P(R), C(Q))$ forms a stabilizing pair if and only if Q stabilizes R in that

$$\begin{bmatrix} I & -Q \\ -R & I \end{bmatrix}^{-1} \text{ exists and belongs to } RH^\infty \tag{11}$$

Moreover, the closed-loop transfer function matrix of $(P(R), C(Q))$ is given by

$$\begin{bmatrix} I & -C(Q) \\ -P(R) & I \end{bmatrix}^{-1} = \begin{bmatrix} I & -C_0 \\ -P_0 & I \end{bmatrix}^{-1} + \begin{bmatrix} M_0 & U_0 \\ N_0 & V_0 \end{bmatrix} \times \begin{bmatrix} I & -Q \\ -R & I \end{bmatrix}^{-1} \begin{bmatrix} 0 & Q \\ R & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_0 & \tilde{U}_0 \\ \tilde{N}_0 & \tilde{M}_0 \end{bmatrix} \in RH^\infty \tag{12}$$

Remarks

1) Note that there is no requirement in this theorem for either R or Q to belong to RH^∞. This allows us to consider the case when the nominal controller does not stabilize the plant $P(R)$.

2) For the simultaneous stabilization of $P_0, P(R)$, there must be a stable controller $Q \in RH^\infty$ stabilizing R (see [3]).

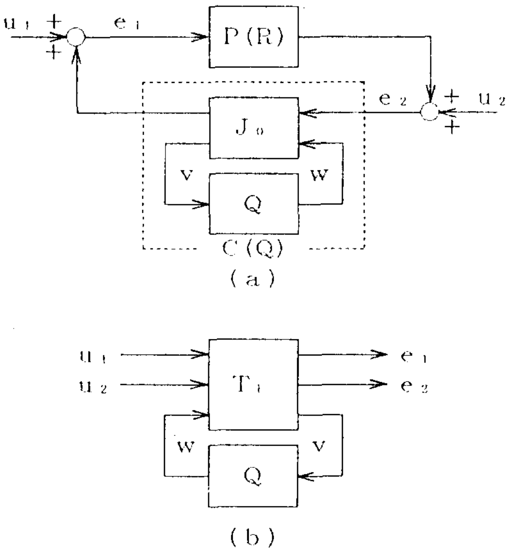


Fig.3 Control system in the presence of plant perturbation.

3) The above results hold for the time-varying case (see [6]).

4) The closed-loop system $(P(R),C(Q))$ can be reorganized as in Fig.3 with $P(R)$ parametrized by R as in (9), J_0 given by (7.a) and T_1 given by

$$T_1=\begin{bmatrix} \begin{bmatrix} I & -C_0 \\ -P(R) & I \end{bmatrix}^{-1} & \begin{bmatrix} M \\ N \end{bmatrix} \\ \begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix} & R \end{bmatrix} \tag{13}$$

Thus information about the plant perturbation R can be deduced by observing the signals w and v (see [7]).

3. Stabilizing controllers for perturbed plants

In this section, we derive a convenient parametrization for the class of all stabilizing controllers in the presence of plant perturbation by using Theorem 1. The perturbations are constrained in such class as the plants are stabilizable by a nominal controller.

Theorem 2.

Suppose the pair (P_0,C_0) is stable and $P(R)$ is the class of plants parametrized by $R\in RH^\infty$ as in (9). (Here we require that $R\in RH^\infty$.) Then, under (2)-(4) and (9), the class of all stabilizing controllers for $P(R)$ can be characterized in terms of an arbitrary $Q(S)$ within the set

$$\begin{aligned} Q(S)&=\{S(I+RS)^{-1}:\ S\in RH^\infty,\ |I+RS|\neq 0\} \\ &=\{(I+SR)^{-1}S:\ S\in RH^\infty,\ |I+SR|\neq 0\} \end{aligned} \tag{14}$$

as

$$\begin{aligned} C(Q(S))&=U(Q(S))V(Q(S))^{-1} \\ &=\tilde{V}(Q(S))^{-1}\tilde{U}(Q(S)) \end{aligned} \tag{15.a}$$

where

$$\begin{aligned} &U(Q(S))=U_0+M_0Q(S), \quad V(Q(S))=V_0+N_0Q(S), \\ &\tilde{U}(Q(S))=\tilde{U}_0+Q(S)\tilde{M}_0, \quad \tilde{V}(Q(S))=\tilde{V}_0+Q(S)\tilde{N}_0 \end{aligned} \tag{15.b}$$

Moreover, the controllers $C(Q(S))$ can be also parametrized in terms of an arbitrary $S\in RH^\infty$ as

$$\begin{aligned} C(Q(S))&=U_1V_1^{-1}, \quad U_1=U_0+MS, \quad V_1=V_0+NS \\ &=\tilde{V}_1^{-1}\tilde{U}_1, \quad \tilde{U}_1=\tilde{U}_0+S\tilde{M}, \quad \tilde{V}_1=\tilde{V}_0+S\tilde{N} \end{aligned} \tag{16}$$

Then, the closed-loop transfer function matrix of $(P(R),C(Q(S)))$ is given by

$$\begin{aligned} \begin{bmatrix} I & -C(Q(S)) \\ -P(R) & I \end{bmatrix}^{-1} &= \begin{bmatrix} I & -C_0 \\ -P_0 & I \end{bmatrix}^{-1} \\ &+ \begin{bmatrix} M & U_0 \\ N & V_0 \end{bmatrix} \begin{bmatrix} 0 & S \\ R & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_0 & \tilde{U}_0 \\ \tilde{N} & \tilde{M} \end{bmatrix} \in RH^\infty \end{aligned} \tag{17}$$

proof: Since R is stable, the set of all stabilizing controllers for $R\in RH^\infty$ is characterized by an arbitrary $S\in RH^\infty$ as in (14), and $Q(S)$ stabilizes R (see [2]). Then, the closed-loop system of $(R,Q(S))$ with (14) can be written as follows:

$$\begin{bmatrix} I & -Q(S) \\ -R & I \end{bmatrix}^{-1} = \begin{bmatrix} I+SR & S \\ R+RSR & I+RS \end{bmatrix} \in RH^\infty \tag{18}$$

and simple manipulation on (12) gives (17). From Theorem 1, the stability of R and S implies that the closed-loop system $(P(R),C(Q(S)))$ is stable, and $C(Q(S))$ stabilizes $P(R)$. It is easy to derive that the representation of $C(Q(S))$ as (15) with (14) can be also written by (9) as (16). □

Remarks

1) There is a requirement that $R,S\in RH^\infty$, but there is no requirement in this theorem for $Q(S)$ to belong to RH^∞. This allows us to consider the case when the controller $C(Q(S))$ does not stabilize the nominal plant P_0.

2) For the simultaneous stabilization of $P_0,P(R)$, there must be that the set of S is the subset of RH^∞ with the constraint $S(I+RS)^{-1}\in RH^\infty$.

3) If $R=0$, that is $P(0)=P_0$, then (14) reduces to S and (15) implies (5).

4) Note that coprime factorizations for C_0 of (3) and $P(R)$ of (9) satisfy the double Bezout identity as

$$\begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & U_0 \\ N & V_0 \end{bmatrix} = \begin{bmatrix} M & U_0 \\ N & V_0 \end{bmatrix} \begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N} & \tilde{M} \end{bmatrix}$$

$$= \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{19}$$

Thus representation of (16) is immediate from the more familiar theory for the class of all stabilizing controllers.

5) The controller $C(Q(S))$ of (15) can be reorganized as in Fig.4(a) with J_0 given by (7.a). Note also that $C(Q(S))$ can be written via (19) as

$$C(Q(S))=C_0+\tilde{V}_0^{-1}S(I+V_0^{-1}NS)^{-1}V_0^{-1} \tag{20}$$

which can be also reorganized as in Fig.4(b,c) with

$$J_1=\begin{bmatrix} C_0 & \tilde{V}_0^{-1} \\ V_0^{-1} & -V_0^{-1}N \end{bmatrix} \in R_P \tag{21.a}$$

$$T_2=\begin{bmatrix} \begin{bmatrix} I & -C_0 \\ -P(R) & I \end{bmatrix}^{-1} & \begin{bmatrix} M \\ N \end{bmatrix} \\ \begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix} & 0 \end{bmatrix} \in RH^\infty \tag{21.b}$$

Then, the associated four closed-loop transfer functions between the u_i and e_i of Fig.4 are affine in S as in (17). Therefore, if we can identify R, then the coprime factorization for the perturbed plant can be given by (9), and the parameter S can be utilized for adaptive tuning or robust controller design.

6) The dual result is immediate on interchanging the role of controller and plant in Theorem 2. Let (P_0,C_0) is stable and $C(Q)$ is the class of controllers parametrized by $Q\in RH^\infty$ as in (5). Then, under (2)-(5), the class of all proper plants stabilizable by $C(Q)$ is characterized in terms of an arbitrary $R(H)$ within the set

$$\begin{aligned} R(H)&=\{H(I+QH)^{-1}:\ H\in RH^\infty,\ |I+QH|\neq 0\} \\ &=\{(I+HQ)^{-1}H:\ H\in RH^\infty,\ |I+HQ|\neq 0\} \end{aligned} \tag{22}$$

as

$$\begin{aligned} P(R(H))&=N(R(H))M(R(H))^{-1} \\ &=\tilde{M}(R(H))^{-1}\tilde{N}(R(H)) \end{aligned} \tag{23.a}$$

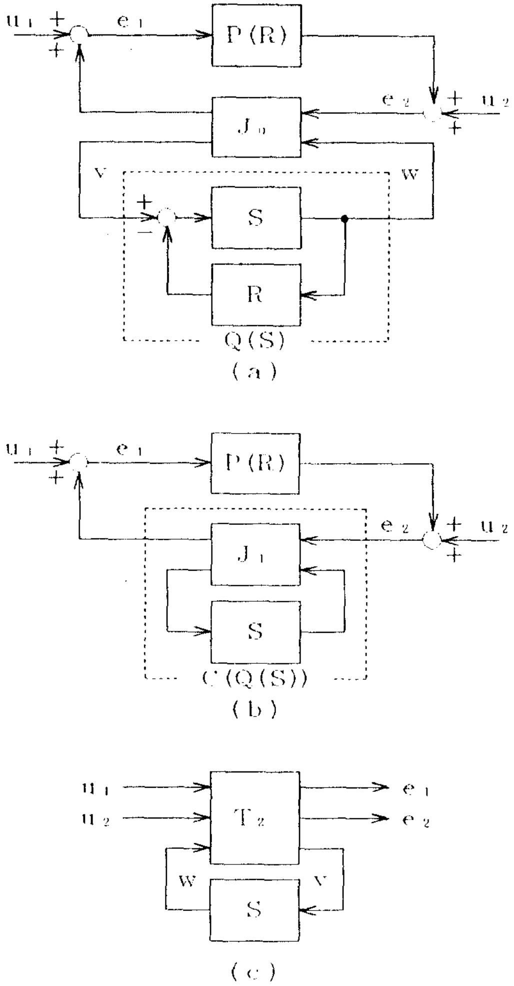
where

$$\begin{aligned} &N(R(H))=N_0+V_0R(H), \quad M(R(H))=M_0+U_0R(H), \\ &\tilde{N}(R(H))=\tilde{N}_0+R(H)\tilde{V}_0, \quad \tilde{M}(R(H))=\tilde{M}_0+R(H)\tilde{U}_0 \end{aligned} \tag{23.b}$$

Moreover, by using the representation of the controller $C(Q)$ of (5), the plant $P(R(H))$ of (23) with (22) can be also parametrized in terms of an arbitrary $H\in RH^\infty$ as

$$\begin{aligned} P(R(H))&=N_1M_1^{-1}, \quad N_1=N_0+VH, \quad M_1=M_0+UH \\ &=\tilde{M}_1^{-1}\tilde{N}_1, \quad \tilde{N}_1=\tilde{N}_0+H\tilde{V}, \quad \tilde{M}_1=\tilde{M}_0+H\tilde{U} \end{aligned} \tag{24}$$

Then, the closed-loop transfer function matrix of $(P(R(H)),C(Q))$ is given by



$R, S\in RH^\infty$

Fig.4 Class of all stabilizing controllers for perturbed plants

$$\begin{bmatrix} I & -C(Q) \\ -P(R(H)) & I \end{bmatrix}^{-1} = \begin{bmatrix} I & -C_0 \\ -P_0 & I \end{bmatrix}^{-1} + \begin{bmatrix} M_0 & U \\ N_0 & V \end{bmatrix} \begin{bmatrix} 0 & Q \\ H & 0 \end{bmatrix} \begin{bmatrix} \tilde{V} & \tilde{U} \\ \tilde{N}_0 & \tilde{M}_0 \end{bmatrix} \in RH^\infty \tag{25}$$

4. Two-degree-of-freedom controllers

In the previous section, we gave a parametrization for the class of all stabilizing one-degree-of-freedom controllers for perturbed plants stabilizable by a nominal controller. In this section, this result is extended in the case of a two-degree-of-freedom control system.

Consider a two-degree-of-freedom control system as shown in Fig.5. Where $P_0\in R_P$ is the nominal plant of (2),

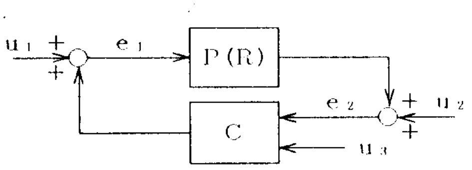


Fig.5 Two-degree-of-freedom control system.

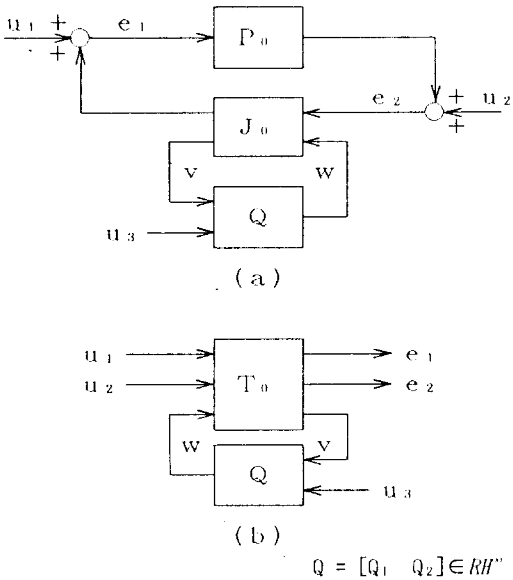


Fig.6 Class of all stabilizing two-degree-of-freedom controllers.

$C=[C_1 \quad C_2] \in R_P$ is a two-degree-of-freedom controller, and u_3 is a reference signal. It is known that the class of all stabilizing two-degree-of-freedom controllers for the nominal plant P_0 can be characterized in terms of arbitrary parameters $Q_1, Q_2 \in RH^\infty$, under (2)-(4), as (see [2])

$$C(Q_1,Q_2) = (\tilde{V}_0+Q_1\tilde{N}_0)^{-1}\left[(\tilde{U}_0+Q_1\tilde{M}_0) \quad Q_2\right] \tag{26}$$

By using (4) and the follwing matrix identity:

$$(A+BC)^{-1} = A^{-1}-A^{-1}B(I+CA^{-1}B)^{-1}CA^{-1} \tag{27}$$

the controller $C(Q_1,Q_2)$ of (26) can be also written as follows:

$$C(Q_1,Q_2) = \left[C_0+D_0 \quad (\tilde{V}_0^{-1}-D_0N_0)Q_2\right] \tag{28.a}$$

where

$$D_0 = \tilde{V}_0^{-1}Q_1(I+V_0^{-1}N_0Q_1)^{-1}V_0^{-1} \tag{28.b}$$

which can be reorganized as in Fig.6 with J_0 and T_0 given by (7), and $Q=[Q_1 \quad Q_2] \in RH^\infty$. The associated six closed-loop transfer functions between the u_i and e_i of Fig.6 are affine in $Q_1, Q_2 \in RH^\infty$ as follows:

$$W(P_0, C(Q_1, Q_2))$$
$$= \left[\begin{bmatrix} I & -C_0 \\ -P_0 & I \end{bmatrix}^{-1} + \begin{bmatrix} M_0 \\ N_0 \end{bmatrix} Q_1 \begin{bmatrix} \tilde{N}_0 & \tilde{M}_0 \end{bmatrix}, \begin{bmatrix} M_0 \\ N_0 \end{bmatrix} Q_2\right] \in RH^\infty \tag{29}$$

Moreover, by using the two-degree-of-freedom controller $C(Q_1,Q_2)$, two specifications could be independently satisfied; one is closed-loop characteristics such as the disturbance rejection, and the other is transfer characteristics with respect to the reference command.

On the other hand, in the case of the presence of plant perturbation, by using Theorem 2 under (3),(9) and (19), the class of all stabilizing two-degree-of-freedom controllers for the perturbed plant $P(R)$ of (9) characterized in terms of new arbitrary parameters $S_1, S_2 \in RH^\infty$ as

$$C(Q(S_1,S_2)) = (\tilde{V}_0+S_1\tilde{N})^{-1}\left[(\tilde{U}_0+S_1\tilde{M}) \quad S_2\right] \tag{30}$$

Setting $C(Q_1,Q_2)=C(Q(S_1,S_2))$, we get a following relation between the (Q_1,Q_2) and (S_1,S_2).

$$[Q_1 \quad Q_2] = (I+S_1R)^{-1}[S_1 \quad S_2] \tag{31}$$

Note also that $C(Q(S_1,S_2))$ of (30) can be written via (19) as

$$C(Q(S_1,S_2)) = \left[C_0+D \quad (\tilde{V}_0^{-1}-DN)S_2\right] \tag{32}$$

where

$$D = \tilde{V}_0^{-1}S_1(I+V_0^{-1}NS_1)^{-1}V_0^{-1} \tag{33}$$

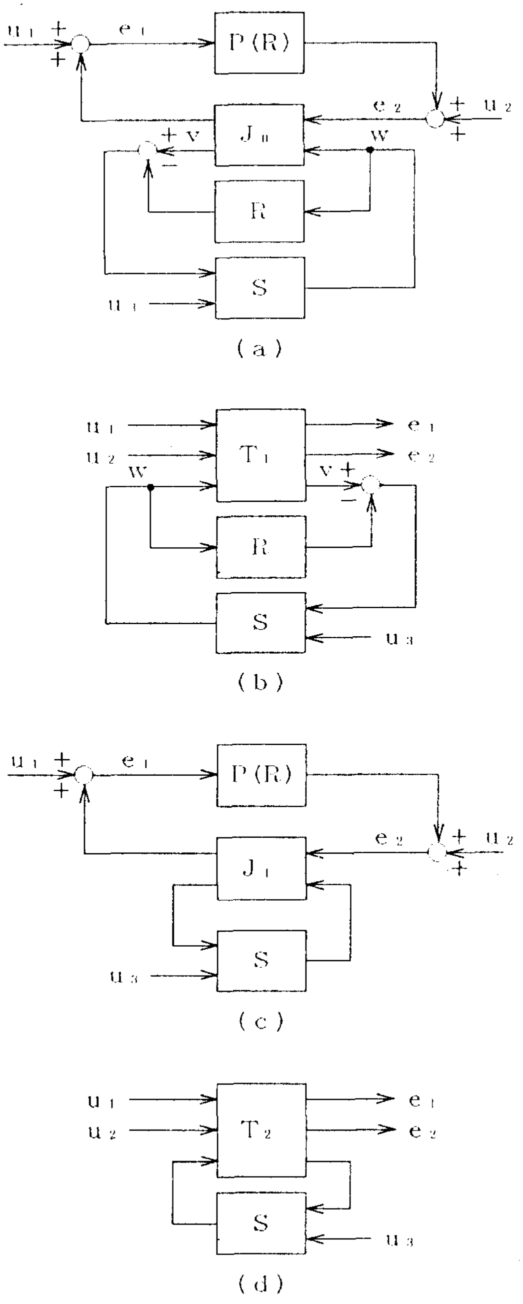
This class can be conveniently reorganized as in Fig.7 with J_0 given by (7.a), T_1 given by (13), J_1 and T_2 given by (21), and $S=[S_1 \quad S_2] \in RH^\infty$. The associated six closed-loop transfer functions between the u_i and e_i of Fig.7 are affine in $S_1, S_2 \in RH^\infty$ as follows:

$$W(P(R), C(Q(S_1,S_2)))$$
$$= \left[\begin{bmatrix} I & -C_0 \\ -P(R) & I \end{bmatrix}^{-1} + \begin{bmatrix} M \\ N \end{bmatrix} S_1 \begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix}, \begin{bmatrix} M \\ N \end{bmatrix} S_2\right] \in RH^\infty \tag{34}$$

This result presents the same characteristics as the case for the nominal plant. That is, two specifications with respect to responses for reference commands and closed-loop characteristics could be independently satisfied.

5. Conclusions

By using a convenient representation of plants stabilizable by a nominal controller, the class of all stabilizing controllers for perturbed plants are characterized in terms of an arbitrary



$R,\ S = [S_1\ \ S_2] \in RH^{\infty}$

Fig.7 Class of all stabilizing two-degree-of-freedom controllers for perturbed plants.

stable proper transfer function. This class is conveniently reorganized in accordance with the case of the nominal plant. Moreover, we can obtain a parametrization of all stable closed-loop transfer functions, which are affine in the free parameter of the controller. The further problem to be done is to investigate adaptive control problems.

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