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THE FUZZY DYNAMIC PROPERTY AND THE FUZZY DYNAMIC RESPONSE ANALYSIS OF SINGLE DEGREE OF FREEDOM SYSTEM

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ABSTRACT: In this paper, we reason about the fuzzy dynamic equation based on L-R type fuzzy namber, analyze and solve the fuzzy dynamic property and the fuzzy dynamic response of singledegree of freedom system. Specific expressions of fuzzy dynamic response are presented.

KEYWORDS: L-R type fuzzy number; L-R type fuzzy function; fuzzy dynamic equation; fuzzy dynamic property; fuzzy dynamic response.

1. Introduction

In finite element analysis for engineering structure, considering the Fuzziness of Loads, boundary conditions and some material properties, We obtained the fuzzy finit element static equilibrium equations. In [2,3], We suggested methods which established and solved the fuzzy finite element static equilibrium eguations,dealt with well fuzziness in static analysis; But to fuzzy dynamic property and fuzzy dynamic response in engineering stucturee, there isn't paper publishered up to today. In this paper, based on [2,3],make a systematic study to fuzzy dynamic property and fuzzy dynaminc response of single degree of freedom system; Reason about the fuzzy dynamic equatioin;Present an optimization method of solving fuzzy dynmic property; Give a method solving the fuzzy dynamic response, and get specific expressions with fuzzy response.

Operation rules with fuzzy numbers have been established form the zadeh's extension principle being introduced fuzzy mathmatics. The extended addition, product, subtraction, Division with L-R type fuzzy numbers see [1].In this paper we name three fuzzy function based on an unite operation fule [1], reason about their expressions.

If $\underline{M}=(m, \alpha, \beta)_{LR}$, then

(a) $\sin\underline{M}=(\sin m, \alpha, \beta)_{LR}$ (1)

(b) $\underline{M}^{1/2}=(m^{1/2}, \alpha \cdot m^{-1/2}, \beta \cdot m^{-1/2})_{LR}$ (2)

(c) $e^{\underline{M}}=(e^{m}, \alpha, \beta)_{LR}$ (3)

2. Fuzzy Dynamic Equation of Singledegree of Freedom System

Based on [1], set up fuzzy variational principles [3], from the principle of minimum potential energy, make fuzzy variation, we reason about the fuzzy finite element dynamic equations:

$$m(\ddot{a},\ddot{a}^{L},\ddot{a}^{R})_{LR}+C(\dot{a},\dot{a}^{L},\dot{a}^{R})_{LR}+(k,k^{L},k^{R})_{LR}\cdot(a,a^{L},a^{R})_{LR}=(R,R^{L},R^{R})_{LR} \quad (4)$$

In [3], reasoned about formula which a L-R type fuzzy number can be decomposed into a mean value and zerofuzzy number. So, according to properties with fuzzy numbers, We obtain:

$$m\ddot{a}+c\dot{a}+ka=R \quad (5)$$

$$m(0,\ddot{a}^{L},\ddot{a}^{R})_{LR}+C(0,\dot{a}^{L},\dot{a}^{R})_{LR}+K(0,a^{L},a^{R})+(0,k^{L},K^{R})_{LR}=(0,R^{L},R^{R})_{LR} \quad (6)$$

Usually, have following form:

$$m\underline{\ddot{a}}+c\underline{\dot{a}}+k\underline{a}=\underline{r} \quad (7)$$

in which, $\underline{a},\underline{k},\underline{R}$ are fuzzy numbers.

3. Analysis of The Fuzzy Dynamic Property

Because of fuzziness of boundary conditions,of some material property, system itself is caused to fuzziness. Neglect resistance,and let R=0, then

$$m\underline{\ddot{a}}+\underline{k}\underline{a}=0 \quad (8)$$

This is free vibration of singledegree of freedom system, Assume its vibration equation:

$$\underline{a}=\underline{x}\sin(\omega t+\theta) \quad (9)$$

in which, $\underline{x}$ is fuzzy displacement, $\underline{\omega}$ is fuzzy circum frequence, θ is initial.From(1),(2),(8),(9), we obtain:

$$\underline{k}\ \underline{x}=\underline{\lambda}\, m\underline{x} \qquad (\underline{\lambda}=\underline{\omega}^{2}) \quad (10)$$

in which, $\underline{\lambda}$, $\underline{x}$ are L-R type fuzzy numbers, and called fuzzy eigenvalue and eigenvector, respectively.Spread out (10) by L-R type fuzzy numbers. We obtain:

$$kx=\lambda mx \qquad (\lambda=\omega^{2}) \quad (11)$$

$$\mathrm{sign}(k)\cdot\mathrm{sign}(x)(0,|k|x^{L}+|x|k^{L},|k|x^{R}+|x|k^{R})_{LR}+\mathrm{sign}(k)\cdot\mathrm{sign}(x)(0,k^{L},k^{R})_{LR}\cdot(0,x^{L},x^{R})_{LR}=\mathrm{sign}(\lambda m)\cdot\mathrm{sign}(x)\cdot(0,|\lambda|mx^{L}+|x|m\lambda^{L},|\lambda|mx^{R}+|x|m\lambda^{R})_{LR}+\mathrm{sign}(\lambda m)\cdot\mathrm{sign}(x)\cdot(0,\lambda^{L}m,\lambda^{R})_{LR}\cdot(0,x^{L},x^{R})_{LR} \quad (12)$$

This is fuzzy eigenvalue and eigenvector problems based on L–R type fuzzy numbers. Of cause,(11)is a mean value equation, and a common eigenvalue and eigenvector problems. It can be solved by JACOBI method or directly.(12) is a equationbased on zero fuzzy number[2]. Let $L(x)=R(x)=\max(0,1-|x|)(p=1)$, then(12) can be transformed into the equation based on closed bounded fuzzy numbers.

$$\mathrm{sign}(k)\cdot\mathrm{sign}(x)[-(|k|x^{L}+|x|k^{L}),(|k|x^{R}+|x|k^{R})]+(1-\lambda_{o})\cdot\mathrm{sign}(k)\cdot\mathrm{sign}(x)[\min(-k^{L}x^{R}-k^{R}x^{L}),\max(k^{L}x^{L},k^{R}x^{R})]=\mathrm{sign}(\lambda m)\cdot\mathrm{sign}(x)[-|\lambda|mx^{L}+|x|\lambda^{L}m),(|\lambda m|x^{R}+|x|\lambda^{R}m)]+(1-\lambda_{o})\cdot\mathrm{sign}(\lambda m)\cdot\mathrm{sign}(x)[\min(-\lambda^{L}mx^{R},-\lambda^{R}mx^{L}),\max(\lambda^{L}mx^{L},\lambda^{R}mk^{R})] \quad (13)$$

Where $\lambda_{o}.\in(0,1)$,is a constant. (13) is a interval equation, it can be transformed into two common equations by the operation rule of interval numbers:

$$f_1(x_1,x_2,x_3,x_4)=q_1 \quad (14\text{-}1)$$

$$f_2(x_1,x_2,x_3,x_4)=q_2 \quad (14\text{-}2)$$

where $x_1=x^{L}$ $x_2=x^{R}$ $x_3=\lambda^{L}$ $x_4=\lambda^{R}$, and $x_i>0$, $i=1,\cdots,4$,

$$\text{Find } X=(x_1,x_2,x_3,x_4)^{T} \quad (15\text{-}1)$$

$$\min F(X)=\{(f_1-q_1)^2+(f_2-q_2)^2\} \quad (15\text{-}2)$$

$$\text{S.t. } f_j(x)+\delta_j>q_j \quad j=1,2 \quad (15\text{-}3)$$

$$x_j> \quad i=1,\cdots,4$$

Where δ_j, is a non–negative parameter.

4. Analysis and Solving of Fuzzy Responses

Form [2] seeing, Supposing a binary L–R type fuzzy function $\underline{f}(x)=(f(x), s(x),t(x))_{LR}$, the expression of integiration of L–R type fuzzy function is :

$$\underline{I}(a,b)=(\int_a^b f(x)dx,\ \int_a^b S(x)dx,\ \int_a^b t(x)dx)_{LR} \quad (16)$$

In this paper,we define a L–R type fuzzy periodic function. Definitionl: a L–R type fuzzy fuction $f(x)$ is L–R type fuzzy periodic function if there exist a real number T, with

$$L\left(\frac{f(x+T)-y}{s(x+T)}\right)=L\left(\frac{f(x)-y}{s(x)}\right) \text{ for } y<\min(f(x),f(x+T))$$

$$R\left(\frac{y-f(x+T)}{t(x+T)}\right)=R\left(\frac{y-f(x)}{t(x)}\right) \text{ for } y>\max(f(x),f(x+T))$$

As algebraic operators with L–R type fuzzy numbers are closed, by the extensiion principle, we yield fourier expansion with $f(x)$.

$$\underline{f}(x)=\frac{\underline{A}_O}{2}+\sum_{i=1}^{n}[\underline{A}_j\cos(j\omega t)+\underline{B}_j\sin(j\omega t)]_{LR} \quad (17)$$

Where

$$\underline{A}_O=[\frac{1}{T}\int_a^b f(t)dt,\ \frac{1}{T}\int_a^b s(t)dt,\ \frac{1}{T}\int_a^b t(t)dt, \quad (18)$$

$$\underline{A}_j=[\frac{2}{T}\int_a^b f(t)\cos(j\omega t)dt,\ \frac{2}{T}\int_a^b s(t)\cos(j\omega t)dt,\ \frac{2}{T}\int_a^b t(t)\cos(j\omega t)dt]_{LR} \quad (19)$$

$$\underline{B}_j=[\frac{2}{T}\int_a^b f(t)\sin(j\omega t)dt,\ \frac{2}{T}\int_a^b s(t)\sin(j\omega t)dt,\ \frac{2}{T}\int_a^b t(t)\sin(j\omega t)dt]_{LR} \quad (20)$$

4.1 Fuzzy Response of Fuzzy System Under Deterministic Loads

Assume that a Binary Force Concerning time acts on singledegree of reedom system, from [5] seeing, this system causes the response.

$$X(t)=\frac{1}{m\overline{\omega}_a}\int_0^t f(t)\cdot e^{-h\overline{\omega}(t-\tau)}\sin(\overline{\omega}_a(t-\tau))]dt \quad (21)$$

Where

$$\overline{\omega}^2=\frac{k}{m},\ \frac{c}{m}=2h\overline{\omega},\ \overline{\omega}_a=\sqrt{1-h^2}\,\underline{\overline{\omega}}$$

Consider the fuzziness of system, W_a is fuzzy number, so the response is also fuzzy set.

By the extension principle, obtain the fuzzy response of fuzzy system. Let $\underline{\overline{\omega}}=(\overline{\omega},\overline{\omega}^{L},\overline{\omega}^{R})_{LR}$, then

$$x(t)=\Big(\frac{\int_0^t f(\tau)e^{-h(t-\tau)\overline{\omega}}\cdot\sin(\sqrt{1-h^2}\,\overline{\omega})dz}{m\sqrt{1-h^2\omega}},$$

$$\frac{\int_0^t f(z)e^{-h(t-\tau)\overline{\omega}}\cdot\sin(\sqrt{1-h^2}\,\overline{\omega})dt\cdot\overline{\omega}^{R}\ \overline{\omega}^{R}}{m\sqrt{1-h^2}\cdot\overline{\omega}^2}$$

$$+\frac{\int_0^t[e^{-h(t-\tau)\overline{\omega}}\cdot\sqrt{1-h^2}\,\overline{\omega}^{L}+e^{-h(t-\tau)\overline{\omega}^{L}}\sin(\sqrt{1-h^2}\,\overline{\omega})]dt}{m^2\overline{\omega}\sqrt{(1-h^2)}},$$

$$\frac{\int_0^t f(z)e^{-h(t-\tau)\overline{\omega}}\cdot\sin(\sqrt{1-h^2}\,\overline{\omega})dt\cdot\overline{\omega}^{L}}{m\sqrt{(1-h^2)}\cdot\overline{\omega}^2}+$$

$$\frac{\int_0^t(e^{-h(t-\tau)\overline{\omega}}\cdot\sqrt{1-h^2}\,\overline{\omega}^{R}+e^{-h(t-\tau)\overline{\omega}^{R}})\cdot\sin\sqrt{1-h^2}\,\overline{\omega})]dt}{m\sqrt{(1-h^2)}\cdot\overline{\omega}}\Big)_{LR} \quad (22)$$

4.2 Fuzzy Response of Deterministic System Under Fuzzy Loads.

4.2.1 Load $\underline{f(t)}$ Being Periodic Fuzzy Function

Because $\underline{f(t)}$ is periodic function,from [4],We have:

$$x(t)=\sum_{j=1}^{n}\frac{[a_j\cos(j\omega t-\psi_j)+b_j\sin(j\omega t-\psi_j)]}{k[(1-\lambda_j)^2+(2\zeta\lambda_j)^2]^{1/2}} \quad (23)$$

When $\underline{f(t)}$ is L–R type fuzzy periodic function, by the extension principle, we obtain:

$$x(t)=\sum_{j=1}^{n}\frac{[\underline{A}_j\cos(j\omega t-\psi_j)+\underline{B}_j\sin(j\omega t-\psi_j)]}{k[(1-\lambda_j)^2+(2\zeta\lambda_j)^2]^{1/2}} \quad (24)$$

Where $\underline{A}_j$, $\underline{B}_j$ are (19),(20), respectively.

4.2.2 Load f(t)is Fuzzy Non-periodic Function

From 4.1, the fuzzy response of the system can be expressed:

$$\underline{x}(t)=(\frac{1}{m\overline{\omega}_d}\int \delta f(\tau)\exp[-h\overline{\omega}(t-\tau)]\sin[\overline{\omega}_a(t-\tau)]dt,$$

$$\frac{1}{m\overline{\omega}_d}\int \delta s(\tau)\exp[-h\overline{\omega}(t-\tau)]\sin[\overline{\omega}_a(t-\tau)]dt,$$

$$\frac{1}{m\overline{\omega}_d}\int \delta t(\tau)\exp[-h\overline{\omega}(t-\tau)]\sin[\overline{\omega}_a(t-\tau)]dt)_{LR} \quad (25)$$

4.3 Fuzzy Response of Fuzzy system Under Fuzzy Loads

As fuzzy loads act on fuzzy system, the fuzzy response can be expressed as following form:

$$\underline{X}(t)=(\int \delta g_0(t,\tau)d_\tau,\ \int \delta g_1(t,\tau)d_{d\tau},\ \int \delta g_2(t,\tau)d_\tau)_{LR} \quad (26)$$

Where g0(t, τ), g1(τ), g2(t, τ) are the mean values, the left and right spreads of fuzzy function g(t, τ), respectively.

Whereas

$$g(t,z)=\frac{1}{m\overline{\omega}_a}\underline{f}(t)\exp[-\zeta\,\overline{\omega}(t-\tau)]\sin(\underline{\overline{\omega}}_a p(t-\tau)) \quad (27)$$

Let $\overline{\omega}=(\overline{\omega},\overline{\omega}^L,\overline{\omega}^R)_{LR}$, $f(\tau)=(f(\tau),s(\tau),t(\tau))^{LR}$, then

$$g(t,z)=\frac{P(t,\tau)}{(m\sqrt{1-h^2}\,\overline{\omega},m\sqrt{1-h^{2p}}\,\overline{\omega}^L,m\sqrt{1-h^2}\,\overline{\omega}^R)_{LR}} \quad (28)$$

Wher $\underline{P}(t,\tau)=\underline{f}(\tau)\cdot\exp[-\zeta\,\underline{\overline{\omega}}(t-\tau)]\cdot\sin(\overline{\omega}_a(t-\tau)]$ (29)

$$\underline{P}(t,\tau)=f(\tau)\exp\{[-\zeta(t-\tau)]\overline{\omega}\}\cdot\sin[\sqrt{1-h^2}\,\overline{\omega}(t-\tau)+f(\tau)\cdot\exp[-\zeta(t-\tau)\overline{\omega}]\cdot[0,\sqrt{1-h^2}(t-\tau)\overline{\omega}^L,\sqrt{1-h^2}(t-\tau)\overline{\omega}^R)_{LR}+f(\tau)\cdot\sin(\sqrt{1-h^2}\,\overline{\omega}(t-\tau)]\{0,-\zeta(t-\tau)\overline{\omega}^L\}_{LR}+\exp[-\zeta(t-\tau)\overline{\omega}]\cdot\sin(\sqrt{1-h^2}\,\overline{\omega}(t-\tau)]\cdot(0,s(\tau),t(\tau))_{LR} \quad (30)$$

fuzzy response can be expressed as following From(30),We obtain the mean Value of P(t, τ), the left and right spreads of P(t, τ),So we can obtain g0(t, τ),g1(t, τ),g2(t, τ), based on the extended Division with two L-R type numbers.

5. Conclusion

In this paper, we have proposed some ideas, methods dealing with kinds of fuzziness in dynamic analysis of structure, extend dynamic analysis of structure from deterministic territiory to fuzzy territiory. Because of cmplication of problem, in this paper, We only consider the case of singledgree of freedom system, We will publish other paper about maltidegree of freedm system.

References

[1] Dubois,D.,Prade, H.,Fuzzy sets and systems: Theory and Applications. New York. 1980.
[2] Caihua Wang, Hengshan Zhu, A method of solving the fuzzy equilibrium equation based on the L-R type fuzzy number, IEEE. International conference on fuzzy system, San Diego, California March 8-12, 1992.
[3] Caihua Wang,Wei He, Establishment of the fuzzy equlibrium equation based o n L-R type fuzzy number, Master's thesis of chongqing University, 1992.
[4] Dayishen Yan,Vibration Theory,Japan, 1980