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Mobile Robot Indoor Map Making Using Fuzzy Numbers and Graph Theory

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Abstract: In this paper, we present a methodology to model an indoor environment of a mobile robot using fuzzy numbers and to make a global map of the robot environment using graph theory. we describe any geometric primitive of robot environment as a parameter vector in parameter space and represent the ill-known values of the parameterized geometric primitive by means of fuzzy numbers restricted to appropriate membership functions. Also we describe the spatial relations between geometric primitives using graph theory for local maps. For making the global map of the mobile robot environment, the correspondence problem between local maps is solved using a fuzzy similarity measure and a Bipartite graph matching technique.

1. Introduction

When an autonomous mobile robot operates in an indoor environment, the description of its surroundings in the form of a map is essential to determine its configuration(position and orientation) as well as to accomplish a given task successfully. There are many different ways to model the indoor environment of a mobile robot depending on the domain in which the model is used such as localization, homing, navigation, path planning, obstacle avoidance, and so on. Typically, geometric primitives such as points, lines, and surfaces are generally used for the map of the natural environment of a mobile robot[1].

Although various sensory information has been used for the map making of the mobile robot environment, there are many cases that sensory observations of the robot environment are fuzzy quantities[2]. These fuzzy quantities are mathematical models of vaguely perceived or imprecisely defined quantitative pieces of information of the robot environment, viewed as ill-bounded sets of possible values.

These ill-bounded sets of possible values can be represented by means of fuzzy numbers restricted to appropriate membership functions by virtue of the possibility theory[3]. A fuzzy number is not a measurement but a subjective valuation[4]. But we can regard it as a possibility distribution function on the universe of discourse. So the ill-known values of geometric primitives of the robot environment can be represented by means of fuzzy numbers restricted to appropriate membership functions.

In this paper, we shall represent a geometric primitive of the robot environment by a parameterization function $g(p)$ and describe the associated sensor observation uncertainty by a possibility distribution function on the parameter vector p. Also we describe the spatial relations between geometric primitives using graph theory. For making the global map of the mobile robot environment, the correspondence problem between local maps is solved using a simple fuzzy similarity measure and a graph matching technique. This map making method is useful to manipulate the mobile robot in an uncertain indoor environment because the action process of the mobile robot is not usually precise.

In the following section we describe an uncertain geometric environment modeling method using fuzzy numbers. In section 3 we deal with a local map making method for the robot environment using graph theory. Section 4 contains a global map making method using a graph matching technique. Concluding remarks are addressed in section 5.

2. Representation of the Uncertain Geometric Environment Using Fuzzy Numbers

Representation of geometric primitives by parameterization technique has been used in the field of stereo vision and multisensory information fusion[5,6]. In this paper we describe a geometric primitive as a parameter vector p and a parameterization vector function g such that:

$$g(x, p) = 0 \tag{1}$$

where g can be interpreted as a model of the physical geometric primitive that maps a set of points (region) $x \subseteq \mathbf{R}^n$ in Euclidean n-space to a point $p \in \mathbf{R}^m$ in parameter space.

Now we represent the uncertainty of geometric primitives by assigning appropriate fuzzy numbers $\tilde{p}$ to the parameter vector $\boldsymbol{p}$. This uncertainty representation is simpler and more comprehensive and intuitive than the usual probability density function(p.d.f) representation because the observed real sensor data have some imprecision caused by measurement error as well as variations caused by dynamic situation of robot environment. Here the term *fuzzy number* is used to indicate a normalized convex fuzzy set $\tilde{M}$ of the real line $\mathbf{R}$ such that:

a. There exists exactly one $x_0 \in \mathbf{R}$ with $\mu_{\tilde{M}}(x_0) = 1$

b. $\mu_{\tilde{M}}(x)$ is piecewise continuous

where x is the genuine value of $\tilde{M}$ and x_0 is called the mean value of $\tilde{M}$. And the meaning of the fuzzy number $\tilde{M}$ is that it has the value of *approximately* (or *about*) x_0.

The most crucial element in the representation of uncertain geometric primitives by fuzzy numbers is to determine the shapes of the membership functions of fuzzy numbers. An overview of the properties for constructing the membership functions and the mathematical form of the membership functions is well given in [7], especially Civanlar[8] proposed a method to obtain the fuzzy membership function from the p.d.f. For the sake of computational efficiency and ease of data acquisition, we use triangular fuzzy numbers for representing the uncertainties of the parameterized geometric primitives. For example, on some line image, we can fit straight line to a set of edge pixels by the Hough transformation. Then the line parameter r and θ can be represented by a 2-dimensional accumulator array like Fig. 1(a). We can represent these line parameters r and θ using triangular fuzzy numbers like Fig. 1(b).

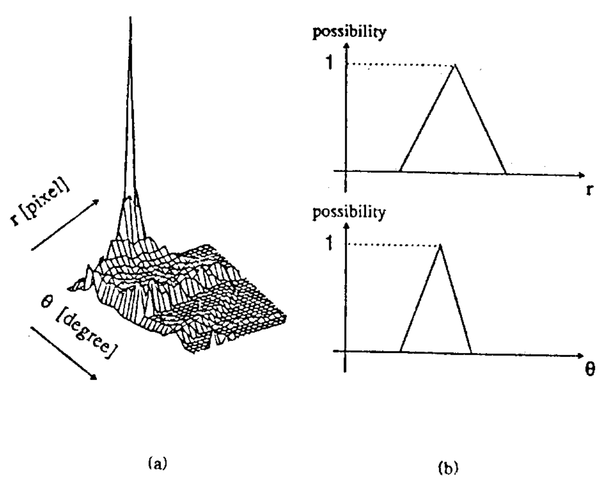


Fig. 1. Uncertainty representation using triangular fuzzy numbers.
(a)Line parameter representation using Hough transformation
(b)Triangular fuzzy number representation

In many sensor-based robotic applications, especially for integrating the information of the mobile sensor systems or the distributed sensor networks, consistent interpretation about geometric primitives between different coordinate frames is crucial. Thus we must consider how the uncertainty of geometric primitives in a coordinate frame is interpreted when it is described with respect to other coordinate frames. Moreover, in many applications of robotics, more specifically in mobile robots, the coordinate transformation cannot be determined without uncertainties. All rotation angles and translations of the transformation are measured using sensors or determined through calibration process. But measurements and calibration are always subject to error and such errors are often unavoidable and varying according as how the sensor is used. Thus we must take the transformation uncertainty into account for describing the propagation of parameter uncertainty between coordinate frames. We had already dealt with these problems, which are the representation of the uncertainty about the coordinate transformation and the manipulation of uncertain quantities between coordinate frames, in [9,10] using a simple fuzzy arithmetic.

3. Representation of the Robot Environment Using Graph Theory

We assume that the indoor environment of a mobile robot is composed of a set of geometric primitives. Then the spatial relations between geometric primitives are important to construct a map of the robot environment. The representation of the relations between geometric primitives can be well described by a graph using their topological properties. In fact, all geometric primitives parameterized like in the previous section have several topological relations with their neighbor geometric primitives.

In this section we explain only the local map of the robot environment with one sensor at a fixed position. The global map making method will be addressed in the next section.

We define each parameterized geometric primitive as a *vertex* of the graph and we connect between vertexs by an *edge* if there is a connection between geometric primitives. Also we assume that the world coordinate frame and sensors are *virtual vertexs* and we connect between sensor and geometric primitives as well as between world coordinate frame and sensor by *virtual edges*. And we label the coordinate transformation($\tilde{T}$) on the virtual edge defined between world coordinate frame and sensor(if the sensor is vision, the perspective transformation should be included).

Therefore, the local map of the mobile robot environment is composed of the sensor coordinate frame represented by homogeneous transformation which has fuzzy quantity, geometric primitives and their parameter vectors represented by fuzzy numbers, and their connected relations. For example, an indoor environment of the mobile robot shown in Fig. 2(a) can be represented like as Fig. 2(b) using the proposed graph representation method.

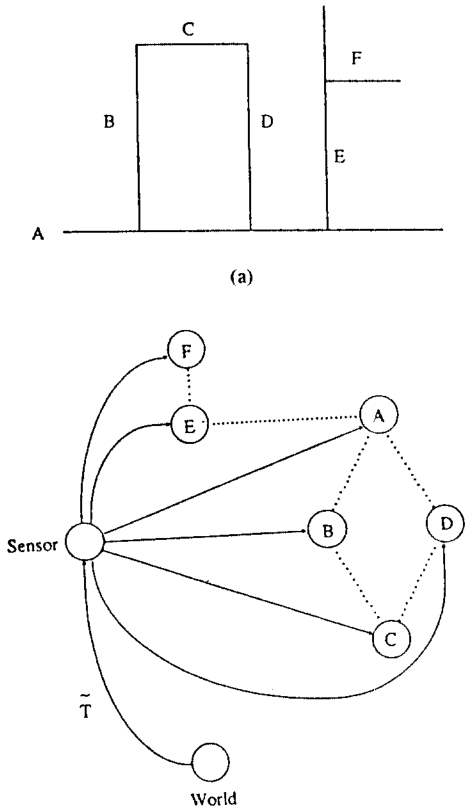


Fig. 2. A graph representation of a mobile robot environment.
(a)An example of the environment
(b)Graph representation

4. Mobile Robot Indoor Map Making Using Graph Matchings

For making the global map of an indoor environment of a mobile robot, the correspondence problem between local maps obtained from several sensors must be firstly solved. Now we consider that two sensors(sensor i and j) are used for the coincident measurement[11] and the local maps obtained from sensors i and j(or one sensor moves from the i-th coordinate frame to the j-th coordinate frame) are represented using a graph like in Fig. 3. Then we can regard the correspondence problem between local maps as a Bipartite graph matching and an optimal assignment problem. This problem can be easily solved using the Kuhn-Munkres algorithm and Hopcroft and Karp procedure[12].

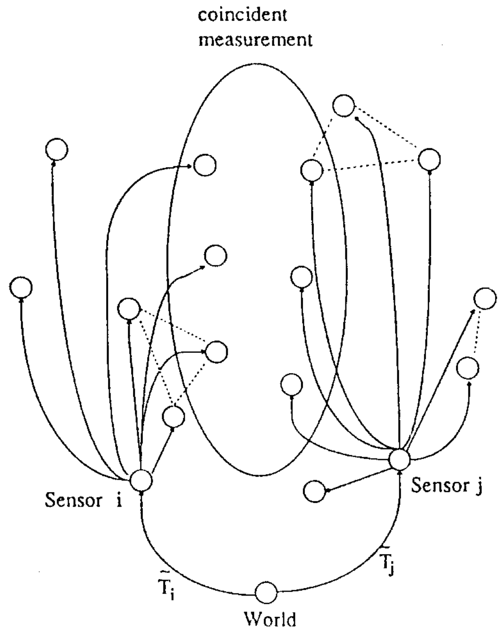


Fig. 3. Graph representation of the coincident measurement from two sensors.

Among the coincident measurement vectors represented by fuzzy numbers, which are obtained from sensor i and sensor j, we match the vertexs of sensor i and the vertexs of sensor j so that all vertexs are matched. In this case, each parameterized vector is the same geometric primitive as a matter of course. So, there is a need to define a *similarity measure* between the vertexs of sensor i and those of sensor j for matching.

In order to determine the similarity between the vertexs(which represent the parameterized geometric primitives described by fuzzy numbers), we propose a modified version of *dissemblance index*[4,13] as a similarity measure of two fuzzy numbers $\tilde{M}$ and $\tilde{N}$ such that(Fig. 4):

$$SM = \lambda\Delta_1 + (1-\lambda)\Delta_* \tag{2}$$

where

$0 \le \lambda \le 1$,

$\Delta_1 = \dfrac{1}{n+1}\sum_{k=0}^{n}\Delta(M_{\alpha_k}, N_{\alpha_k}),\ \alpha_0 = 0,\ \alpha_n = 1,$

$\Delta_* = \Delta(M_1, N_1),$

$\Delta(M_{\alpha_k}, N_{\alpha_k}) = (|a_1 - b_1| + |a_2 - b_2|)/2(\beta_2 - \beta_1),$

and $[a_1, a_2]$ and $[b_1, b_2]$ are two real intervals about the M_{α_k}(α-cut of $\tilde{M}$ at level α_k) and N_{α_k}, respectively. Also

$[\beta_1, \beta_2]$ is the narrowest interval that surrounds M_{α_k} and N_{α_k} for all $\alpha_k \in [0, 1]$. Therefore $\Delta(M_{\alpha_k}, N_{\alpha_k})$ is a normalized distance between two intervals M_{α_k} and N_{α_k}.

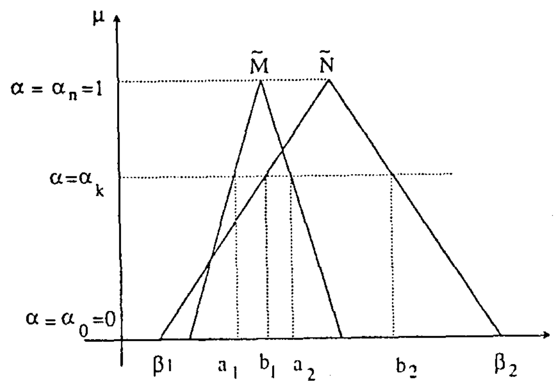


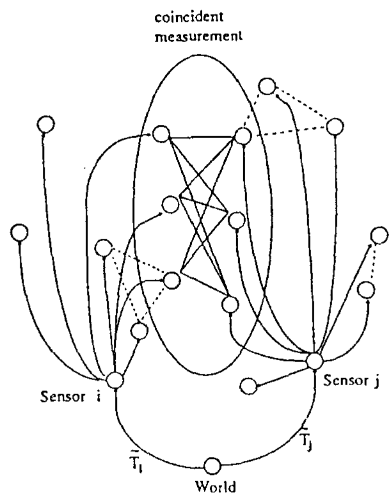
Fig. 4. A similarity measure between two fuzzy numbers.

In eq. (2), Δ_1 is a normalized summation of the dissemblance of two fuzzy numbers at each α_k level and Δ_* is a normalized separation of the centers of two fuzzy numbers. Therefore, we can take account of not only the dissemblance between the shapes of two fuzzy numbers but also the separation of the centers of two fuzzy numbers according to the choice of the appropriate λ value.

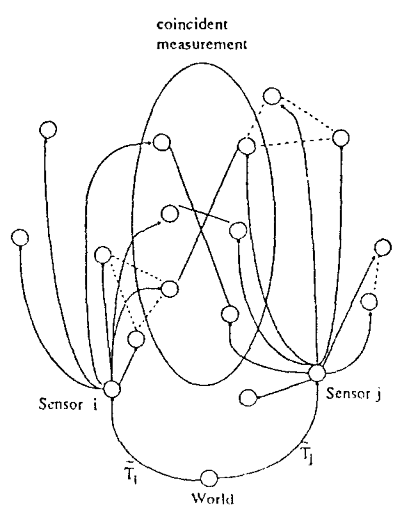
Using this similarity measure we can obtain the weight matrix between the vertexs of sensor i and the vertexs of sensor j. After this weight matrix obtaining procedure, we can perform the graph matching procedure using the Kuhn-Munkres algorithm. From the algorithm, we consider the vertoxs which generate the maximum weight perfect matching as the correspondence vertexs. Fig. 5(a) shows an example of the Bipartite graph matching and Fig. 5(b) shows an example of the correspondence vertexs after the maximum weight matching procedure.

For global map making, all correspondence vertexs must be combined in one vertex of the global map(graph). There are many combination techniques such as weighted average, MLE, Kalman filter, Bayesian, Dempster-Shafer, and so on. The traditional mathematical combination methods usually require a sufficient amount of statistical data, so that some kind of distribution governing the process must be assumed. However the map making problem for a mobile robot(or in unstructured environment), there are often not enough data to assume a probability distribution or accept subjective probability assumptions.

Thus a fuzzy number approach seems to be appropriate to combine the correspondence vertexs. A fuzzy number approach had already proposed in [14] by us named as *combination using fuzzy weighted average*. The fuzzy number approach can provide a relatively simple mathematical method to combine the correspondence vertexs with some advantages over traditional combination techniques. Moreover, it can take the subjective weighting(subjective importance) into consideration in uncertain situations.



(a)



(b)

Fig. 5. Graph matching and correspondence.
(a)Bipartite graph matching
(b)Correspondence vertexs

5. Conclusions

In this paper, we have proposed a map making methodology of an indoor environment of a mobile robot using fuzzy numbers and graph theory. Any geometric primitive of the robot environment represented by a fuzzy-numbered parameter vector and the spatial relations between geometric primitives represented by a graph. Also the correspondence problem for the global map making was solved using a fuzzy similarity measure and a graph matching algorithm.

We think that the proposed map making method meight be useful to many application areas of the mobile robot, especially to manipulate it in an uncertain environment, because the action process of the mobile robot is not usually precise. We also think that further researches are needed on the global map making of the indoor environment especially in the representation of the relation between geometric primitives using not only topological properties but also geometrical properties such as parallel, vertical, end position of line, and so on.

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