'93 KACC (1993.10.20~22)

Nonlinear Interaction and Dynamic Compensators

Shintaro Ishijima and Akira Kojima

Tokyo Metropolitan Institute of Technology
6-6 Asahigaoka Hino-City Tokyo 191, JAPAN

Abstract

The main difference between a linear system and a nonlinear system is the existence of direct interactions between input signals. These interactions will be classified into three types, (1) self-interaction among different order terms of control signals, (2) static mutual interactions between the control signals, and (3) dynamic interactions through the coefficient vector fields of the control variables. In this paper, we will show that interactions of type (2) and (3) can be avoided by applying an appropriate dynamic compensator, while the interaction of type (1) is fatal.

1. Introduction

If the number of inputs is greater than or equal to the number of states, almost the all control problems will be trivial or at least easily solvable even if the system is highly nonlinear. Let us call such a system a homeomorphic system. We have shown that a linear controllable system can be always decomposed into homeomorphic subsystems with "virtual inputs" and the pole assignment problem will be reduced to the trivial pole placement problems for each homeomorphic subsystem. Such an observation has motivated us to study the decomposition problem of nonlinear systems. Up to now, we have shown that a nonlinear system can be decomposed to the full-controlled subsystems under a quasi-coordinate. Here a system which is attainable by using only the coefficient vector fields of input variables has been called a full-controlled system and if the system is linear, a full-controlled system is nothing but a homeomorphic system. in this paper, we will show that in some cases, a nonlinear system can be further decomposed by applying a dynamic compensator. A dynamic compensator can synchronize the input signals to avoid the direct interactions among the input signals and consequently an augmented system admits the further decomposition. The interactions among the input signals can be classified into three categories, (1) self-interaction among high order terms of control signals, (2) static mutual interactions between the control signals and (3) dynamic interactions through the coefficient vector fields of the control variables. It is shown that the interactions of type (2) and (3) can be avoided by applying an appropriate dynamic compensator, while the interaction of type (1) is fatal. A sufficient condition for the existence of the self interaction (1) is derived and it is shown that the interactions (2) and (3) can be avoided by applying a dynamic compensator.

2. Decomposition under Quasi-coordinate

Suppose the system can be described by the following nonlinear differential equation.

$$\dot{x} = f(u, x) \tag{1}$$

Here $x(t) \in R^n$ is the state vector and $u(t) \in R^r$ is the input vector. And suppose that $f(u, x)$ is an analytic map from $R^r \times R^n$ to R^n and it can be expanded as follows

$$f(u, x) = f^0(x) + f^1(x)u + f^2(x)u^{[2]} + \cdots \tag{2}$$

where $u^{[k]}$ is a vector of which elements are k order terms of the elements of u, $u^{[k]} = (u_1^k, \ldots, u_r^k, u_1 u_2^{k-1}, \ldots)^T$ and f^i, $i = 0, 1, \ldots$ are analytic matrices.

Let X be a vector field and $\mathcal{D}$ be a distribution on R^n. $\mathcal{D}$ is called X invariant if for any vector field $Y \in \mathcal{D}$, it holds that $[X, Y] \in \mathcal{D}$. Let L^u be a distribution generated by the vector fields f^j, $j = 1, \ldots$ and it will be called the input distribution. Let L_0 be the smallest involutive distribution which contains L^u and f^0 invariant. As is well known if at x_0, $dim L_0(x) = n$, then the system is strongly attainable at x_0. The system which satisfies $\dim \bar{L}^u(x) = n$ where $\bar{L}^u$ is the involutive closure of L^u is called a full-controlled system. In the following we will assume that the system is strongly attainable at almost all point $x \in R^n$. Note that the set of singular points at where the system is not strongly attainable is the invariant set.

Now let $y = \Phi(x)$ be a change of variables where Φ is an analytic map from U, a neighborhood of $x_0 \in R^n$ to $V \subset R^n$, a neighborhood of $y_0 = \Phi(x_0)$. If Φ is nonsingular at x_0, that is $\partial\Phi(x_e)/\partial x$ is full rank, then Φ can be regarded as the local coordinate transformation. Let us consider the next system.

$$\begin{cases} \dot{x}_1 = x_1 u \\ \dot{x}_2 = {x_1}^3. \end{cases} \tag{3}$$

This system is not feedback linearizable at the origin because the linear approximation of this system at the origin is not controllable and the necessary condition of feedback linearizability is not satisfied. However if we apply the change of variables,

$$\begin{cases} y_1 = {x_1}^3 \\ y_2 = x_2 \\ v = 3{x_1}^3 u \end{cases} \tag{4}$$

the system is transformed to

$$\begin{cases} \dot{y}_1 = v \\ \dot{y}_2 = y_1. \end{cases} \tag{5}$$

This means that the system is linearized formally by the feedback transformation (4). The trick is that the transformation (4) is singular at the origin and never satisfy the conditions for the coordinate transformation. However, such a transformation will be convenient for the analysis of nonlinear systems.

Definition 1

A change of variables $y = \Phi(x)$ will be called a quasi-coordinate transformation at x_0 if Φ is homeomorphic at almost all points in U, the neighborhood of x_0. y will be called a quasi-coordinate.

It is known that an involutive distribution $\mathcal{D}$ can be written as,

$$\mathcal{D}_r = \text{span}\{\partial/\partial x_1, \ldots, \partial/\partial x_d\}$$

under an appropriate local coordinate $(x_1, \ldots, x_d, \ldots, x_n)$ in the neighborhood U of the point x_0, if $\mathcal{D}$ is nonsingular at x_0. Let us consider the case where $\mathcal{D}$ is singular at x_0. The integer defined by,

$$k = \max_{x \in U} \text{ dim span } \mathcal{D}(x)$$

will be called the generic dimension of an analytic distribution $\mathcal{D}$. Let D_s be the set of singular points of $\mathcal{D}$,

$$D_s = \{x \in U | \text{dim span } \mathcal{D}(x) < k\}$$

Definition 2

Suppose an analytic distribution $\mathcal{D}$ of generic dimension k is defined in the neighborhood $U \subset R^n$ and D_s is the singular set of $\mathcal{D}$ in U. $\mathcal{D}$ is called nonsingularly covered by ν covers or simply it has nonsingular cover at x_0 if there exists ν open sets $\mathcal{O}^1, \ldots, \mathcal{O}^\nu$ and ν quasi-coordinate transformations $\Phi^1, \ldots, \Phi^\nu$ which satisfy the following conditions.

(i)

$$\sum_{i=1}^{\nu} \mathcal{O}^i + D_s = U$$

(ii) Φ^i is analytic homeomorphism from the open set $\mathcal{O}^i$ to the open set $V^i \subset R^n$. There exits a nonsingular distribution defined on the closure $\bar{V^i}$ of V^i which satisfies that at every $x \in \mathcal{O}^i$,

$$\text{span } {\mathcal{D}_r}^i(\Phi(x)) = \text{span } (\Phi)_* \mathcal{D}(x).$$

(iii) Every vector field $X \in (\Phi^i)_* \mathcal{D}$, there exist analytic functions ${\alpha^i}_1, \ldots, {\alpha^i}_k$ such that

$$X(p) = \sum_{j=1}^{k} {\alpha_k}^i(p) {Y^i}_{(p)}$$

where ${Y^i}_j$, $j = 1, \ldots, k$ are bases vector fields of ${\mathcal{D}_r}^i$.

Based on this concept, we can prove the next theorem.

Theorem 1

Suppose the system (1) is strongly attainable at almost all points. There exists a local quasi-coordinate system under which the system can be decomposed to the following form.

$$\begin{cases} \dot{z}_1 = f_1(u_1, z_1, \ldots, z_q) & u_1 = u(\text{real input}) \\ \dot{z}_2 = f_2(u_2, z_2, \ldots, z_q) & u_2 = z_1 \\ \cdots & \cdots \\ \dot{z}_q = f_q(u_q, z_q) & u_q = z_{q-1} \end{cases} \tag{6}$$

where $z_i(t) \in R^{n_i}$ and $\sum_{j=1}^{q} n_j = n$ and each subsystem is full-controlled.

We will call such a decomposition the full-controlled virtual decomposition and call u_j, $j = 1, \ldots, q$ as virtual inputs. Note also that we will call a decomposition of the form (6) where each subsystem is not necessarily full-controlled as a virtual decomposition.

Example 1

Consider a bilinear system,

$$\begin{aligned} \dot{x}_1 &= x_2 u \\ \dot{x}_2 &= x_1 + x_1 u. \end{aligned} \tag{7}$$

We can easily show that the distribution $\mathcal{D} = \{x_2\partial/\partial x_1 + x_1\partial/\partial x_2\}$ can be nonsingularly covered by 4 covers. For instance, on $R_1^+ = \{(x_1, x_2) | x_1 > \delta|x_2|\}$, the quasi-coordinate transformation,

$$\begin{aligned} y_1 &= x_2 \\ y_2 &= x_1^2 - x_2^2 \end{aligned}$$

will transform the system into the full-controlled virtual decomposition form.

$$\begin{aligned} \dot{y}_1 &= (1+u)\sqrt{y_1^2 + y_2} \\ \dot{y}_2 &= -2y_1\sqrt{y_1^2 + y_2} \end{aligned} \tag{8}$$

3. Interactions and Compensators

Generally, the virtual full-controlled decomposition for a linear control system,

$$\dot{x} = Ax + Bu \tag{9}$$

is coincides with the Hessenberg type decomposition.

$$\begin{aligned} \dot{x}_1 &= B_0 u + A_{11}x_1 + \ldots + A_{1q}x_q \\ \dot{x}_2 &= A_{21}x_1 + \ldots + A_{2q}x_q \\ &\ldots \\ \dot{x}_q &= A_{qq-1}x_{q-1} + A_{qq}x_q \end{aligned} \tag{10}$$

where $x_i(t) \in R^{n_i}$ and $u(t) \in R^r$. The characteristics of this decomposition are as follows.

(L1) $r = n_1 \leq n_i \leq n_{i+1}, \quad i = 1, \ldots, q-1$

(L2) $\text{rank} A_{i,i-1} = n_i$

However for a nonlinear system, these properties are not satisfies due to the coupling among the virtual inputs signals. Such interactions can be classified as follows.

(N1) self interactions due to the existence of higher order terms

(N2) static interactions among the virtual input signals

(N3) dynamic interactions through the coefficient vector fields

Now we will define the interactions as follows.

Definition 3

In the full-controlled virtual decomposition (6), if $n_{j-1} < n_j$ for some $j \in (1, 2, \ldots, q)$, then we will call that the subsystem j has the interactions among the virtual inputs.

The examples of each interactions can be given as follows.

(N1)

$$\begin{cases} \dot{x}_1 = u \\ \dot{x}_2 = u^3 \end{cases} \tag{11}$$

(N2)

$$\begin{cases} \dot{x}_1 = u_1 \\ \dot{x}_2 = u_1 u_2 \\ \dot{x}_3 = u_2 \end{cases} \tag{12}$$

(N3)

$$\begin{cases} \dot{x}_1 = u_1 \\ \dot{x}_2 = x_1 u_2 \\ \dot{x}_3 = x_2 u_2 \end{cases} \tag{13}$$

Now consider the system,

$$\begin{aligned} \dot{u} &= v \\ \dot{x} &= f(u, x) \end{aligned} \tag{14}$$

defined on R^{n+r} and define a series of distribution $\{\mathcal{H}_j\}$, $j = 1, \ldots$ as follows.

$$\begin{aligned} \mathcal{H}_0 &= \{\ \partial/\partial u_1, \ldots, \partial/\partial u_r\ \} \\ \mathcal{H}_j &= \{\ [f(u,x)\partial/\partial x, \bar{\mathcal{H}}_{j-1}] + \bar{\mathcal{H}}_{j-1}]\ \} \end{aligned} \tag{15}$$

For the existence of the self interactions, we can prove the next theorem.

Theorem 2

For the system (1), if there exists some $j \in (1, \ldots, q)$ such that

$$[\bar{\mathcal{H}}_{j-1}, \mathcal{H}_j] \not\subset \mathcal{H}_j$$

then there exists the self interactions in the virtual subsystem j.

Now let us consider the static interactions (N2). It will be clear that this type of interactions can be reduced to the dynamic interaction (N3) by applying a dynamic compensator. Indeed, for the system (12), applying a dynamic compensator,

$$\begin{cases} \dot{y}_1 = v_1 \\ u_1 = y_1 \end{cases}$$

we will get the augmented system

$$\begin{cases} \dot{y}_1 = v_1 \\ \dot{x}_1 = y_1 \\ \dot{x}_2 = y_1 u_2 \\ \dot{x}_3 = u_2. \end{cases} \tag{16}$$

This shows that the static interactions can be transformed to dynamic interactions (N3). Hence it will be suffice to study the dynamic interactions. For this purpose, let us consider the next 2 inputs system,

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2. \tag{17}$$

By applying the dynamic compensator,

$$\begin{cases} \dot{y}_1 = v_1 \\ \cdots \\ \dot{y}_p = y_{p-1} \end{cases} \tag{18}$$

we will get the next augmented system.

$$\begin{array}{ll} \dot{y}_1 = v_1 & \dot{z}_1 = g_1(z_1, \ldots, z_n, y_p)u_2 \\ \cdots & \cdots \\ \dot{y}_p = y_{p-1} & \dot{z}_p = g_p(z_{p-1}, \ldots, z_n, y_p) \\ & \dot{\tilde{z}}_p = G_{p+1}(y_p, z_p, \ldots, z_n) \end{array} \tag{19}$$

where $\tilde{z}_p = (z_{p+1}, \ldots, z_n)^T$ and we have assumed that there is no self interactions. The equation (19) shows that if we select p appropriately, we can avoid the dynamic interactions. It will be easy to extend this result to the general system with more than 3 inputs and we will get the next theorem.

Therem 3

If there exists no self interactions in the system (1), then we can get the decomposition with the properties (L1) and (L2) by applying an appropriate dynamic compensator.

This is almost same as to say that if the system has no self interactions then the system is formally linearizable (linearizable under a quasi-coordinate).

4. Conclusion

In this paper, we have shown that the nonlinear system has nonlinear interactions among the input signals and the dynamic compensator can play an important role to avoid such interactions and to get the finer decomposition. based on this results, we can define the measure which shows how fine the decomposition is. Such information about the structure of the system will be a great help to design controllers of the nonlinear systems.

References

[1] S.Ishijima and A.Kojima, Practical Stabilization of the Variable Length Pendulum by Irreducible Virtual Decomposition, Proceedings of IFAC'93, 1993

[2] S. Ishijima and Y.F.Zheng, Dynamic Decomposition of Nonlinear Control Systems, Proceedings of MTNS-91l, 1991