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Error Analysis for Time-in-Flight Laser Range Finder with Multiple Tone Amplitude Modulation

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Abstract

The error analysis for the **Time-in-Flight Laser Range Finder with Multiple Tone Amplitude Modulation** relevant to the phase detection error is made. The distance can be estimated to solve the formulae which express the relationship between the absolute distance from the range finder to the object and the wavenumbers and the phases of the modulated waves by the optimization technique. The main cause of the estimation error can be considered as the **phase detection error** induced from the amplitude modulator and the phase detector. To clarify the phase detection error and the optimal amplitude frequency set, the numerical analysis are made.

1. Introduction

The time-in-flight laser range finder with amplitude modulation has the advantages of the *compact* and *calibration-free* system compared with the triangulation based range finder such as slit/spot ray range finder [1, 2]. But this is used almost as the range finding system its detectable range is fixed such as the hazard detector mounted on the automated vehicle, since the range finder cannot detect the absolute range [3, 4].

To detect the absolute range through the range finder, the multiple tone amplitude modulation has been proposed [2]. The absolute range can be detected as the solution of the simultaneous equation which show the relationship between the numbers of the modulated waves and the range [2]. Although through the multiple tone amplitude modulation, the absolute range can be detectable, it has to be clarify the optimal modulation frequency set by considering the systematic error. The dominant systematic error is the phase dispersion in the modulator, especially in the case of the Acoust-Optic modulator [3]. In this study, the simulation scheme to calculate the phase of the returned wave by considering the phase dispersion for the 3D measurement of the **large structure** such as ships, buildings and so on. By using the simulation scheme, the error of the estimated range is clarified and the optimal frequency set is selected.

2. Formulation of the Range Finding

The geometrical configuration of the range finder is shown in Fig. 1.

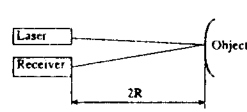

Fig. 1 The geometrical configuration of the range finder.

The amplitude of the transmitted light from the laser is modulated by the sine wave which has the certain frequency and the phase of the received light which is reflected the from the object in the distance of R from the laser/receiver is detected. This relationship is expressed by the following *distance equation.*

$$R = \frac{c\varphi}{4\pi f} + \frac{cN}{2f}, \qquad (1)$$

where c, φ, f and N are lightspeed, the detected phase, the modulation frequency and the wavenumber, respectively. Since in the distance equation there are two unknown variables, Range: R and wavenumber: N, while the phase is the only known variable, it is difficult to estimate the absolute range: R.

To estimate the absolute range, the *Two-tone amplitude modulation method* were proposed [2]. In the method, the two kinds of the phase which corresponds to the two kinds of modulation frequency are detected for each point of the object. In other words, in the method the absolute range is the solution of the following simultaneous range equation.

$$\begin{cases} R = \dfrac{c\varphi_1}{4\pi f_1} + \dfrac{cN_1}{2f_1} \\ R = \dfrac{c\varphi_2}{4\pi f_2} + \dfrac{cN_2}{2f_2} \end{cases} \qquad (2)$$

where the subscript 1 and 2 are correspond to the modulation frequency. In this simultaneous equation, there are three unknown variables N_1, N_2 and R and two known variables φ_1 and φ_2. So this becomes the underdetermine problem. In this study, the two methods to solve the equation are proposed.

2. 1 The constant wavenumber method

To remove the ambiguity of the wavenumber, the two frequencies are selected to satisfy the condition which the wavenumbers are the same ($N_1 = N_2$). Under this condition, the simultaneous distance equation can be solved as follows.

$$R = \frac{c}{4\pi}\frac{\varphi_1 - \varphi_2}{f_1 - f_2}. \qquad (3)$$

2. 2 The optimization method

Since the wavenumbers are unknown, it is practical to estimate the range which satisfy the difference between the absolute ranges calculated from the two range equations under the certain wavenumber range. This method is to minimize the following cost function with the constraint of wavenumbers.

$$J = [R(i) - R(j)]^2 = \left[\frac{c}{4\pi}\left(\frac{\varphi_1}{f_1} - \frac{\varphi_2}{f_2}\right) + \frac{c}{2}\left(\frac{i}{f_1} - \frac{j}{f_2}\right)\right]^2 \qquad (4)$$
$$(N_{1\min} \le i \le N_{1\max},\ N_{2\min} \le j \le N_{2\max}),$$

where the subscript min and max means the lower and upper limit os the wavenumber which are previously defined.

3. Error Analysis

The main cause of the estimation error can be the phase error in the modulator and detector. In this study, the error analysis relevant to the phase error is made for the establishment of the optimal frequency set definition.

3. 1 The effect of the quantization of the detected phase

Since the A/D converter can be used for phase detection, the quantization affects the accuracy. In the case of M levels quantization from 0 to 2π of the phase, the range resolution ΔR is expressed as follows.

$$\Delta R = \frac{c}{2Mf}. \qquad (5)$$

Table 1 shows the relationship between the frequency and the range resolution in the case of 10 bit quantization (M=1024).

Table 1 The relationship between the frequency and the range resolution

Frequency [MHz]	Resolution [m]
1	1.46×10^{-1}
10	1.46×10^{-2}
100	1.46×10^{-3}

3. 2 The constant wavenumber method

The constant wavenumber method has the constraint for the frequency set to satisfy the wavenumbers in each case of two modulation frequencies are the same. The frequency difference can be defined by the range as follows. And this relationship is shown in Fig. 2.

$$|f_1 - f_2| = \frac{c}{2R} \,. \qquad (6)$$

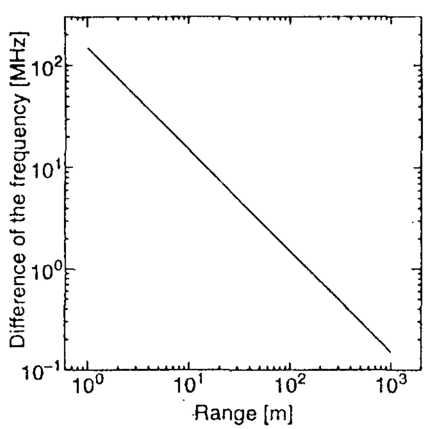

Fig. 2 The relationship between the range and the frequency difference

These mean that it is necessary to select the small difference frequency set to estimate the far range. And this fact shows that in the case of the far range estimation, the estimation error becomes large because the estimation error is proportional to $1/(f_1 - f_2)$.

If the phase error exists, the frequency constraint will be violated in certain range. Such *critical* range R_c is expressed by the following equation.

$$R_c = \frac{ck}{2f} \,, \quad (k = 0, 1, \ldots) \,. \qquad (7)$$

To select the lower frequency can reduce the numbers of the critical ranges, however, it make the range resolution large. So it has to select the optimum frequency set by considering the phase error.

3. 3 The optimization method

The main cause of the estimation error induced from the phase error in the optimization method is the mis-selection of the wavenumber, in other words, this is the conversion to the undesirable point. This occurs more frequently in higher frequency. To avoid this, there are two ways, first one is to select the lower frequency and second one is to select the narrow wavelength range. Since the first way has the disadvantage which the range resolution becomes large and the second way has the disadvantage which the detectable range becomes small, the frequency set has to be determine by considering the degree of phase error.

4. The Combined Method

To reduce the estimation error in the optimization method, the reduction of the wavenumber range is useful. So to reduce the wavenumber range, the constant wavenumber method can be used as the preprocessing, it is named the *combined method*. The combined method is useful except around the critical range in the constant wavenumber method. To avoid the critical range, the *Quad-tone amplitude modulation method* which use the four frequencies will be useful. The estimation error of the combined method will be almost same as that of the optimization method.

5. Numerical Experiment

To clarify the availability and the limitation of the proposed methods, the numerical experiment is conducted under the following condition.

·Measurement range: 0 - 100 [m].

·Modulation frequency: f_1=19.6 [MHz], f_2=20.9 [MHz].

·Phase error: ±0.1, ±0.05, ±0.01 [rad.].

·Interest range: 10, 50, 99 [m].

·Method: The constant wavenumber method, The optimization method, The combined method.

·Range of wavenumbers: 0 - 30 (for the optimization method).

In this case, the critical ranges are 7.65 x n (n = 0, .., 13) [m] for 19.6 [MHz] and 7.28 x n (n = 0, ..., 13) for 20.9 [MHz].

Tables 2 (a - c) show the RMS error of the range estimation in the cases of (a) the constant wavenumber method, (b) the optimization method and (c) the combined method respectively.

Table 2 The RMS error of the range estimation.

(a) The constant wavenumber method.

Phase error [rad.]	Interest Range [m] 10	50	99
±0.1	1.6×10^{0}	1.6×10^{0}	8.3×10^{1}
±0.05	8.2×10^{-1}	8.2×10^{-1}	8.3×10^{1}
±0.01	1.6×10^{-1}	1.6×10^{-1}	8.3×10^{1}

(b) The optimization method

Phase error [rad.]	Interest Range [m] 10	50	99
±0.1	7.8×10^{1}	7.8×10^{1}	7.8×10^{1}
±0.05	7.0×10^{1}	7.0×10^{1}	7.0×10^{1}
±0.01	2.6×10^{1}	2.6×10^{1}	2.6×10^{1}

(c) The combined method

Phase error [rad.]	Interest Range [m] 10	50	99
±0.1	5.3×10^{-2}	5.3×10^{-2}	5.3×10^{-2}
±0.05	2.6×10^{-2}	2.6×10^{-2}	2.6×10^{-2}
±0.01	5.3×10^{-3}	5.3×10^{-3}	5.3×10^{-3}

For all interest ranges, the estimation error from the combined method is smallest and satisfactory, this is because the reduction of the wavelength range eliminates the undesirable points. This fact shows the usefulness of the combined method.

6. Conclusions

The previous results lead the following conclusion. The solution of the simultaneous range equation in the case of the range finder with two-tone amplitude modulation by the combined method is useful, and to avoid the critical range, it will be useful that the quad-tone amplitude modulation which the frequencies are optimally selected.

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(All texts are written in Japanese.)