Identification of Hard Bound on Model Uncertainty In Frequency Domain

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Abstract: In this paper, we investigate a set-membership identification approach to the quantification of an upper bound of model uncertainty in frequency domain, which is required in the H_∞ robust control system design. First we formulate this problem as a set-membership identification of a nominal model error in the presence of unknown noise input with unknown bound, while the ordinary set-membership approaches assume that an upper bound of the uncertain input is known. For this purpose, the proposed algorithm includes the estimation of the bound of the uncertain input. Thus the proposed method can obtain the hard bound of the model error in frequency domain as well as a parametric lower-order nominal model. Finally numerical simulation results are shown to confirm the validity of the presented algorithm.

1. Introduction

Recently system identification for robust control system design has collected much research interests [1]. For instance, the H_∞ control system design needs the knowledge on both nominal model and its frequency domain model error bound. Then the model error quantification should be carried out, while conventional identification schemes aimed mainly at estimation of only nominal models. For this purpose, first, H_∞ identification schemes minimizing the H_∞ error norm have been investigated and several kinds of worst error bounds were established [2][3]. However these error bounds cannot link directly with the determination of the uncertainty bound or weighting function in frequency domain for controller design. Second, the set membership identification has also been developed to estimate a parameter set of the nominal model on the condition that the model error bounds are given a priori [4]-[6]. In [4], a bound of the parameters of the input-output model was given as an ellipsoidal bound on the assumption that the upper bound of model uncertainty is given a priori in frequency domain. However, this scheme could not link with the H_∞ control design. On the other hand, if the upper bound of model uncertainty in time domain is given as a prior information, the model error bound in frequency domain can be estimated via the set-membership identification method [5][6], which is more suitable to the controller design. Third, the model validation approach has been proposed to investigate the problem how to find out the smallest set of the model error and the unknown noise input, given a nominal model and a finite input-output data sequence [7]-[9]. This interesting approach, however, cannot give the upper bound of model uncertainty but a minimal model set which does not invalidate the given data. The fourth is a statistical approach in which unmodeled dynamics is treated as a realization of a stochastic process described by a parametrized probability density function [10]. This method is different from the above three approaches in that it gives a soft or statistical bound of model uncertainty. Comparison of these approaches are summarized in Table 1.

In this paper, we propose a new identification scheme for a hard bound of unmodeled dynamics in frequency domain, which will be linked directly with the H_∞ control design. We formulate this problem as a set-membership identification of a nominal model error in the presence of unknown input with an unknown bound, while the ordinary set-membership approaches assume that an upper bound of the uncertain input is known. For this purpose, the proposed algorithm includes the estimation of the bound of the uncertain input. Hence, in this scheme we do not need the assumption that the bound of disturbances and the model error bound in time domain are known a priori, which is different from [4]-[6]. Now we can apply the identification of the model error bound even when the nominal model is described by a transfer function or a state space model, which is also a new aspect of the proposed method. Finally validity of the proposed scheme is studied in numerical simulations with comparison to other approaches.

2 New Scheme for Qualification of Model Error Bound

As shown in Fig.1, the true input-output description $G^o(z)$ is assumed to be given by

$$G^o(z) = G(z,\theta^o) + \Delta G(z) \tag{1}$$

where $G(z,\theta^o)$ is a stable nominal model and $\Delta G(z)$ is an additive model uncertainty.

The nominal model is normally chosen a lower-order parametric model. In this paper we treat with the transfer function model described by

$$G(z,\theta) = \frac{B_\theta(z)}{A_\theta(z)} = \frac{b_1 z^{-1} + \cdots + b_n z^{-n}}{1 + a_1 z^{-1} + \cdots + a_n z^{-n}} \tag{2}$$

where $\theta = (a_1, \cdots, a_n, b_1, \cdots, b_n)^T$. The order of the system n is chosen lower than that of the unknown true plant. Other choices of the nominal model are the FIR model and the Laguerre-Kautz model, which are all numerator models which have unknown parameters in only the numerator. The proposed method is applicable to (2) as well as this numerator type of nominal model.

Table 1 Schemes for identification of model error bound

Algorithms	Posteriori information (Purpose)	Prior information: Nominal model	Prior information: Model error	Prior information: Noise	Refs.
Set-membership identification	Hard bound in frequency domain	FIR model Laguerre model Kautz model	FIR model $D(z)$ known	Upper bound known	[5][6]
		Transfer function State-space model	FIR model $D(z)$ known	Upper bound *unknown*	This paper [11]
		Nonpsrsmetric model	Frequency domain shape	Upper bound known	[6]
	Hard bound in parameters of transfer function	Transfer function	$D(z)$ known $\|\Delta\|_\infty \le 1$	Upper bound known	[4]
Stochastic embedding	Soft bound in frequency domain	FIR model Laguerre model Kautz model	FIR model with stochastic parameters	Probability known	[10]
H_∞ identification	Hard bound in frequency domain	Nonpsrsmetric model		Upper bound known	[2][3]
Model Validation	Identification of nominal model and model error invalidating finite data	Transfer function	Transfer function $\|\Delta\|_\infty \le 1$ $D(z)$ known	Upper bound known	[7][8][9]

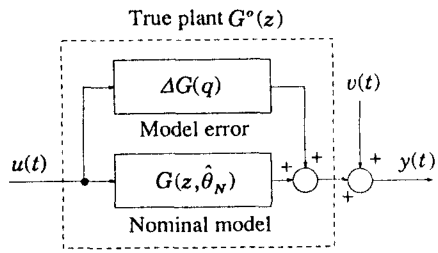

Fig.1 Identification of nominal model and its model error bound

The model error $\Delta G(z)$ is assumed to be described by

$$\Delta G(z) = D(z)\Delta(z) \tag{3}$$

where $D(z)$ is the known stable transfer function and $\Delta(z)$ is the *unknown* stable transfer function. In this paper, $D(z) = 1$, $\Delta^o(z)$ is given by

$$\Delta G(z) = \Delta^o(z) = \sum_{i=1}^{\infty} \delta_i z^{-i} \tag{4}$$

where $\{\delta_1, \delta_2, \cdots\}$ are unknown. Therefore, we take a nominal model error described by the FIR model as

$$\Delta(z) = \sum_{i=1}^{m} \delta_i z^{-i} \tag{5}$$

where $\delta = (\delta_1, \delta_2, \cdots, \delta_m)^T$.

The input-output data is obtained via the noise-corrupted true input-output relation as

$$y(t) = G^o(z)u(t) + v(t) \tag{6}$$

where $u(t)$ is a known input with the known bound c_u, and $v(t)$ is an unknown but bounded zero-mean noise where the bound c_v is *unknown*, that is,

$$|u(t)| \le c_u, \quad |v(t)| \le c_v \tag{7}$$

The assumption that c_v is *unknown* is one of the feature of this paper, while almost all other approaches assumed that c_v is *known*.

The identification purpose is to give the uncertainty bound $M_N(e^{j\omega}) > 0$ of the estimated nominal model such that

$$\left|G^o(e^{j\omega}) - G(e^{j\omega}, \hat{\theta}_N)\right| \le M_N(e^{j\omega}) \quad \forall \omega \tag{8}$$

Now the problem to be solved is to obtain $\hat{\theta}_N$ and the *minimum* bound of $M_N(e^{j\omega})$ using a *finite* number of input-output data, on the condition that the nominal model and the nominal model error are given by (2) and (5) respectively, n and m are given and c_u is known but c_v is *unknown*.

The above problem is rather difficult. In this paper we formulate the problem as follows: Given sufficiently large number N of input-output data, estimate the nominal model parameter $\hat{\theta}_N$ and an upper bound $M_N(e^{j\omega})$ in (8).

3. Set-Membership Identification Approach

3.1 Identification model of model error

The proposed scheme for identification of the model error bound is depicted in Fig.2 [11]. The first step is to estimate the parameters θ of the nominal reduced-order model by applying the prediction error method in which the squares sum of the error $e(t) = y(t) - \hat{y}(t)$ is minimized where $\hat{y}(t) = G(z,\hat{\theta})\ u(t)$. Filtering the input $u(t)$ and the output $y(t)$ with low-pass filters is effective in reduction of bias error of the nominal model identification in the frequency domain [12].

The second step is to take a set-membership approach to determine an upper bound of model error. From Fig.2 we can represent the input-output relation as

$$\begin{aligned}
e(t) &= G^o(z)u(t) + v(t) - G(z,\hat{\theta})u(t) \\
&= (G(z,\theta) + \Delta G(z))u(t) - G(z,\hat{\theta})u(t) + v(t) \\
&= \Delta(z)u(t) + [\Delta G(z) - \Delta(z)]u(t) \\
&\qquad + (G(z,\theta) - G(z,\hat{\theta}))u(t) + v(t) \qquad (9)
\end{aligned}$$

By defining the uncertainty term $w(t)$ as

$$\begin{aligned}
w(t) &= [\Delta G(z) - \Delta(z)]u(t) \\
&\quad + (G(z,\theta) - G(z,\hat{\theta}))u(t) + v(t) \qquad (10)
\end{aligned}$$

we have the description of the model uncertainty as

$$e(t) = \Delta(z)u(t) + w(t) \qquad (11)$$

$$|w(t)| \le c \qquad (12)$$

Now the problem is reduced to the set-membership (SM) identification for (11) and (12), though the bound c in (12) is unknown.

3.2 Ordinary set-membership identification

We will review the ordinary set membership identification scheme based on the optimal bounding ellipsoid algorithm, and then go on to extend it to the case in which the upper bound of the noise input is unknown.

(11) is rewritten as

$$e(t) = \varphi_\delta^T(t)\delta + w(t), \quad |w(t)| \le c \qquad (13)$$

where $\delta = (\delta_1, \delta_2, \cdots, \delta_m)^T$ and $\varphi_\delta(t) = (u(t-1), u(t-2), \cdots, u(t-m))^T$. Let S_t be a convex polytope defined by $S_t = \{\delta : (e(t) - \varphi_\delta^T(t)\delta)^2 \le c^2\}$. If the input-output data $\{u(t),\ e(t)\}$ are available, the true parameters δ should be included in the intersection of the subsets $\{S_t\}$. By using the optimal bounding ellipsoid algorithm [4], we can calculate the ellipsoid Ω_t which out-bounds the intersection of the

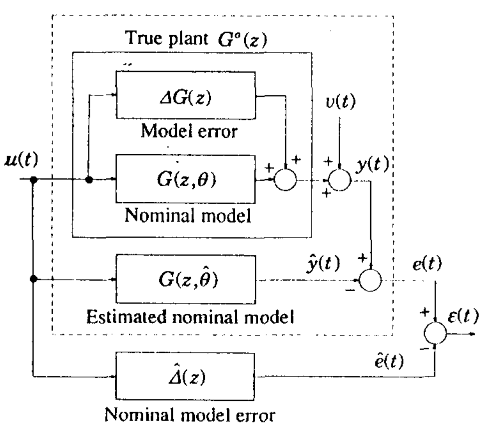

Fig.2 Schematic diagram of the proposed method of identification of the nominal model as well as the model error via the set-membership identification approach.

previous ellipsoid Ω_{t-1} and the polytope S_t iteratively as

$$\Omega_t = \{\delta : (\delta - \hat{\delta}(t))^T P_\delta^{-1}(t)(\delta - \hat{\delta}(t)) \le 1\} \qquad (14)$$

where the center value $\hat{\delta}(t)$ and the matrix $P_\delta(t)$ relating with the volume of the ellipsoid are updated as follows:

Step 1: Let the initial conditions be $\hat{\delta}(0) = 0,\ P_\delta(0) = \alpha I$ where α is a sufficiently positive large.

Step 2: Solve the next algebraic equation with respect to $\lambda(t)$ as

$$\alpha_1(t)\lambda^2(t) + \alpha_2(t)\lambda(t) + \alpha_3(t) = 0 \qquad (15)$$

where $\alpha_1(t) \equiv (m-1)c^2G^2(t)$, $\alpha_2(t) \equiv G(t)\{(2m-1)c^2 - G(t) + \varepsilon^2(t)\}$, $\alpha_3(t) \equiv m(c^2 - \varepsilon^2(t)) - G(t)$, $\varepsilon(t) = e(t) - \varphi_\delta^T(t)\hat{\delta}(t-1)$ and $G(t) \equiv \varphi_\delta^T(t)P_\delta(t-1)\varphi_\delta(t)$. If $\alpha_3(t) < 0$, use the most solution $\lambda(t)$ of (15). If $\alpha_3(t) \ge 0$, set $\lambda(t) = 0$.

Step 3: Update the parameter set by

$$\hat{\delta}(t) = \hat{\delta}(t-1) + \lambda(t)Q(t)\varphi_\delta(t)\varepsilon(t) \qquad (16)$$

$$Q(t) = P_\delta(t-1) - \lambda(t)\frac{P_\delta(t-1)\varphi_\delta(t)\varphi_\delta^T(t)P_\delta(t-1)}{1 + \lambda(t)G(t)} \qquad (17)$$

$$P_\delta^{-1}(t) = \frac{1}{\sigma^2(t)}Q^{-1}(t) \qquad (18)$$

$$\sigma^2(t) = 1 + \lambda(t)c^2 - \frac{\lambda(t)\varepsilon^2(t)}{1 + \lambda(t)G(t)} \qquad (19)$$

Now, if the upper bound c of the uncertainty term $w(t)$ is given, we can obtain the ellipsoidal region (14) recursively by using the above algorithm.

3.3 Scheme for estimating unknown bound c

It is noted in (9) that the uncertain term $w(t)$ consists of the unknown true unmodeled dynamics $\Delta G(z)$, unknown nominal model $G(z,\theta)$ and the random noise $v(t)$. On the assumptions of the stability of the identified system and the boundedness in (7), $w(t)$ becomes also bounded. Therefore, in this section, we will give an effective scheme for estimating an upper bound c satisfying that $|w(t)| \le c$.

Before showing a scheme for evaluating c, we clarify what factors the upper bound c is effected on. From (10) the uncertainty term $w(t)$ depends on the truncation error in the FIR model $\Delta(z)$ for describing $\Delta G(z)$, the identification error of the nominal model $G(z,\theta) - G(z,\hat{\theta})$, and the random noise $v(t)$. If the truncation error is outbounded a priori as

$$\sum_{k=m+1}^{\infty} |\delta_k| \le \kappa_m \qquad (20)$$

and it holds that $\hat{\theta}_N \to \theta\ (N \to \infty)$, then the upper bound c can be evaluated by

$$|w(t)| \le c = c_u\kappa_m + c_v \qquad (21)$$

Let $\hat{c}$ be an estimate of the upper bound c. We shall define two bounds, an upper and lower bound for admissible $\hat{c}^2$;

$$c_U^2(t) \equiv \hat{\varepsilon}^2(t) + \frac{\hat{G}(t)}{m} \qquad (22)$$

$$c_L^2(t) \equiv \hat{\varepsilon}^2(t) \qquad (23)$$

respectively. It is obvious that $c_L^2(t) < c_U^2(t)$ for all t.

The quantity · is calculated by replacing c with $\hat{c}$ in the SM identification algorithm above given. If the estimate

$\hat{c}^2$ is less than $c_U^2(t)$, the updating of the parameter set is carried out at time t. Therefore, $\hat{c}^2$ should be chosen so that $\hat{c}^2 \le c_U^2(t)$ for the updating. On the other hand, if c^2 is larger than $c_L^2(t)$, it holds that $\hat{\sigma}^2(t) \ge 1$. Thus $\hat{c}^2$ should be chosen such that

$$c_L^2(t) < \hat{c}^2 < c_U^2(t)$$

Now we take an estimate of the upper bound as

$$\hat{c}^2 = \sup_{t \ge M_1} \hat{\varepsilon}^2(t) \ge c_L^2(t) \tag{24}$$

where M_1 is so large that $\hat{\varepsilon}^2(t)$ is not influenced by the initial conditions. Thus we have;

Lemma: If $\hat{c}$ is chosen to satisfy (19), then $\hat{c}$ is an upper bound such that

$$\hat{c}^2 \ge w^2(t) \ \textit{for all}\ t \ge M_1 \tag{25}$$

Proof: From (24) we have

$$\begin{aligned}\hat{c}^2 &\ge \sup_{t \ge M_1} (e(t) - \varphi_\delta^T(t)\hat{\delta}(t-1))^2 \\ &= \sup_{t \ge M_1} [(\varphi_\delta^T(t)\tilde{\delta}(t-1))^2 + w^2(t) \\ &\qquad + 2(\varphi_\delta^T(t)\tilde{\delta}(t-1))w(t)]\end{aligned} \tag{26}$$

When $\varphi_\delta^T(t)\tilde{\delta}(t-1)$ and $w(t)$ have the same sign, the RHS of (26) has the maximum. Then we have

$$\hat{c}^2 \ge \sup_{t \ge M_1} w^2(t)$$

that leads to (25).

4 Quantification of frequency domain error bound

By using the estimate $\hat{c}^2$ given by (19), we can calculate the ellipsoid region Ω_t specifying the parameter set of δ via the set-membership identification as

$$(\delta - \hat{\delta}(t))^T P_\delta^{-1}(t)(\delta - \hat{\delta}(t)) \le 1 \tag{27}$$

Let the discrete Fourier transform (DFT) of $\hat{\delta}(t)$ and δ denoted by $\hat{\Delta}(e^{j\omega})$ and $\Delta(e^{j\omega})$ respectively, which are represented by $\Delta(e^{j\omega}) = \varphi^T(e^{j\omega})\delta$ and $\hat{\Delta}(e^{j\omega}) = \varphi^T(e^{j\omega})\hat{\delta}(t)$, where $\varphi^T(e^{j\omega}) = (e^{-j\omega}, \cdots, e^{-j\omega m})$. By using the method given in [5], we can compute numerically the corresponding ellipsoid in the complex plane as

$$\begin{pmatrix}\mathrm{Re}[\Delta(e^{j\omega}) - \hat{\Delta}(e^{j\omega})] \\ \mathrm{Im}[\Delta(e^{j\omega}) - \hat{\Delta}(e^{j\omega})]\end{pmatrix}^T \Phi^{-1}(e^{j\omega}) \begin{pmatrix}\mathrm{Re}[\Delta(e^{j\omega}) - \hat{\Delta}(e^{j\omega})] \\ \mathrm{Im}[\Delta(e^{j\omega}) - \hat{\Delta}(e^{j\omega})]\end{pmatrix} \le 1 \tag{28}$$

where

$$\Phi(e^{j\omega}) = \begin{pmatrix}\mathrm{Re}\,\varphi^T(e^{j\omega}) \\ \mathrm{Im}\,\varphi^T(e^{j\omega})\end{pmatrix} P_\delta(t) \left(\mathrm{Re}\,\varphi(e^{j\omega}) \quad \mathrm{Im}\,\varphi(e^{j\omega})\right)$$

Thus, the nominal model-error in frequency domain is given by $\hat{\Delta}(e^{j\omega})$ (by the center of the ellipsoid) and the upper bound of the model error is given by the maximum distance of the ellipsoid from the origin. Thus, we can give the bound $M_N(e^{j\omega})$ in (8) by the envelope of the calculated ellipsoids (28).

$$\begin{aligned}&\left|G^o(e^{j\omega}) - G(e^{j\omega}, \hat{\theta}_N)\right| \\ &\le \left|\hat{\Delta}(e^{j\omega})\right| + \text{Ellipsoid Bound} = M_N(e^{j\omega})\end{aligned} \tag{29}$$

5. Numerical Examples

5.1 3rd-order exponentially decaying system

We consider the same example as treated in [5], which is given by

$$G^o(s) = \frac{250}{(s+1)(s+5)(s+10)} \tag{30}$$

The input-output data is sampled with the sampling interval $T = 0.2$ and with the zero-order hold. The input signal $u(t)$ is a uniformly $[-1, 1]$ distributed white noise, and the disturbance input $v(t)$ is also uniformly $[-0.01, 0.01]$ distributed.

In the first example, we have chosen the first-order ARX model as the nominal model, and the nominal model error (FIR model with truncated order $m = 35$). Fig.3 indicates that the estimated upper bound $\hat{c}$ of c given by the proposed scheme is actually the bound of the uncertain input $w(t)$. Fig.4 shows the Nyquist curves of the true plant $G^o(e^{j\omega})$ and the estimated nominal model error $\hat{\Delta}(e^{j\omega})$ which is given by the center of the ellipsoids, in which the uncertainty bound is given by the size of the ellipsoids. The ARX model was identified by applying the prediction error method. Fig.5 illustrates the Bode plots of the true plant and the estimated nominal model (the first-order ARX model).

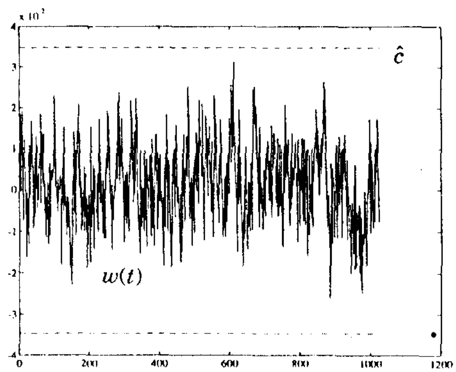

Fig.3 The estimated upper bound $\hat{c}$ of $w(t)$ obtained by the proposed scheme, and the actual value of $w(t)$.

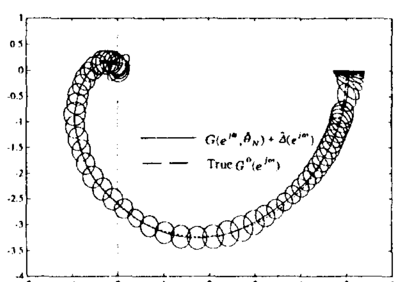

Fig.4 Nyquist plots of the true plant and the estimated of $G(e^{j\omega}, \hat{\theta}_N) + \hat{\Delta}(e^{j\omega})$ and ellipsoidal bounds.

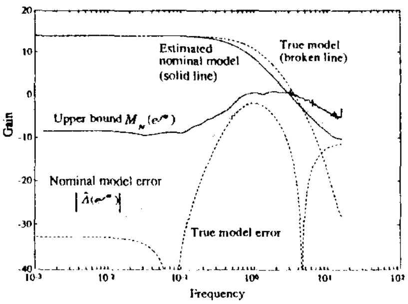

Fig.5 Bode plots of the estimated upper bound of model error $M_N(e^{j\omega})$.

As shown in Fig.5, there exists the model error in high frequency range which is also plotted by the broken line as the true model error. This model error is of course unknown but even if the disturbance bound c_v and the model error are unknown, the proposed scheme can give the nominal model error $\hat{\Delta}(e^{j\omega})$ and the upper bound of the model error $M_N(e^{j\omega})$. The proposed scheme does not utilize any prior information on the identified model.

If we adopt the second-order ARX mode as the nominal model, we can reduce the model error in high frequency range. In this case the size of the ellipsoids can also be reduced as shown in Fig.6. Fig.7 plots the Bode diagrams of the upper bound of the model error $M_N(e^{j\omega})$ which is rather improved.

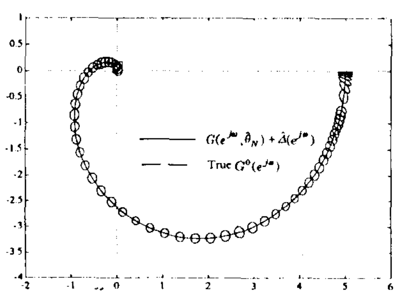

Fig.6 Nyquist plots when the nominal model is the second-order ARX model.

When we employed the FIR model as the nominal model, we sometimes encounter with the instability problem of identification. Normally we pass the input-output data to a lowpass filter to obtain the nominal model. In the case of the FIR nominal model, this filtering brings about the ill-conditioning of the input autocorrelation matrix. As we have repoted in [14], we should better introduce the regularization parameters to the input autocorrelation matrix. The optimal choice of these parameters can improve the mean squares error of the impulse response estimate. In this examle, the optimal regularization scheme was adopted to obtain the stable estimate of the nominal model in Fig.8.

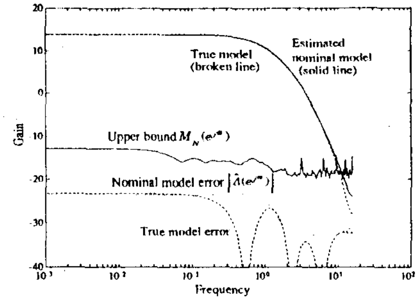

Fig.7 Bode plots of the estimated upper bound of model error $M_N(e^{j\omega})$ when the nominal model is the second-order ARX model.

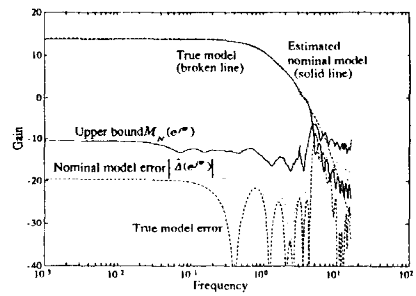

Fig. 8 Bode plots of the estimated upper bound of model error $M_N(e^{j\omega})$ when the nominal model is the finite impulse response model.

5.2 7th-order damped oscillating system

Let the true model be described by

$$G^o(s) = \frac{1}{s+1}\left(1+0.05(s+1)\prod_{i=1}^{3}\frac{\omega_i^2}{s^2+2\varsigma_i\omega_i s+\omega_i^2}\right) \quad (31)$$

We took the sampling interval as 0.5 to obtain the discrete-time model. The Bode plot of the magnitude is given by the broken line in Fig.9. The gain magnitude has three spectral peaks, while we take the nominal model described by a simple first-order ARX model

$$G(z,\theta) = \frac{bz^{-1}}{1+az^{-1}}$$

The parameters a and b in the nominal model are obtained by use of the ordinary prediction error method in which we filtered the input-output data with a low-pass filter. The gain magnitude of the estimated nominal model is also shown by the solid line in Fig.9. Fig.10 shows the Bode plots of the model error, however, the results in the low frequency range is not numerically accurate. We have reported why the accuracy cannot be attained and proposed the efficient algorithm based on the decimation. The estimated error bound is very much improved by the decimation scheme as shown in Fig.11.

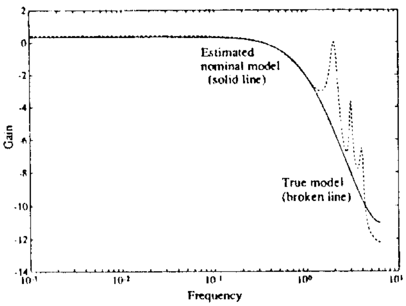

Fig.9 Bode plot of the true plant and the estimated nominal model (the first-order ARX model)

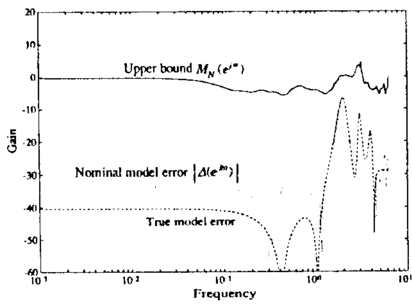

Fig.10 Bode plot of the upper bound of model error (Decimation is not employed)

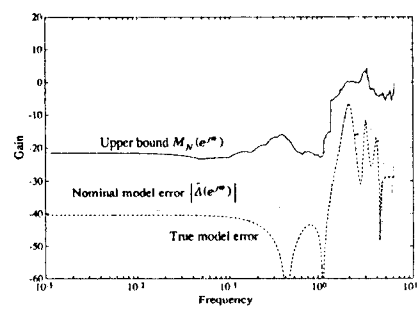

Fig.11 Bode plot of the upper bound of model error (Decimation is employed)

6. Conclusions

This paper have presented a new practical scheme for identifying the frequency-domain model error bound without using any prior information on the model uncertainty and the disturbance input but using only accessible input-output data. The proposed scheme includes the estimation of the upper bound of the uncertain input.

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