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유색잡음 매개변수가진과 외부가진을 받는 확률 시스템의 응답해석

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(RESPONSE ANALYSIS OF A STOCHASTIC UNDER PARAMETRIC AND EXTERNAL EXCITATION HAVING COLORED NOISE CHARACTERISTICS)

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ABSTRACT

Interaction between system and disturbance results in system with time-dependent parameter. Parameter variation due to interaction has random characteristics. Most of the randomly varying parameters in control problem is regarded as white noise random process, which is not a realistic model. In real situation those random variation is colored noise random process.

Modified F-P-K equation is proposed to get the response of the random parametric system using some correction factor. Proposed technique is employed to obtain the colored noise parametric system response and confirmed via Monte-Carlo Simulation.

INTRODUCTION

The general theory of analysis of linear time-invariant stochastic system having white noise is well developed and documented.[1,2,3,4] For white noise excitations the theory gives a direct evaluation of the response statistics. The probabilistic description of the response of dynamic systems involving time variation in the inertia coefficient and subjected to external random excitation was examined[5,6], which have been carried out through the application of stochastic Volterra integral equations of the second kind[7].

There are lot of physical system which has random input with induced system parameter variation having random character.

One class of such problems is response and stability of helicopter rotor blades in atmospheric turblent flow[8]. The behavior of such systems is usually described by liner time-invariant differential equations with non-homogeneous random terms. Those non-homogeneous stochastic system with random parameter variation is referred to as parametric stochastic system[9]. When the parametric excitation is white noise process, the response of the system constitutes a Markov process and the response transition probability density function can be determined by solving the Fokker-Plank equation. The solutions of the Fokker-Flank equation have been obtained for a certain class of first order systems.[10,11]

The probability density for a class of piecewise linear system subject to simultaneous parametric and external random white processes was derived.[12,13] Also the probability density of a special class of second order linear system involving white noise parametric and external exitations.[14,15] If the excitations is not white noise process, the response can be approximated by a Markov vector by introducing shaping linear filters between a white noise input and the system itself. However, this approach may result in a rather complicated system for which the corresponding Fokker-Planck equation may not be solved analytically. In this paper, some modification factor from the ratio between colored noise and white noise is adapted and reformed Fokker-Planck equation is prposed to obtain the response statistics of stochastic system excited simultaneous parametric and external excitation.[16]

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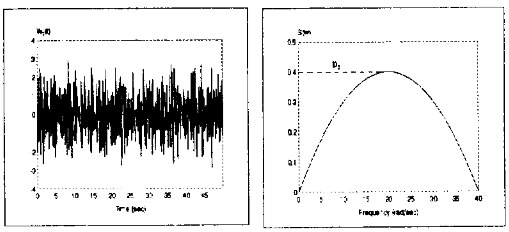
PROBLEM FORMULATION

A single degree-of-freedom system with randomly varying stiffness is considered as follows :

$$\ddot{q} + 2\zeta\omega\dot{q} + \omega^2(1+W_1(t))q = W_2(t) \quad (1)$$

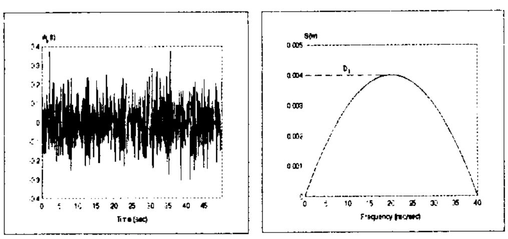
The system has parametric random noise $W_1(t)$ under external random input $W_2(t)$.

Both $W_1(t)$ and $W_2(t)$ are independen Gaussian colored random process.



a) External random process $W_2(t)$ b) Power spectral density

Fig. 1 Colored noise external random excitation $W_2(t)$



a) Parametric random process $W_1(t)$ b) Power spectral density

Fig. 2 Colored noise parametric random excitation $W_1(t)$ $(D_1 = D_2/100)$

First consider $W_1(t)$ and $W_2(t)$ are white noise Gaussian random process.

Markov vector approach is used as follows. Introducing the coordinate transformation (2)

$$x_1 = q$$
$$x_2 = \dot{q} \quad (2)$$

the system equation becomes as equation (3).

$$dx_1 = x_2 dt$$
$$dx_2 = [-\omega^2 x_1 - 2\zeta\omega x_2]dt - x_1\omega^2 dB_1(t) + dB_2(t) \quad (3)$$

where $W_i(t) = \frac{d}{dt} B_i(t)$

and $E[dB_i^2(t)] = 2D_i dt$

D_i : White noise power spectral density

The Fokker-Planck equation (4) is used to get the transition probability density function.

$$\frac{\partial}{\partial t} P(\mathbf{x},t) = \sum_{i=1}^{2} \frac{\partial}{\partial x_i} a_i P(\mathbf{x},t) + \sum_{j=1}^{2}\sum_{k=1}^{2} \frac{\partial}{\partial x_j}\frac{\partial}{\partial x_k} b_{jk} P(\mathbf{x},t) \quad (4)$$

where $a_i(\mathbf{x}, t)$: First incremental moments of the Markov process $\mathbf{x}(t)$

$b_{jk}(\mathbf{x}, t)$: Second incremental moments of the Markov process $\mathbf{x}(t)$

Final form of dynamic moment equation, using (4), becomes (5).

$$\frac{d}{dt} E[x_1^k x_2^l] = f(E[x_1^m x_2^n], \zeta, \omega, D_1, D_2) \quad (5)$$

It shoud be noticed that the equation (5) is based on the white noise which has constant power spectral density D_1, D_2.

Since in reality most of the random process in physical system is non-white, it is not realistic approach to analysis the stochastic system under the assumption.

PROPOSED METHODOLOGY

As is seen previously most of the analysis on the stochastic system have used white noise assumption. However the noise is colored random process.

Concept that colored noise is approximated summation of the band-limited white noise is the essence of the proposed technique as in Fig 3.

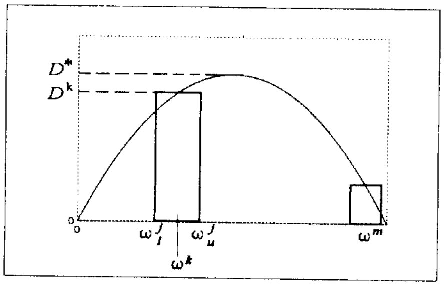


Fig. 3 Colored noise consist of band-limited white noises

Thus following system of equation (6) is obtained.

$$\dot{m}_{01}{}^{k} = m_{10}{}^{k}$$
$$\dot{m}_{10}{}^{k} = -\omega^2 m_{10}{}^{k} - 2\zeta\omega\, m_{01}{}^{k}$$
$$\dot{m}_{11}{}^{k} = -2\zeta\omega\, m_{11}{}^{k} - \omega^2 m_{02}{}^{k} + m_{20}{}^{k}$$
$$\dot{m}_{02}{}^{k} = 2\, m_{11}{}^{k} \qquad (6)$$

$$\dot{m}_{20}{}^{k} = -2\,\omega^2 m_{11}{}^{k} + 2\,\omega^4 D_1{}^{k} m_{02}{}^{k} - 4\zeta\omega\, m_{20}{}^{k} - 2\,D_2{}^{k}$$

$m_{ij}{}^{k}$ are evaluated for each band-limited spectral density D^k

where $m_{ij} = E[x^i \dot{x}^j]$

Also, the power spectral density of parametric random process $D_1{}^{k}$ is smaller one as (7)

$$D_1{}^{k} = \frac{D_2{}^{k}}{N} \qquad (7)$$

where N is some number.

It is well known that the relation between wide band white noise, band-limited white noise and cut off frequencies is as equation (8)[1].

Also I_k defined as modification factor due to the k-th band-limited white noise.

$$I_k = I\left(\frac{\omega^k{}_u}{\omega_n}, \zeta\right) - I\left(\frac{\omega^k{}_l}{\omega_n}, \zeta\right) \qquad (8)$$

where

$$I\left(\frac{\omega^k}{\omega_n}, \zeta\right) = \frac{1}{\pi}\tan^{-1}\frac{2\zeta\left(\frac{\omega^k}{\omega_n}\right)}{1-\left(\frac{\omega^k}{\omega_n}\right)^2} + \frac{\zeta}{2\pi\sqrt{1-\zeta^2}}\, ln\frac{1+\left(\frac{\omega^k}{\omega_n}\right)^2+2\sqrt{1-\zeta^2}\left(\frac{\omega^k}{\omega_n}\right)}{1+\left(\frac{\omega^k}{\omega_n}\right)^2-2\sqrt{1-\zeta^2}\left(\frac{\omega^k}{\omega_n}\right)}$$

ω_n : system natural frequency

ω_u^k : upper cut-off frequency of k-th band limited white noise

ω_l^k : lower cut-off frequency of k-th band limited white noise

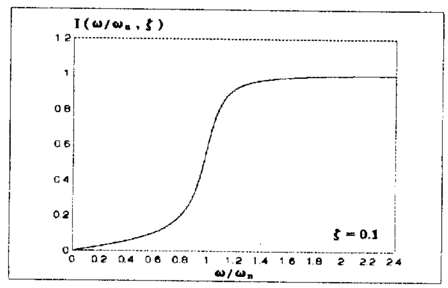


Fig.4 Integral factor of band-limited white noise compared to wide band white noise.

The integrtal factor $I\left(\frac{\omega}{\omega_n}, \zeta\right)$ shown in Fig. 4 is adopted as a correction factor.

Thus modified dynamic moment equation is reformed.

Since linear system is considered, final form of system response becomes as (9)

$$m_{ij} = \sum_{k=1}^{m} I_k\, m_{ij}{}^{k} \qquad (9)$$

RESULTS

Stochastic response of the system under simultaneus parametric and external colored n oise excitation obtained from modified dynamic moment equation. Also Mote-Carlo stochastic simulations is performed to confirm the results.

Fig.5 shows the mean square response of the system having no parametric excitation.

The response obtained using the proposed methodology reveals good agreement with simulation in the relatively smaller random exitation level.

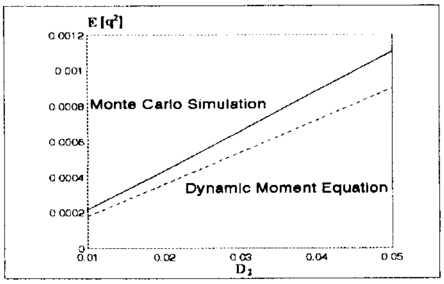


Fig.5 Mean square response for the system when D_1 = 0.

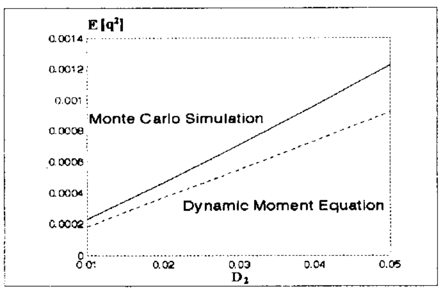


Fig.6 mean square response of the system when D_1 = D_2 / 20.

Fig.6 shows the system response under simultaneous parametric and external colored noise excitation. Modified dynamic moment equation gives reasonally good results.

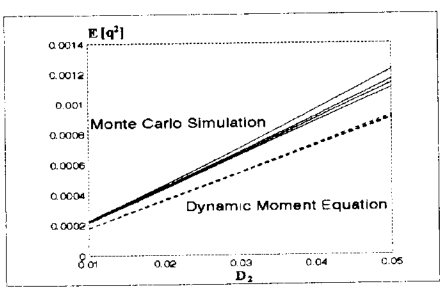


Fig.7 Comparison of response trends under the effect of level of parametric excitation level

In Fig.7 the system mean square response becomes large as the parametric excitation level increases.

However, comparing with the simulation, the proposed technique is less sensitive to the degree of random excitaion level.

CONCLUSION

Proposed technique is attempted for the first time to analysis the simultaneous parametric and external colored noise system.

Integral factor is successfuly adopted to modify the dynamic moment equation derived from Fokker-Planck equation.

The response obtained from modified dynamic moment equation shows quite close to the one obtained via Monte-Carlo stochastic simulation. However, as the level of random exitation is applied on the system is increased the response reveals discrepancy.

Intensive study is being carried on to overcome the limit of the proposed methodology and to verify the idea.

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