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Orbital Maneuvers by Using Feedback Linearization Method

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Abstract

A method for obtaining optimal orbital maneuvers of a space vehicle has been developed by combining feedback linearization method with the elegance of the Lambert's theorem. To obtain solutions to nonlinear orbital maneuver problems, The full nonlinear equations of motion for space vehicle in polar coordinate system are transformed exactly into a controllable linear set in Brunovsky canonical form by using feedback linearization by choosing position vector as fully observable output vector. These equations are used to pose a linear optimal tracking problem with a solutions to Lambert's problem and a linear analytical solution of continuous low thrust problem as reference trajectories.

1. Introduction

An orbital maneuver is defined as the change of the shape and/or orientation of an orbit by using natural and/or artificial perturbations.

The trajectory optimization problems for space vehicle have been investigated for more than three decades by many researchers. The methodology to solve the problems can be classified into two types, direct and indirect methods.

One of them is the direct parameter optimization method in which the continuous control is approximated by sequence of constant parameters. The solution of an optimization problem using this method requires mathematical programming, either linear or nonlinear. The other type consists of indirect methods based on the necessary conditions for optimality from the calculus of variations application of these results in Two-Point, Boundary-Value, Problems (TPBVP's), that also must generally be solved numerically except for especially simple cases. The resulting TPBVP is very hard to solve for some problems due to its sensitivity to initial guesses of costate variables.

For the case of indirect methods as applied to orbit transfer, extensive practical and theoretical work was done by Lawden [1]. Kelly [2] solved this type of TPBVP by using a gradient method. The shooting method was used by Melbourne [3]. McCue [4] used a quasi-linearization method to solve the more difficult nonlinear trajectory problem.

Since Krener[5] introduced and solved the linearization problem with only a local change of coordinates in the state space, many researchers have investigated the conditions under which nonlinear dynamical systems can be transformed locally into linear, controllable systems. The theoretical basis for feedback linearization method was developed by Isidori, et al. [6], and Hunt, et al. [7]. It has been applied successfully to a automatic flight control problem of a helicopter by Meyer, et al.[8]. It, also, has been applied to solve problems involving robot arm manipulation[9], and position control of a PM stepper motor [10].

In this work, a new approach based on advanced nonlinear control theory in which feedback linearization is used to develop nonlinear feedback control laws to cancel the nonlinear dynamics resulting from a linear equivalent model by means of state transformation and nonlinear feedback introduced and investigated.

A linear optimal tracking scheme by using the feedback linearization is introduced to obtain solutions of orbital maneuver problems. The reference trajectories were chosen an impulsive solutions to Lambert's Problem for high-thrust orbital maneuvers and linear analytical closed form solutions of linear system for low-thrust orbital maneuvers.

2. Equations of Motion

Optimal orbital maneuver problems of space vehicles have been usually studied by assuming for the dynamical system a two point masses with only the perturbation of two-body motion due to finite-thrust acceleration, which was not constrained in any manner.

The dynamic model chosen in this work is a space vehicle that moves around the earth in the gravitational fields and a variable thrust acceleration. Both the earth and a space vehicle are assumed as point masses.

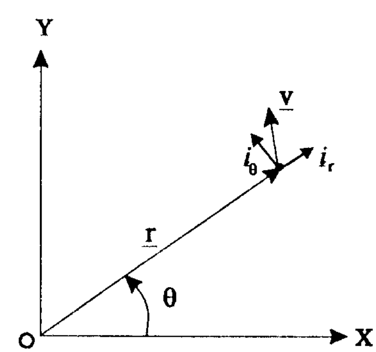

Fig.1 The Motion of Space Vehicle in Polar Coordinate System

As shown in Fig. 1, the two-dimensional inertial coordinate system, O_{xy}, with origin at the center of the earth, is used as basic reference coordinate system. The well-known equations of motion governing a space vehicle thrusting in a gravitational field in a rotating local polar coordinate system are

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2} + u_r \tag{1}$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = u_\theta \tag{2}$$

Where, r is the radial distance from the center of the earth to space vehicle, θ is the polar angle from the reference point, u_r and u_θ are thrust acceleration components in each unit vector direction respectively. we can write the equations of motion as a vector form,

$$\dot{\underline{x}} = \underline{f}(\underline{x}) + \underline{\underline{g}}(\underline{x})\underline{u}. \tag{3}$$

where $\underline{x} = [x_1, x_2, x_3, x_4]^T = [r, \theta, \dot{r}, \dot{\theta}]^T$ and $\underline{u} = [u_1, u_2]^T = [u_r, u_\theta]^T$.

Singularity-Free Equations of Motion

Using Lagrange's planetary equations[11] may give some benefits for solving orbital maneuvers with the system assumed as two-body point mass. One may not need to integrate equations of motion for pure two-body motion but just need to propagate true anomaly. By the way, this form of equations of motion cannot be used maneuvers of equatorial or circular orbit due to singularities among them. Therefore, singularity-free equations of motion are introduced as,

$$\begin{aligned}
\frac{da}{dt} &= \frac{2a^2}{h}\left[(P_2 \sin L - P_1 \cos L)U_r + WU_\theta\right] \\
\frac{dP_1}{dt} &= \frac{h}{\mu}\left[-\cos L U_r + \frac{P_1 + \sin L(1+W)}{W}U_\theta\right] \\
\frac{dP_2}{dt} &= \frac{h}{\mu}\left[\sin L U_r + \frac{P_2 + \cos L(1+W)}{W}U_\theta\right] \\
\frac{dL}{dt} &= \frac{\mu^2}{h^3}W^2
\end{aligned} \tag{4}$$

where $P_1 = e\sin\omega$, $P_2 = e\cos\omega$, $L = \omega + f$, $h = \sqrt{\mu a(1-e^2)}$, and $W = 1 + P_1 \sin L + P_2 \cos L$.

These full nonlinear equations of motion may be linearized with respect to intermediary circular orbit. The linear analytical solution to the trajectory optimization problem for the space vehicle with the singularity-free linearized system was obtained by Lee[12].

3. Feedback Linearization

Feedback linearization is the technique of transforming the nonlinear system into an equivalent controllable linear form by using state and feedback transformations. The actual system and controller remain nonlinear. References [13] [14] provide detail theoretical background about it.

Before moving into the story of feedback linearization, let's take brief review of mathematical preliminary. For a single-input, single-output (SISO) nonlinear system of the form,

$$\dot{\underline{x}} = \underline{f}(\underline{x}) + \underline{g}(\underline{x})u \tag{5}$$

$$y = h(\underline{x}) \tag{6}$$

where a vector functions, $\underline{f}: R^n \rightarrow R^n$, $\underline{g}: R^n \rightarrow R^n$, are C^∞ vector fields and $h: R^n \rightarrow R^n$ is a C^∞ scalar function, the Lie derivatives of $h(\underline{x})$ along the direction of vector $\underline{f}(\underline{x})$ is a scalar function defined as

$$L_{\underline{f}} h = \nabla h \cdot \underline{f} \tag{7}$$

The Lie derivative may be taken recursively,

$$L_{\underline{f}}^0 h = h$$

$$L_{\underline{f}}^i h = L_{\underline{f}}(L_{\underline{f}}^{i-1} h) = \nabla(L_{\underline{f}}^{i-1} h) \cdot \underline{f} \quad \text{for i = 1, 2, ...} \tag{8}$$

The Lie bracket of $\underline{f}$ and $\underline{g}$ is a third vector field defined as

$$[\underline{f}, \underline{g}] = \nabla \underline{g} \cdot \underline{f} - \nabla \underline{f} \cdot \underline{g} \tag{9}$$

The Lie bracket $[\underline{f}, \underline{g}]$ is usually written as $ad_{\underline{f}}\underline{g}$. The Lie bracket may be defined recursively by

$$ad_{\underline{f}}^0 \underline{g} = \underline{g}$$

$$ad_{\underline{f}}^i \underline{g} = [\underline{f}, ad_{\underline{f}}^{i-1} \underline{g}] \quad \text{for i = 1, 2, ...} \tag{10}$$

Relative Degree

Let's consider a single-input single-output nonlinear system in Eqs. (5) and (6) with dimension n. There exists an output function $h(\underline{x})$ with relative degree κ at a point $\underline{x}_0$ if and only if the following conditions are satisfied:

a) matrix $[\underline{g}(\underline{x}_0), ad_{\underline{f}}\underline{g}(\underline{x}_0), \ldots, ad_{\underline{f}}^{\kappa-1}\underline{g}(\underline{x}_0)]$ has rank κ, (11)

b) distribution $\Delta = \text{span}\{\underline{g}(\underline{x}), ad_{\underline{f}}\underline{g}(\underline{x}), \ldots, ad_{\underline{f}}^{\kappa-2}\underline{g}(\underline{x})\}$ is involutive near $\underline{x} = \underline{x}_0$. (12)

The relative degree, κ, is characterized as the number of times one has to differentiate the output function in order to have the input u appear explicitly. If there exists an output function $h(\underline{x})$ with relative degree κ and the dimension n is equal to relative degree, then the system can be fully feedback linearizable.

The idea of relative degree can be extended to multi-input, multi-output nonlinear systems as a form,

$$\dot{\underline{x}} = \underline{f}(\underline{x}) + \underline{\underline{g}}(\underline{x})\underline{u} \tag{13}$$

$$\underline{y} = \underline{h}(\underline{x}) \tag{14}$$

where $\dot{\underline{x}}$, $\underline{f}(\underline{x})$ and $\underline{\underline{g}}(\underline{x})$ are nx1 vectors and $\underline{y}$, $\underline{h}(\underline{x})$, and $\underline{u}$ are mx1 vectors. The MIMO nonlinear system of the form (13) and (14) has a set of relative degrees $\{\kappa_1, \ldots, \kappa_m\}$ at a point $\underline{x}_0$ if

$$L_{g_i} L^{k_i}{}_f h_i(\underline{x}) = 0 \tag{15}$$

for all $1 \le j \le m$, for all $1 \le i \le m$, for all $k_i < \kappa_i - 1$, and for all $\underline{x}$ in a neighborhood of $\underline{x}_0$ and the m x m matrix.

$$\underline{\underline{E}}(\underline{x}) = \begin{bmatrix} L_{g_1} L_f^{\kappa_1 - 1} h_1(\underline{x}) & \cdots & L_{g_m} L_f^{\kappa_1 - 1} h_1(\underline{x}) \\ L_{g_1} L_f^{\kappa_2 - 1} h_2(\underline{x}) & \cdots & L_{g_m} L_f^{\kappa_2 - 1} h_2(\underline{x}) \\ \cdots & \cdots & \cdots \\ L_{g_1} L_f^{\kappa_m - 1} h_m(\underline{x}) & \cdots & L_{g_m} L_f^{\kappa_m - 1} h_m(\underline{x}) \end{bmatrix} \quad (16)$$

is nonsingular at $\underline{x}_0$.

The matrices (11) and (16) are equivalent to controllability matrix, $\{\underline{\underline{B}}, \underline{\underline{A}}\,\underline{\underline{B}}, \ldots, \underline{\underline{A}}^{n-1}\underline{\underline{B}}\}$ in linear control theory.

The feedback linearization procedure can be summarized as,

$$\begin{matrix} \dot{\underline{x}} = \underline{f}(\underline{x}) + \underline{g}(\underline{x})\underline{u} \\ \underline{y} = \underline{h}(\underline{x}) \end{matrix} \quad \xleftrightarrow[\underline{z} \,=\, \underline{\Phi}(\underline{x})]{\underline{u} = \underline{\alpha}(\underline{x}) + \underline{\underline{\beta}}(\underline{x})\underline{\upsilon}} \quad \dot{\underline{z}} = \underline{\underline{A}}\underline{z} + \underline{\underline{B}}\underline{\upsilon} \quad (17)$$

The process of feedback linearization is illustrated in (17). A nonlinear equations should have the form of Eq.(13) to be transformed into the controllable linear one by using feedback linearization. If we obtain a proper nonlinear change of state variables defined by $\underline{\Phi}(\underline{x})$, a nonlinear feedback function $\underline{\alpha}(\underline{x})$, and a linear invertible change of coordinates in the input $\underline{\underline{\beta}}(\underline{x})$ such that the input/output behavior of the system is linear and controllable, then Eq.(13) can be transformed into the form,

$$\dot{\underline{z}} = \underline{\underline{A}}\underline{z} + \underline{\underline{B}}\underline{\upsilon} \quad (18)$$

where $\underline{z}$ and $\underline{\upsilon}$ are new state and control variables respectively, and $\underline{\underline{A}}$ and $\underline{\underline{B}}$ are this constant system and control matrices, respectively. This type of transformation is <u>exact</u>, and the control obtained from the system is directly applicable to the nonlinear system without any internal modifications.

For the mathematical model in hand, the output relation, $\underline{y} = \underline{h}(\underline{x})$ is selected as position vector with assumption that position variables are fully observable. The relative degrees of the system is {2,2}, i.e. sum of relative degrees, κ=4 which is equal to system dimension. Therefore, the model is fully feedback linearizable.

A matrix form of the equations of motion which is the canonical form for feedback linearization is

$$\dot{\underline{z}}_1 = \underline{z}_2 \quad (19)$$

$$\dot{\underline{z}}_2 = \underline{f}_2(\underline{z}_1, \underline{z}_2) + \underline{\underline{g}}_2(\underline{z}_1, \underline{z}_2)\underline{u} \quad (20)$$

where

$$\underline{z}_1 = [r, \theta]^T; \; \underline{z}_2 = [\dot{r}, \dot{\theta}]^T. \quad (21)$$

If we let new control variable,

$$\underline{\upsilon} = \underline{f}_2(\underline{z}_1, \underline{z}_2) + \underline{\underline{g}}_2(\underline{z}_1, \underline{z}_2)\underline{u} \quad (22)$$

then, Equations (13a) and (13b) reduce to

$$\dot{\underline{z}}_1 = \underline{z}_2 \quad (23)$$

$$\dot{\underline{z}}_2 = \underline{\upsilon}. \quad (24)$$

The input transformation is

$$\underline{u} = -\underline{\underline{g}}_2(\underline{z}_1, \underline{z}_2)^{-1}\underline{f}_2(\underline{z}_1, \underline{z}_2) + \underline{\underline{g}}_2(\underline{z}_1, \underline{z}_2)^{-1}\underline{\upsilon} \quad (25)$$

and state transformation is

$$\underline{z} = \underline{x}$$

where $\underline{\underline{g}}_2(\underline{z}_1, \underline{z}_2)$ should not be singular.

We can write Eqs.(23) and (24) explicitly as so-called Brunovsky canonical form,

$$\dot{\underline{z}} = \underline{\underline{A}}\underline{z} + \underline{\underline{B}}\underline{\upsilon} = \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{I}} \\ \underline{\underline{0}} & \underline{\underline{0}} \end{bmatrix}\begin{bmatrix} \underline{z}_1 \\ \underline{z}_2 \end{bmatrix} + \begin{bmatrix} \underline{\underline{0}} \\ \underline{\underline{I}} \end{bmatrix}\begin{bmatrix} \upsilon_1 \\ \upsilon_2 \end{bmatrix} \quad (26)$$

where $\underline{z} = [\underline{z}_1^T, \underline{z}_2^T]^T$.

4. Lambert's Theorem

For the two-body, two-point, boundary-value problem, Lambert gave a remarkable theorem that the orbital transfer time depends only on semi-major axis, the sum of the distances of initial and final points of the arc from center of force, and the length of the chord joining these points. i.e.,

$$\sqrt{\mu}(t_2 - t_1) = F(a, r_1 + r_2, c) \quad (27)$$

where $t_2 - t_1$ is the time required to describe the arc from p_1 to p_2 The geometrical configuration of the theorem is shown in Fig. 2.

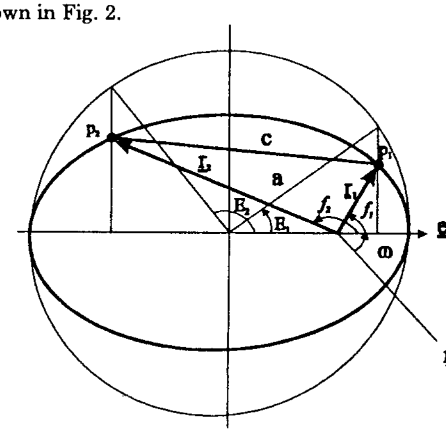

Fig. 2 The Geometry of the Lambert's Problem

Lambert's theorem for elliptic orbits was proven analytically by Lagrange. The Lambert's theorem can be expressed as

$$\sqrt{\mu}(t_2 - t_1) = a^{\frac{3}{2}}[(\alpha - \sin\alpha) - (\beta - \sin\beta)] \quad (28)$$

where $t_2 - t_1$ is the time to traverse the arc from p_1 to p_2, μ is the gravitational parameter, a is the semi-major axis of the transfer orbit, and α and β are variables defined as

$$\alpha = 2\sin^{-1}\left(\sqrt{\frac{s}{2a}}\right) \quad \text{and} \quad \beta = 2\sin^{-1}\left(\sqrt{\frac{s-c}{2a}}\right) \quad (29)$$

where $s = \frac{1}{2}(r_1 + r_2 + c)$ is the semiperimeter of the triangle Op_1p_2.

For a given transfer time Δt, orbital elements may be obtained by Lambert's theorem. A semi-major axis can be obtained by solving,

$$f(a) = \sqrt{\mu}\Delta t - a^{\frac{3}{2}}[(\alpha - \sin\alpha) - (\beta - \sin\beta)]. \quad (30)$$

using a bi-section or Newton-Raphson iterative method. For the Newton-Raphson method,
The bisection method in generally more stable than the Newton-Raphson method. Once the semi-major axis is found the eccentricity of the transfer orbit may calculated from

$$e^2 = 1 - 4\,\frac{(s-r_1)(s-r_2)\sin^2(\frac{\alpha+\beta}{2})}{c^2}, \quad (31)$$

Next, the true anomalies at the ends of the chord may be obtained by using the familiar orbit equation,

$$r = \frac{a(1-e^2)}{(1+e\cos\upsilon)}. \quad (32)$$

5 Linear Optimal Control

A linear optimal control problem [15] may be formulated by using standard procedures from the calculus of variations.

Linear Tracking Problem

A linear regulator problem is generalized into a linear tracking one in which the desired value of state vector is not the origin. That is, it is desired to find the control in such a way as to cause the output $\underline{z}(t)$, to track or follow a desired output state $\underline{r}(t)$. At the same time, it ought to minimize the scalar functional termed the performance index,

$$J = \frac{1}{2}[\underline{z}(t_f) - \underline{z}_{df}]^T \underline{\underline{S}}_f [\underline{z}(t_f) - \underline{z}_{df}] + \frac{1}{2}\int_{t_0}^{t_f}\{[\underline{z}-\underline{r}]^T \underline{\underline{Q}}[\underline{z}-\underline{r}] + \underline{\upsilon}^T \underline{\underline{R}}\underline{\upsilon}\}dt \quad (33)$$

where $\underline{z}_{df}$ is the desired final state and $\underline{\underline{S}}_f$ is a positive semi-definite matrix of weights on final state error. Also, $\underline{\underline{Q}}$ and $\underline{\underline{R}}$, respectively, are a positive semi-definite weighting matrix on the state and a positive definite weighting matrix on the control, during the maneuvers, subject to the dynamic constraint equations,

$$\dot{\underline{z}}(t) = \underline{\underline{A}}\underline{z}(t) + \underline{\underline{B}}\underline{\upsilon}(t). \quad (34)$$

The Hamiltonian for this problem is

$$H = \frac{1}{2}\{[\underline{z}-\underline{r}]^T \underline{\underline{Q}}[\underline{z}-\underline{r}] + \underline{\upsilon}^T \underline{\underline{R}}\underline{\upsilon}\} + \underline{\lambda}^T(\underline{\underline{A}}\underline{z} + \underline{\underline{B}}\underline{\upsilon}) \quad (35)$$

and necessary conditions for the vanishing the first variation of the performance index are

$$\dot{\underline{\lambda}} = -\left(\frac{\partial H}{\partial \underline{z}}\right)^T = -\underline{\underline{Q}}\,\underline{z} - \underline{\underline{A}}^T\underline{\lambda} + \underline{\underline{Q}}\,\underline{r} \quad (36)$$

$$\frac{\partial H}{\partial \underline{\upsilon}} = \underline{\underline{R}}\,\underline{\upsilon} + \underline{\underline{B}}^T\underline{\lambda} = \underline{0} \quad \Rightarrow \quad \underline{\upsilon} = -\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\lambda} \quad (37)$$

The Euler-Lagrange Eqs. (34) and (36) along with initial condition for state variables and final conditions for costate variables,

$$\underline{\lambda}(t_f) = \underline{\underline{S}}_f\underline{z}(t_f) - \underline{\underline{S}}_f\underline{z}_{df} \quad (38)$$

from the transversality condition define a standard linear TPBVP. This type of problem can be solved by using a "sweep method." The solution for $\underline{\lambda}(t)$, is of the form,

$$\underline{\lambda}(t) = \underline{\underline{E}}(t)\underline{x}(t) + \underline{e}(t), \quad (39)$$

where $\underline{\underline{E}}(t)$ and $\underline{e}(t)$ are $n\times n$ and $n\times 1$ matrices, respectively, which satisfy the differential equations,

$$\dot{\underline{\underline{E}}} = -\underline{\underline{E}}\underline{\underline{A}} - \underline{\underline{A}}^T\underline{\underline{E}} - \underline{\underline{Q}} + \underline{\underline{E}}\underline{\underline{B}}\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{E}}, \quad (40)$$

and

$$\dot{\underline{e}} = -\underline{\underline{A}}^T\underline{e} + \underline{\underline{E}}\underline{\underline{B}}\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{e} + \underline{\underline{Q}}\underline{r}, \quad (41)$$

subject to the boundary conditions,

$$\underline{\underline{E}}(t_f) = \underline{\underline{S}}_f, \quad (42)$$

and

$$\underline{e}(t_f) = -\underline{\underline{S}}_f\underline{z}_{df}. \quad (43)$$

Eqs. (40) and (41) may be integrated backward from the final time to find $\underline{\lambda}(t_0)$. Then, Eqs. (34) and (36) are integrated forward to obtain the solution.

5. Results

The example chosen in this work is the orbital rendezvous problem between neighboring two low-earth circular orbits in the same orbital plane. The problem is to find the trajectories and control law that minimize ΔV required for the rendezvous. Its boundary conditions in local polar coordinate system are shown in Table. 1.

The rendezvous time is the same with coasting time required by an intermediary orbit to reach from the initial to final polar angle. The intermediary orbit was taken as the average of the initial and final circular orbit.

Table1. Boundary Conditions

	Initial Conditions	Final Conditions
Radius (km)	6778.0	7378.0
Polar Angle (rad)	0	2.949606
Radial Velocity (km/sec)	0	0
Angular Rate (rad/sec)	1.131400E-3	9.962324E-4
Time (seconds)	0	2782.026

This example gives the solution orbit to Lambert's problem which is close to fundamental orbit defined as an orbit connecting two spatial points with minimum eccentricity.

The solution orbit to Lambert's problem for the example in terms of orbital elements is given in Table 2.

Table2. Lambert Orbit

	Orbital Elements
Semi-Major Axis (km)	7078.033
Eccentricity	0.042581
Inclination (deg)	0.0
Longitude of the Ascending Node(deg)	Not Defined
Longitude of Argument of Perigee(deg)	-5.6684
True Anomaly at t = 0 (deg)	5.6684
True Anomaly at t = 2782.026 (deg)	174.6684

Lambert Orbit Follower

A numerical solution to the nonlinear TPBVP has been obtained using standard shooting method for the example as basic reference to compare the solutions from other methods. The time histories of thrust accelerations and state variables for nonlinear and Lambert orbit follower are shown in Figs. 3 through 7. Also, Lambert orbit is obtained using Lambert's theorem and plotted to show how closely the solution from new method follows it.

An optimal linear tracking by using solution to a Lambert's impulsive thrust TPBVP is devised as a reference orbit so that the feedback linear system tracks the solution trajectory. That is, "Lambert orbit follower" is introduced. A time-varying Q-matrix that weights the state variables in performance index was used to obtain a control law as thrust-coast-thrust shape and avoid steep peaks at the both ends. The weighting matrices for optimal linear tracking problem are selected as

$$\underline{\underline{Q}}(t) = \varepsilon(t) \times Diag\left[9 \times 10^{-11}, 7.4 \times 10^{-16}, 3.53 \times 10^{-6}, 8.0 \times 10^{-5}\right]$$

$$\underline{\underline{S}}_f = Diag\left[1 \times 10^{-5}, 3 \times 10^{-7}, 0.7, 0.7\right], \quad \text{and} \quad \underline{\underline{R}} = Diag[1,1]$$

where a time-varying scalar function, $\varepsilon(t)$ is shown in Fig. 8.

Total cost to each method is shown in Table 3 in terms of ΔV. It shows that the Lambert orbit follower method requires less cost than continuous thrust solution.

Table 3. List of total costs, ΔV for Each Method.

Methods	ΔV (km/sec)
Lambert's Problem (Impulsive)	0.321066
Lambert Orbit Follower	0.418849
Nonlinear Optimal Solution	0.489265
Linear Optimal Solution	0.493580
Linear Trajectory Follower	0.585527

ΔV for Lambert's problem is that corresponding to two-impulsive solution and obtained geometrically. As shown in Fig. 4, total thrust acceleration obtained from Lambert orbit follower for midcourse remains almost zero. In other word, it behaves like thrust-coast-thrust solution.

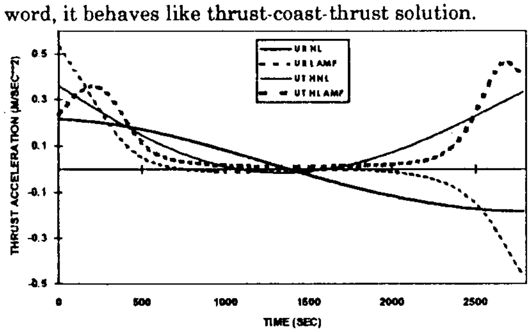

Fig. 3 Time histories of thrust accelerations.

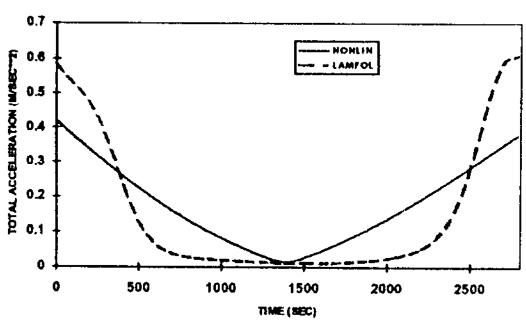

Fig. 4. Time histories of total thrust accelerations.

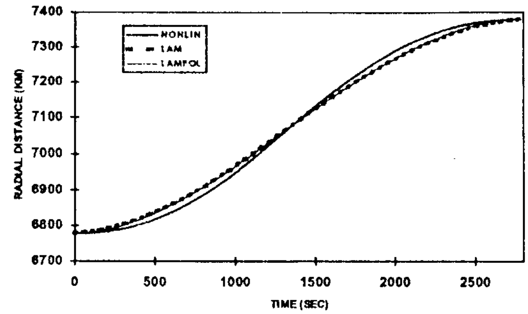

Fig. 5 Time histories of radial positions.

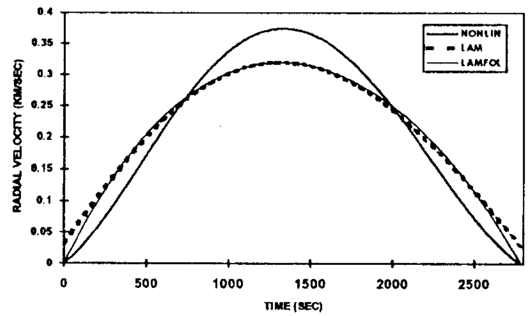

Fig. 6 Time histories of radial velocities.

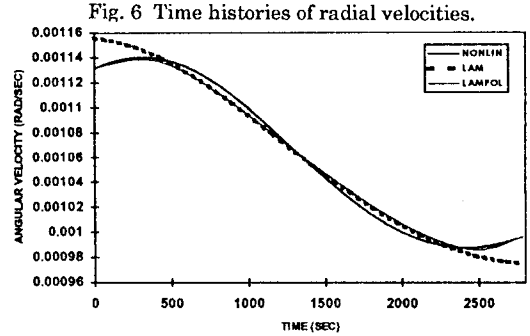

Fig. 7 Time histories of angular rates.

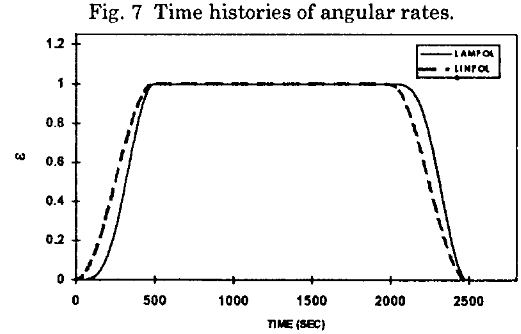

Fig. 8 Time histories of scalar function, $\varepsilon(t)$.

Linear Trajectory follower

The full nonlinear singularity-free Lagrange Planetary equation was linearized about the intermediary orbit and obtained the linear analytical solution with the linearized system equations. This solution is a good approximation of nonlinear solution for the orbital maneuvers between neighboring orbits with reasonable maneuver time. Therefore, a nonlinear solution that tracks this linear solution as in a guidance problem was obtained the system. Weighting matrices are selected as

$$\underline{\underline{Q}}(t) = \varepsilon(t) \times Diag[9.13\times10^{-12}, 2\times10^{-15}, 3.2\times10^{-6}, 1.333\times10^{-5}]$$

$\underline{\underline{S}}_f = Diag[1\times10^{-5}, 3\times10^{-7}, 0.73, 0.73]$, and $\underline{\underline{R}} = Diag[1,1]$

where a time-varying scalar function, $\varepsilon(t)$ is shown in Fig. 8. Results from the three different methods are presented in Figs. 9 through 12, and linear trajectory follower was reasonably good as far as linear and nonlinear solution in state variables. Even though it requires little bigger ΔV, It requires reasonable amount

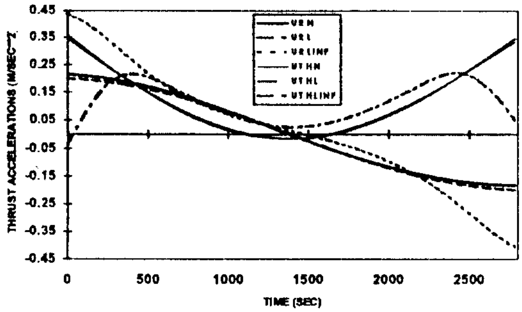

Fig. 9 Time histories of thrust accelerations.

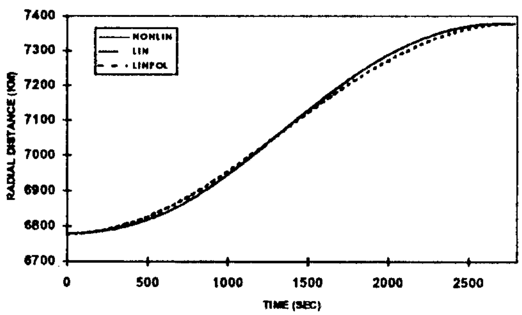

Fig. 10 Time histories of radial positions.

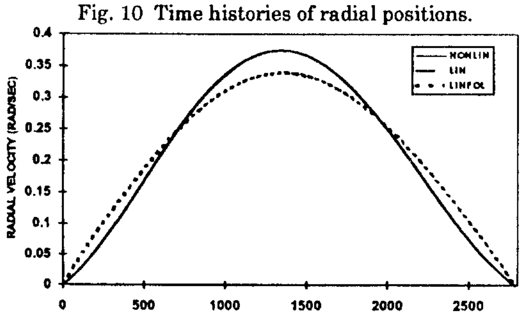

Fig. 11 Time histories of radial velocities.

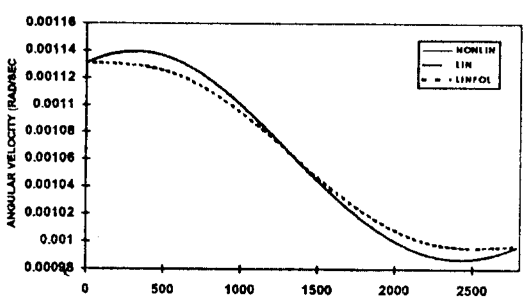

Fig. 12 Time histories of angular velocities .

of ΔV as it is shown in Table 3. Thrust acceleration of the linear trajectory follower, however, shows some deviations from both linear and nonlinear solutions at the both ends.

6. Conclusion

The feedback linearization method has been combined with solutions to Lambert's Problem and with linear analytical solution to obtain methods for solving low-thrust, low-earth, orbital maneuvers.

As expected, the Lambert orbit follower method produced a solution that is competitive with the two-impulse solution and the linear solution follower method produced a trajectory that requires reasonable amount of total velocity magnitude change. Although not exactly a thrust-coast-thrust trajectory, the Lambert orbit follower solution is very similar and corresponds to a total velocity magnitude change less than that for the continuous thrust optimal solution. Solutions using both methods are relatively easy to obtain.

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