'93 KACC (1993.10.20~22)

New Guidance Law for Air-to-Air Missile

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Abstract

In this paper, a new guidance law for a short-range air-to-air missile with constant thrust is presented. It is essentially based on the concept of proportional navigation. First, the theoretical guidance law is derived. Then, we show the technique for practical implementation of the guidance law. By a computer simulation, it is shown that the new guidance law gives better performance than the conventional proportional navigation.

1 Nomenclature

F = desired acceleration command vector
A_M = missile axial acceleration vector
A_T = target axial acceleration vector
V_M = missile velocity vector
V_T = target velocity vector
R = relative distance vector
σ = line-of-sight(LOS) angle
$\dot{\sigma}$ = line-of-sight(LOS) angle rate vector
θ = missile flight-path angle
$\dot{\theta}$ = missile flight-path angle rate vector
ϕ_M = missile flight-path angle to LOS
ϕ_T = target flight-path angle to LOS
μ = $\phi_T - \phi_M$
N = navigation constant
N_e = effective navigation constant
T_{hr} = missile thrust vector
I_{sp} = specific impulse
m = missile mass
g = gravitational acceleration
t = fight time
t_{sp} = specific time
t_{go} = time-to-go
t_f = total flight time

2 Introduction

It is well known that the conventional proportional navigation (PN) is one of the most effective guidance laws when missile and target velocities are constant. [1-3] But in fact, a missile has an acceleration due to thrust during boost phase. In the previous paper,[4] one of the authors presented the guidance law for a missile with constant acceleration. In practice, however, the acceleration is not constant but is increasing by thrust with its own profile of each missile, and missile acceleration and velocity must change significantly as Figure 1. For instance, a short range air-to-air missile (AAM) has the axial acceleration that is increasing by thrust and the axial deceleration due to air drag. Also, a target may have constant axial acceleration. These kinds of axial accelerations seriously influence the performance of the missile guided by PN because of their varying velocities.

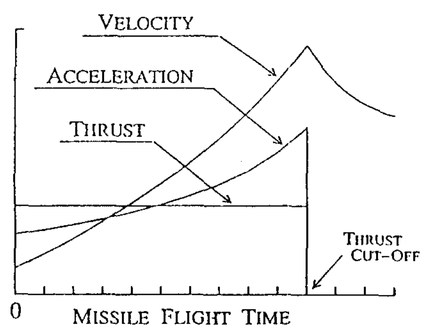

Figure 1 Thrust, Acceleration and Velocity of AAM

So we derive a new guidance law for a missile with constant thrust and constant acceleration against a target with constant acceleration. Thought the new guidance law gives the theoretical acceleration to guide a missile on a collision course, it is very difficult to implement it on most existing tactical missiles. Therefore, this paper shows the technique for practical implementation of the guidance law. Finally, the performance of the guidance law presented is compared with that of PN using simulation studies on the simple AAM model.

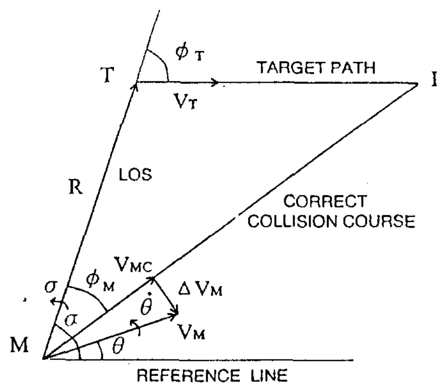

Figure 2 Intercept Geometry

3 Derivation of a New Guidance Law

Figure 2 shows the intercept geometry of a missile intercepting a target. M and T represent the actual positions of a missile and a target, respectively, at time t. From Figure 2 the line-of-sight (LOS) rate is given by the following vector equation:

$$\dot{\sigma} = \frac{\mathbf{R} \times (\mathbf{V}_T - \mathbf{V}_M)}{R^2} \tag{1}$$

Assuming that a missile is flying with thrust and constant acceleration A_M and a target is flying with constant acceleration A_T, let the triangle ITM be a collision triangle — that is, I indicates the intercept point. $\mathbf{V}_{MC}$ is the correct missile velocity vector to obtain a collision at I and $\Delta\mathbf{V}_M$ is the deviation of missile velocity vector from $\mathbf{V}_{MC}$. Then we have

$$\mathbf{V}_M = \mathbf{V}_{MC} + \Delta\mathbf{V}_M \tag{2}$$

Substituting Eq.(2) into Eq.(1), we obtain

$$\dot{\sigma} = \frac{\mathbf{R} \times (\mathbf{V}_T - \mathbf{V}_{MC})}{R^2} + \frac{\mathbf{R} \times (-\Delta\mathbf{V}_M)}{R^2} \tag{3}$$

The first term of the right side of Eq.(3) represents the correct LOS rate when the missile flies along the collision course. The second term is the deviation of the LOS rate from the correct one. If a missile is guided with a flight-path rate in proportion to the deviation of the LOS rate, assuming no missile dynamic lag, the missile flight-path rate becomes

$$\dot{\theta} = N \frac{\mathbf{R} \times (-\Delta\mathbf{V}_M)}{R^2} \tag{4}$$

where N is the navigation constant. Then the required lateral acceleration command for a missile to fly along the correct collision course is given by

$$\begin{aligned} \mathbf{F} &= \dot{\theta} \times \mathbf{V}_M \\ &= -N \frac{(\mathbf{V}_{MC} - \mathbf{V}_M) \times \mathbf{R}}{R^2} \times \mathbf{V}_M \end{aligned} \tag{5}$$

From Figure 2, $\mathbf{V}_{MC}$ can be written as

$$\mathbf{V}_{MC} = V_M \left(\frac{\sin\phi_M \mathbf{V}_T}{\sin\phi_T V_T} + \frac{\sin\mu \mathbf{R}}{\sin\phi_T R} \right) \tag{6}$$

Substituting Eq.(6) into Eq.(5), we obtain

$$\mathbf{F} = \frac{N}{R^2} \left\{ (\mathbf{V}_M - \frac{V_M \sin\phi_M}{V_T \sin\phi_T} \mathbf{V}_T) \times \mathbf{R} \right\} \times \mathbf{V}_M \tag{7}$$

Now, if a missile is flying on the correct collision course, that is,

$$\mathbf{V}_M = \mathbf{V}_{MC} \tag{8}$$

missile flight range is

$$V_M t_{go} + \frac{A_M}{2} t_{go}^2 + I_{sp} g \left\{ t_{go} - (t_{sp} - t_{go}) \log \left| \frac{t_{sp}}{t_{sp} - t_{go}} \right| \right\}$$

and target flight range is

$$V_T t_{go} + \frac{A_T}{2} t_{go}^2$$

As the collision triangle gets closed, we have

$$\begin{aligned} &\mathbf{V}_M t_{go} + \frac{\mathbf{A}_M}{2} t_{go}^2 + \frac{\mathbf{T}_{hr}}{T_{hr}} I_{sp} g \left\{ t_{go} - (t_{sp} - t_{go}) \log \left| \frac{t_{sp}}{t_{sp} - t_{go}} \right| \right\} \\ &= \mathbf{R} + \mathbf{V}_T t_{go} + \frac{\mathbf{A}_T}{2} t_{go}^2 \end{aligned} \tag{9}$$

where

$$t_{sp} = \frac{I_{sp} m}{T_{hr}} \tag{10}$$

From Figure 2, the component of Eq.(9) perpendicular to **R** is given by

$$\left[V_M t_{go} + \frac{A_M}{2}t_{go}^2 + I_{sp}g\left\{t_{go} - (t_{sp} - t_{go})log\left|\frac{t_{sp}}{t_{sp} - t_{go}}\right|\right\}\right]\sin\phi_M = \left(V_T t_{go} + \frac{A_T}{2}t_{go}^2\right)\sin\phi_T \quad (11)$$

Dividing Eq.(11) by $t_{go}\sin\phi_T$, we obtain

$$V_M\left[1 + \frac{A_M t_{go}}{2V_M} + \frac{I_{sp}g}{V_M}\left\{1 - \frac{t_{sp} - t_{go}}{t_{go}}\log\left|\frac{t_{sp}}{t_{sp} - t_{go}}\right|\right\}\right]\frac{\sin\phi_M}{\sin\phi_T} = V_T\left(1 + \frac{A_T t_{go}}{2V_T}\right) \quad (12)$$

Eq.(12) can be rewritten as

$$\frac{V_M\sin\phi_M}{V_T\sin\phi_T} = \frac{1 + \epsilon_T}{1 + \epsilon_{M1} + \epsilon_{M2}} \quad (13)$$

where

$$\epsilon_T = \frac{A_T t_{go}}{2V_T} \quad (14)$$

$$\epsilon_{M1} = \frac{A_M t_{go}}{2V_M} \quad (15)$$

$$\epsilon_{M2} = \frac{I_{sp}g}{V_M}\left\{1 - \frac{t_{sp} - t_{go}}{t_{go}}\log\left|\frac{t_{sp}}{t_{sp} - t_{go}}\right|\right\} \quad (16)$$

Substituting Eq.(13) into Eq.(7), we obtain

$$\mathbf{F} = \frac{N}{R^2}\left\{(\mathbf{V}_M - k\mathbf{V}_T) \times \mathbf{R}\right\}\times\mathbf{V}_M \quad (17a)$$

where

$$k = \frac{1 + \epsilon_T}{1 + \epsilon_{M1} + \epsilon_{M2}} \quad (17b)$$

This is the new guidance law for a missile with thrust to intercept a target. We need much information to compute **F**, because Eq.(14) has target speed, acceleration and t_{go}; Eq.(15) consists of missile speed, acceleration and t_{go}; Eq.(16) has information about missile thrust. If both a missile and a target have no thrust and no acceleration, that is, constant velocity, ϵ_T, ϵ_{M1} and ϵ_{M2} become zero and Eq.(17a) reduces to a conventional proportional navigation. Since the value of t_{go} is required in order to compute ϵ_T, ϵ_{M1} and ϵ_{M2}, we need to derive the equation for t_{go}. From Figure 2, we have

$$\left[V_M t_{go} + \frac{A_M}{2}t_{go}^2 + I_{sp}g\left\{t_{go} - (t_{sp} - t_{go})\log\left|\frac{t_{sp}}{t_{sp} - t_{go}}\right|\right\}\right]\cos\mu = R\cos\phi_T + V_T t_{go} + \frac{A_T}{2}t_{go}^2 \quad (18)$$

$$\left[V_M t_{go} + \frac{A_M}{2}t_{go}^2 + I_{sp}g\left\{t_{go} - (t_{sp} - t_{go})\log\left|\frac{t_{sp}}{t_{sp} - t_{go}}\right|\right\}\right]\sin\mu = R\sin\phi_T \quad (19)$$

Squaring both sides of Eqs.(18) and (19) and adding them, we obtain

$$\left[V_M t_{go} + \frac{A_M}{2}t_{go}^2 + I_{sp}g\left\{t_{go} - (t_{sp} - t_{go})\log\left|\frac{t_{sp}}{t_{sp} - t_{go}}\right|\right\}\right]^2 = R^2 + 2R(V_T t_{go} + \frac{A_T}{2}t_{go}^2)\cos\phi_T + (V_T t_{go} + \frac{A_T}{2}t_{go}^2)^2 \quad (20)$$

Assuming that V_M, V_T, A_M, A_T, R and ϕ_T are measured and I_{sp}, T_{hr} and m are known, t_{go} and t_{sp} can be computed, ϵ_T, ϵ_{M1} and ϵ_{M2} can be obtained from Eqs.(14), (15) and (16) and the required guidance acceleration commands are computed from Eq.(17). We call this guidance law *the true guidance method.*

4 Realization of a New Guidance Law

Since it is very difficult to realize Eq.(17) on an existing AAM, we consider the real mechanization of the guidance law.

4.1 On-Board Approximation

Eq.(17) gives the desired missile acceleration for collision course, when V_M, V_T, A_M, A_T and R are measured and t_{go} is computed from Eq.(20) in real time. But it is difficult to get such varying data on a flying missile, because its body is too small to be equipped with the kind of instruments — Radar, Seeker, Rate-Gyro, INS, High Speed Computer — to measure or calculate all of these variables with. So we estimate the variables using the following on-board approximation:

$$\widetilde{t_{go}} = t_f - t \tag{21a}$$

$$\widetilde{R} = R_0 \frac{t_f - t}{t_f} \tag{21b}$$

$$\widetilde{V_T} = V_{T0} + A_{T0} t \tag{21c}$$

$$\widetilde{V_M} = V_{M0} + A_{M0} t + I_{sp} g \log\left|\frac{\widetilde{t_{sp}}}{\widetilde{t_{sp}} - t}\right| \tag{21d}$$

$$\widetilde{t_{sp}} = t_{sp0} - t \tag{21e}$$

where V_{M0}, V_{T0}, A_{M0}, A_{T0}, R_0, t_f and t_{sp0} are the initial values of V_M, V_T, A_M, A_T, R, t_{go} and t_{sp}. These values are given from the parent aircraft to the missile before fired.

4.2 Simplified Design of a Guidance Law

Let us consider the real mechanization of Eq.(17). The term $\mathbf{V}_M - k\mathbf{V}_T$ in Eq.(17a) can be rewritten as

$$\mathbf{V}_M - k\mathbf{V}_T = \mathbf{V}_M - \frac{1+\epsilon_T}{1+\epsilon_{M1}+\epsilon_{M2}}\mathbf{V}_T$$

$$= \frac{1+\epsilon_T}{1+\epsilon_{M1}+\epsilon_{M2}}(\mathbf{V}_M - \mathbf{V}_T) + \frac{\epsilon_{M1}+\epsilon_{M2}-\epsilon_T}{1+\epsilon_{M1}+\epsilon_{M2}}\mathbf{V}_M \tag{22}$$

Substituting Eq.(22)into Eq.(17a), we obtain

$$\begin{aligned} \mathbf{F} &= (N\frac{1+\epsilon_T}{1+\epsilon_{M1}+\epsilon_{M2}})\frac{(\mathbf{V}_M - \mathbf{V}_T)\times\mathbf{R}}{R^2}\times\mathbf{V}_M \\ &+ (N\frac{\epsilon_{M1}+\epsilon_{M2}-\epsilon_T}{1+\epsilon_{M1}+\epsilon_{M2}})\frac{\mathbf{V}_M\times\mathbf{R}}{R^2}\times\mathbf{V}_M \end{aligned} \tag{23}$$

The first term of Eq.(23)represents proportional navigation (PN) with the navigation constant

$$N\frac{1+\epsilon_T}{1+\epsilon_{M1}+\epsilon_{M2}}$$

and the second term represents pure pursuit navigation (PP) with the navigation gain

$$N\frac{\epsilon_{M1}+\epsilon_{M2}-\epsilon_T}{1+\epsilon_{M1}+\epsilon_{M2}}$$

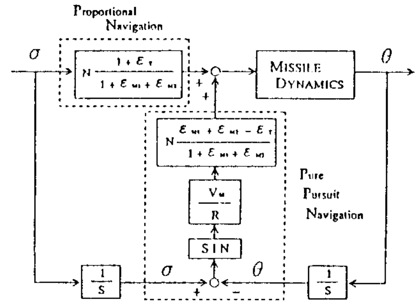

Figure 3 Block Diagram for the New Guidance Law

So we can set up the guidance system from Eq.(23) combining PN with PP as Figure 3. In the previous subsection, we showed the on board approximation. We can use those values and the effective navigation constant N_e, which is defined by

$$N_e = \frac{N V_M \cos\phi_M}{V_C} \tag{24}$$

Using these values, we modify the block diagram of Figure 3 in order to realize the guidance law simply on board. Figure 4 is the resulting block diagram. We call it *the simplified guidance method*. This guidance system can be adapted easily on many air-to-air missiles.

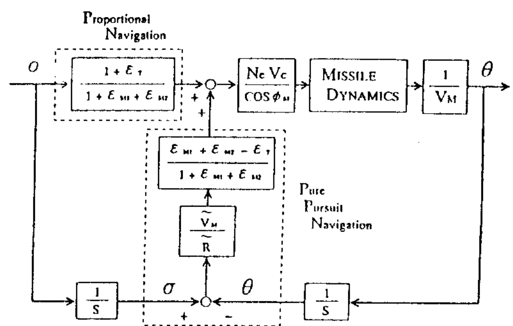

Figure 4 Block Diagram for the Simplified Guidance Method

5 Simulation on an Air-to-Air Missile

Let us apply the true guidance method and the simplified guidance method to the simple model of a short-range air-to-air missile (SRAAM) and compare the results with that achieved with the conventional proportional navigation (PN). Since most SRAAM have solid rocket motors, their thrust can be assumed nearly constant during boost phase and it has deceleration due to air drag. On the other hand, the target acceleration can be considered nearly constant because it is difficult to change it significantly during the short intercept time. Let us assume that both the missile and the target are particles which have mass; their trajectories are limited to two dimensions at $5{,}000m$ altitude; and the total dynamics of the guidance system, including the missile dynamics, a noise filter,etc., is given by a first-order lag with time constant $0.4sec$. And other conditions are the following. The missile deceleration due to air drag is constant at $2.5g$. The acceleration of the target is constant at $0.5g$, and the target initial velocity is $256m/s$ (Mach 0.8). The initial velocity of the missile is also $256m/s$ (Mach 0.8). It has thrust of $1352kg$, the specific impulse is $220sec$, and total mass before firing is $94kg$. The effective navigation constant for the guidance laws was set equal to 5.0.

Figure 5 displays the simulation result where the target is flying straight and the missile is launched along the collision course. The figure shows that the missile guided by the true guidance method flies straight with no acceleration commands. The trajectory achieved with PN is curved and requires large lateral acceleration commands. The trajectory achieved with the simplified guidance law is also slightly curved, but the deviation from the trajectory is much smaller than that with PN. Its required acceleration commands are very small.

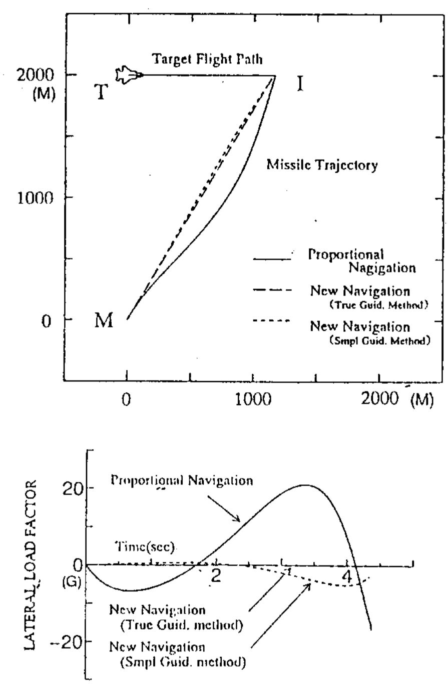

Figure 5 Missile Trajectories and Target Flight Path, and Lateral Load Factor Time History

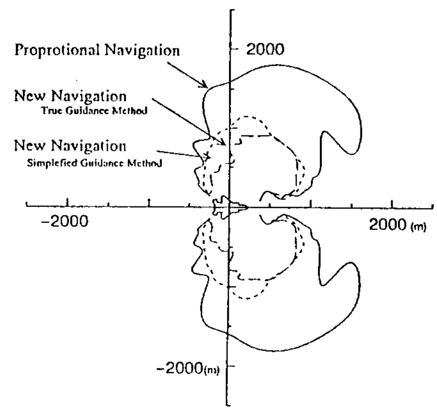

Figure 6 Inner Launch Envelope of AAM

Figure 6 shows the launch boundaries for the miss distance and the maximum lateral load factor specific as $3meter$ and $30\,g$. Here it is assumed that the target is flying straight and the missile is launched against the target without lead angle (zero off-bore sight angle); other conditions are the same as those described before. The inner launch envelope for SRAAM must simultaneously satisfy at least these boundaries. Figure 6 comperes the three inner envelopes achieved with different guidance laws.

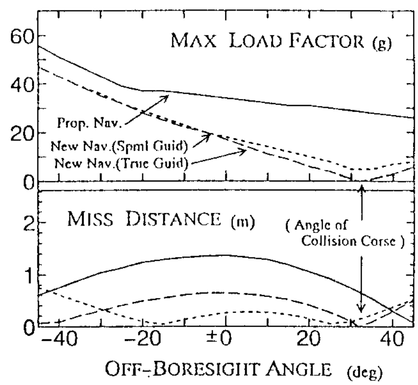

Figure 7 Miss Distance and Maximum Load Factor VS Off-Boresight Angle

Figure 7 shows the miss distance and the maximum lateral load factor varied by the off-bore sight angle of firing from the distance of 1,500*meter* in the 90*deg* aspect angle direction. The figure shows that PN required large lateral acceleration commands to guide missiles. Miss distance by PN is worse than that of the new navigation. The result shows that the simplified method has much better off-boresight ability than proportional navigation and the ability of the true guidance method is the best.

6 Conclusion

A new navigation law for an air-to-air missile (AAM) has been presented. The proportional navigation is one of the most effective guidance laws when missile and target velocities are constant. But in the case of a short-range air-to-air missile, velocities of a missile and a target vary significantly because a missile gets acceleration by its thrust in the boost phase and deceleration due to air drag and also a target has acceleration for its maneuvering. So we study the new guidance law taking missile thrust and other information into account in order to improve the performance of AAM guidance. From the simulation studies, the following results are obtained. The AAM's trajectory achieved with the proportional navigation is quite curved and large acceleration commands are required to intercept a target. But the missile guided by the new guidance law flies nearly straight and hit the target with little acceleration commands. Miss distance by the new guidance law is smaller than that by the proportional navigation law. The inner launch envelope has shown that the new guidance law provide an overall performance improvement over proportional navigation.

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