ACOUSTICAL PROPERTIES OF UNDERWATER BUBBLE LAYER WITH TRANSITION SUBLAYERS

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Abstract

In the ocean bubble layers play a significant role in the sound propagation as well as sound generation. Most of early works dealt with acoustic properties of the bubble layers with sharp flat boundaries. Bubble layers with transition sublayers are more likely in the ocean. In this paper a theory of sound propagation through plane bubble layers with transition sublayers at both borders was developed. It shows that the reflection and transmission coefficients depend on the thickness of transition sublayers. The theory with thicker transition sublayers shows weaker resonance properties of bubble layer. It gives better presentation for the peculiar behavior of the experimental data than that with sharp flat boundaries.

I. Introduction

In the ocean bubble clouds can be generated by various processes of ocean surface agitation. Breaking waves are major generating sources of bubble clouds. Such bubble clouds can be formed in the ocean upper layer down to tens of meters by Langmuir circulation, turbulence and other mechanisms [1]. It was already well known that bubbles work as sound scatterers in the ocean. However, it was not clearly understood how sound waves interact with bubbles in the ocean. The bubbles in the ocean are also just recently recognized as one of the most efficient sources of underwater ambient noise over wide frequency range.

One of the noise generating mechanisms around several kilohertz frequency range can be explained with resonance oscillations of individual bubbles in ocean. However, such individual bubble resonance cannot explain the generating mechanism of several hundred hertz frequency range. In 1985 Prosperetti [3] and Carey [4] independently suggested that the collective oscillations of bubble clouds might explain the generating mechanisms of ocean ambient noise in the low frequency range. In 1989 Yoon *et al.* [5-7] experimentally proved that bubble clouds have their own resonance frequencies and can generate much lower-frequency sound rather than that of individual bubble resonance. They described the resonance behavior of cylindrical bubble columns as a possible source of natural underwater sound.

An acoustic role of bubbles in the sea is not limited only to sound generation. They can also have influence on sound propagation. In recent paper Yoon and Choi [8] reported a study of the sound propagation through plane bubble layer in water. They investigated the resonance frequencies and transmission properties of plane bubble layer. Some discrepancy between theoretical and measured transmission coefficients was observed. The discrepancy

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may be caused by introducing bubble layer with sharp flat boundaries and equal-sized bubble distribution in their theoretical calculation. In the experiment it is practically very difficult to create bubble layers with such boundaries. In this paper we investigate acoustical properties of underwater bubble layer with transition sublayers considering more likely bubble layers in the ocean.

II. Theoretical approach

The liquid-bubble mixture is described with averaged equations that lead to an effective Helmholtz equation for the average pressure field P,

$$\nabla^2 P + k_m^2 (P - P_o) = 0 \tag{1}$$

where P_o is the undisturbed static pressure and k_m is the wave number in the bubbly mixture given by

$$k_m^2 = \frac{\omega^2}{c_o^2} + 4\pi\omega^2 \int_0^\infty \frac{a f(a)\, da}{\omega_o^2 - \omega^2 + 2ib\omega} \tag{2}$$

where ω is the angular frequency of sound wave, c_o is the sound speed in the pure liquid, a is the bubble radius, $f(a)$ is the bubble number density, b is the damping constant, ω_o is the resonance frequency for the bubble with radius a. This expression is valid for low void fraction of the bubbly mixture, where the void fraction is defined as the ratio of gas volume fraction to bubbly mixture volume. The resonance frequency ω_o of bubble radius a is given by

$$\omega_o = \frac{1}{a}\sqrt{\frac{3\gamma P_o}{\rho}} \tag{3}$$

where ρ is the liquid density and γ is the ratio of the specific heats, *i.e.*, 1.4 for air.

If the sound wave frequency is much lower than the bubble resonance frequencies, *i.e.*, $\omega \ll \omega_o$, then Eq. (2) becomes

$$k_m^2 = \frac{\omega^2}{c_o^2} + \frac{\rho\omega^2}{\gamma P_o}\beta. \tag{4}$$

Here, $\beta = \frac{4}{3}\pi \int_0^\infty a^3 f(a)\, da$ = void fraction.

For low frequencies the sound speed only depends on the void fraction. For bubbly mixture near ocean surface the sound speed c_m can be written as:

$$c_m = \frac{1500}{\sqrt{1 + 1.6 \cdot 10^4 \beta}}. \tag{5}$$

If all the bubbles have the same size, then $f(a) = n_o \delta(a - a_o)$, and Eq. (2) is reduced to

$$k_m^2 = \frac{\omega^2}{c_o^2} + \frac{4\pi\omega^2 a_o n_o}{\omega_o^2 - \omega^2 + 2ib\omega}. \tag{6}$$

The spatial bubble distribution in the ocean is usually homogeneous within a certain depth and then sharply decreases beyond the depth. Such depth was about 6 m for wind speed 11 m/s in the ocean [9,10]. Let us consider the plane bubble layer with transition sublayers. To simulate a more likely bubble layer in the ocean, the sound speed profile shown in Fig. 1 is selected and can be described by

$$\frac{c_o^2}{c^2(z)} = 1 + N\left(\frac{e^{2z/h}}{1 + e^{2z/h}}\right)\left(\frac{e^{2(d-z)/h}}{1 + e^{2(d-z)/h}}\right), \tag{7}$$

where $N = (k_m/k_o)^2 - 1$, for low frequencies $N = 1.6 \cdot 10^4 \beta$, d is the layer thickness, h is the characteristic thickness of transition layer.

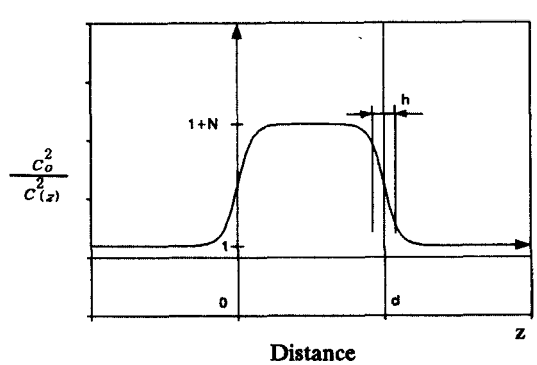

Fig.1. Typical sound speed profile in underwater bubble layer with transition sublayers around $z = 0$ and $z = d$.

c_o^2/c^2 becomes approximately one half of N at the transition sublayer thickness h around $z=0$ or $z=d$.

When $h \ll d$, the profile in the single transition sublayer can be described with that for the well-known Epstein transition sublayer, for which the sound speed profile is given by

$$\frac{c_o^2}{c^2(z)} = 1 + N\frac{e^{2z/h}}{1+e^{2z/h}} \qquad (8)$$

The Epstein's exact solutions in reference 11 give transmission and reflection of monochromatic acoustic wave incident on the single transition layer. For incident plane wave of

$$P_{z<0} = A_i e^{i(\omega t - k_o z)}, \qquad (9)$$

the transmission T_{12} and reflection R_{12} coefficients at the single transiton sublayer from medium 1 to medium 2 are given by

$$R_{12} = \frac{\Gamma(-iS)\Gamma[(iS/a)(1+\sqrt{1+N})]}{\Gamma(iS)\Gamma[(-iS/a)(1-\sqrt{1+N})]} \cdot \frac{\Gamma[1+(iS/a)(1+\sqrt{1+N})]}{\Gamma[1-(iS/a)(1-\sqrt{1+N})]}, \qquad (10)$$

$$T_{12} = 1 + R_{12}, \qquad (11)$$

where Γ is the Gamma function and the parameter $S=k_o h$ is the characteristic thickness of transition layer which is expressed in the scale of sound wavelength. If $S \ll 1$, then the asymptotic expansion of the gamma function can be used and Eq.(10) becomes

$$R_{12} = -\left(\frac{\sqrt{1+N}-1}{\sqrt{1+N}+1}\right) \cdot \left[1+2i\Upsilon S - S^2\left(2\Upsilon^2 + \frac{\pi^2}{6}\sqrt{1+N}\right)\right], \qquad (12)$$

where $\Upsilon=0.577$ is the Euler-Mascheroni constant. If S becomes very small and negligible, Eq.(12) is transformed to the well-known formula (13) describing the reflection coefficient from sharp flat layer [11]:

$$R_{12} = -\frac{\sqrt{1+N}-1}{\sqrt{1+N}+1}. \qquad (13)$$

The sound field inside the layer can be presented as a sum of two oppositely propagating plane waves the propagation constant of which is equal to k_m everywhere inside the layer when $h << d$. Applying the boundary conditions for both transition sublayers we can obtain the reflection coefficient R from the whole layer

$$R = \frac{R_{12} + R_{23}\exp(-2ik_m d)}{1+R_{12}R_{23}\exp(-2ik_m d)}. \qquad (14)$$

From the transposition relations of $R_{23}=-R_{12}$ and $T=1+R$ the reflection and transmission coefficients from the whole layer can be given by

$$R = R_{12}\frac{1-\exp(-2ik_m d)}{1-R_{12}^2\exp(-2ik_m d)}, \qquad (15)$$

$$T = \frac{(1-R_{12}^2)\exp(-ik_m d)}{1-R_{12}^2\exp(-2ik_m d)}. \qquad (16)$$

For very thin transition sublayers, *i.e.*, $S=0$ the reflection coefficient R is determined by Eq.(15) with Eq.(13) are reduced to the corresponding relations presented in reference 8 for the plane bubble layer with sharp flat boundaries.

III. Comparison between theory and experiment

The experimental measurements were carried out in a laboratory tank. The bubble layers were produced with a linear bubble generator with compressed air supply system. The thickness of bubble layers was 6 cm and the void fractions were 0.05 % and 0.1 %, respectively. The average bubble radius was about 0.3 mm that corresponds to the bubble resonance frequency about 11 kHz. Theoretical curves and experimental data were presented in Figs. 2 and 3 for the void fractions 0.05 % and 0.1 %, respectively. The lines, a and b, in Figs. 2 and 3 are the theoretical transmission coefficients for the bubble layer with the transition sublayer thicknesses of $h=0.3\,d$ and $h=0.1\,d$, respectively. The line c is the

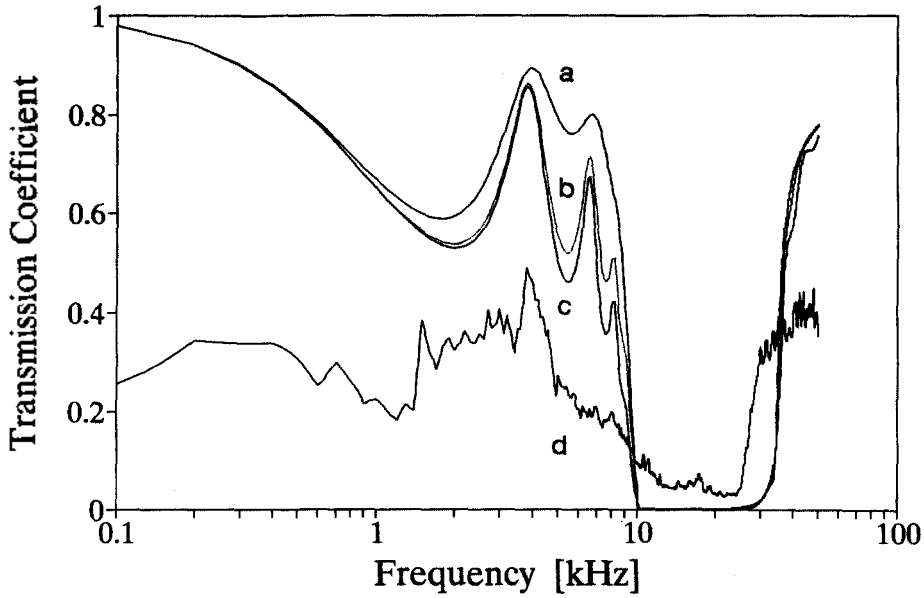

Fig.2. Comparison between the theoretical (a and b for the layer with transition sublayers with thickness of 0.3d and 0.1d, respectively, and c for sharp flat boundaries) and measured transmission coefficients (line d) for the layer with void fraction 0.05 %.

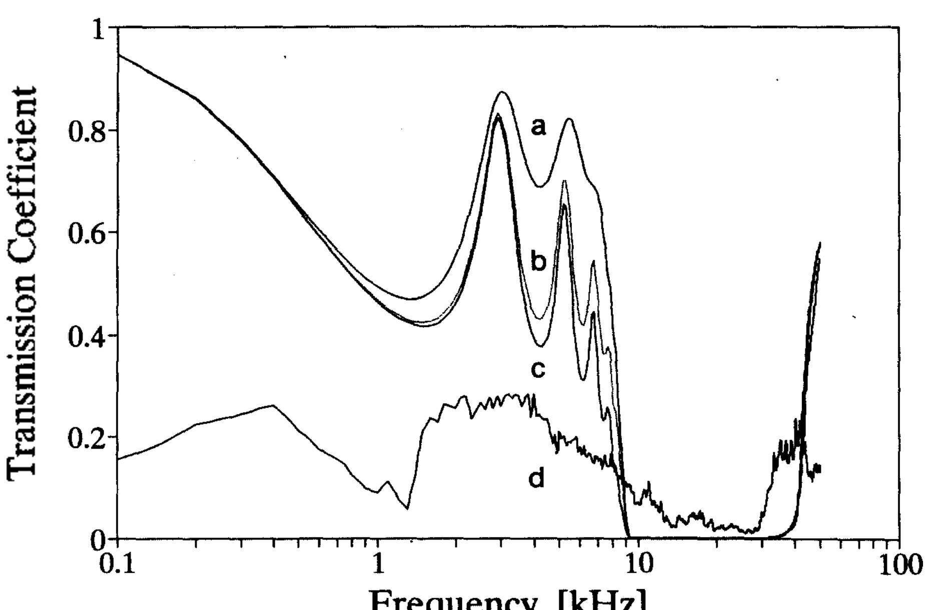

Fig.3. Comparison between the theoretical (a and b for the layer with transition sublayers with thickness of 0.3d and 0.1d, respectively, and c for sharp flat boundaries) and measured transmission coefficients (line d) for the layer with void fraction 0.1 %.

theoretical calculation for the layer with sharp flat boundaries. The line d is the measured data. One can see that the transmission coefficient for the bubble layer with transition sublayers has no such strong resonance effects while the layer with sharp boundaries has. The theoretical prediction with the transition sublayers shows better peculiar behavior of the experimental data than that without the transition sublayers.

V. Discussion

We investigated sound propagation through the bubble layers considering that with transition sublayers. Such bubble layers with transition sublayers are typical in the ocean. It was theoretically shown that the reflection and transmission coefficients depend on the thickness of transition sublayers. The resonance properties of bubble layer with rather thick subblayers are not so well presented compared with those of the bubble layers with the sharp flat boundaries. The theory developed in this work gives better agreement with the experimental data than the theory without transition sublayers and allows to consider the acoustical parameters of the ocean subsurface bubble layers. However, the case of singe bubble size distribution and normal acoustical wave incidence on the bubble layer was only considered. The results of this work can be extended to the cases of wide bubble size distributions and oblique incidence of sound waves.

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