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# Trajectory Generation for Contour Control of Mechatronics Servo Systems Subjected to Torque Constraints

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## Abstract

The actuator saturation defects the countour control performance of mechatronics servo systems. In this paper, trajectory generation of contour control of the mechatronics servo system is developed taking into account of the constraint of the torque in the system. By using the generated trajectory, the torque constraint and assigned working accuracy are satisfied and the accurate contour control performance is achieved.

## 1. Introduction

Many mechatronics servo systems such as industrial robot arms and NC (numerical control) machines are operating in actual production lines and assembly lines, etc. Contour control of mechatronics servo systems is implemented in wide fields such as grinding and cutting. Both, the performance at high speed and high accuracy are required for the industrial mechatronics servo systems. We have already experienced that the following trajectory could coincide with an objective trajectory only at low speed operations. The following locus deviates from the objective locus at high speed operations. The torque saturation of the servo motors of the actuators is a big problem because it causes the deterioration of contour control performance. The actuator saturation problem was investigated[1, 2, 3]. These controllers are almost feedback type and require changes of control strategy and/or hardware of the servo systems. In industrial applications, these changes are not so easily acceptable. On the other hand, the trajectory generation for an interactive robot operation is proposed[4], but the trajectory generation taking into account of the constraint of the torque in the system has not been investigated.

In this paper, a method of trajectory generation within the torque constraints for contour control of the mechatronics servo systems is proposed. The generated trajectory is within the torque constraints of the servo motors and it is satisfied with the assigned working accuracy. By using the generated trajectory and the inverse dynamics compensation, the accurate contour control performance is achieved.

## 2. Problem Statement

The main objective of the contour control of mechatronics servo systems is coincidence of the following locus with the objective locus. In industrial applications, the objective locus is given beforehand and the tangent velocity $V$ of the objective trajectory is a constant. We introduce assumptions for the contour control of the mechatronics servo system as follows;

1. The objective locus is approximated by the combination of lines and circles.
2. Working accuracy ($\epsilon$), which means the allowable error between the objective locus and the following locus, is given.
3. The maximum acceleration ($\alpha_{max}$) within the torque constraints of the servo motors is known.

Under the above assumptions, we develop a method of the trajectory generation so that the generated trajectory is within the torque constraints. It is satisfied with the working accuracy and the tangent velocity of the generated trajectory keeps the tangent velocity $V$ as long as possible. The contour control of the two dimensional mechatronics servo system (XY table) is considered. The servo motors of the actuators are controlled, independently and the interference term of the two axis could be neglected. The block diagram contained the torque saturation for one axis of the XY table is shown in Fig. 1. The dynamics of the system for one axis is described as

$$\ddot{y}(t) = \mathrm{sat}(K_v(K_p(u(t) - y(t)) - \dot{y}(t))) \tag{1}$$

where

$$\mathrm{sat}(x) = \begin{cases} \alpha_{max} & (\alpha_{max} < x) \\ x & (-\alpha_{max} \le x \le \alpha_{max}) \\ -\alpha_{max} & (x < -\alpha_{max}) \end{cases} \tag{2}$$

and $K_p$ and $K_v$ means the position loop gain and the velocity loop gain, respectively. Within the torque constraint, the dynamics of the servo system is described by

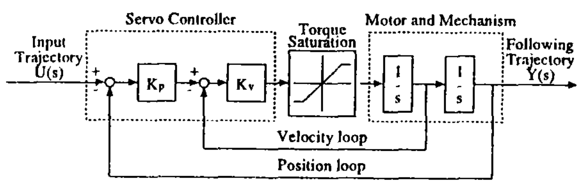


Figure 1 Block diagram of the mechatronics servo system contained with the torque saturation

the second order model in the frequency domain as

$$Y(s) = \frac{K_p K_v}{s^2 + K_v s + K_p K_v} U(s). \tag{3}$$

We generate the trajectory so that the torque saturation does not occur, i.e., the dynamics is described as equation (3).

## 3. Contour Control with Torque Saturation

### 3.1 Concept for Contour Control with Torque Saturation

The trajectory of the mechatronics servo systems is generated so that the mechatronics servo systems are operated within the torque constraint. Figure 2 shows the structure of the contour control of the mechatronics servo system. We generate the trajectory $(w_x(t), w_y(t))$ for the mechatronics servo system within the torque constraint and the error of the generated locus $(w_x, w_y)$ from the objective locus $(r_x, r_y)$ is satisfied with the working accuracy. The tangent velocity of the generated trajectory $(w_x(t), w_y(t))$ keeps the tangent velocity $V$ as long as possible. The input of the servo system $(u_x(t), u_y(t))$ is derived from the modification of the generated trajectory $(w_x(t), w_y(t))$ by using the inverse dynamics of the mechatronics servo system. Then, the following trajectory of the mechatronics servo system $(x(t), y(t))$ could coincide with the generated trajectory $(w_x(t), w_y(t))$ and the following locus $(x, y)$ is satisfied with the assigned working accuracy.

### 3.2 Trajectory Generation within the Torque Constraint

The trajectory within the torque constraint is generated as following procedure (see Fig. 2).

1. When the objective locus has corners, the corners are approximated by circles so that the approximation error is within the working accuracy. Thus, the generated locus $(w_x, w_y)$, which is satisfied with the working accuracy, is derived.
2. A radius of the circle, which is contained in the approximated locus, is set to $r$ and the minimum radius $r_{min}(= V^2/\alpha_{max})$ within the torque constraint is calculated by using the maximum acceleration $\alpha_{max}$ and the tangent velocity $V$. Here, the maximum magnitude of the acceleration vector $(\ddot{w}_x(t), \ddot{w}_y(t))$ is set to $\alpha_{max}$ instead of the acceleration of each axis. It leads us an easy derivation of the trajectory generation and the acceleration of the generated trajectory is satisfied with the torque constraint.
   (a) $r \geq r_{min}$: The trajectory $(w_x(t), w_y(t))$ is generated so that the tangent velocity is fixed at $V$.
   (b) $r < r_{min}$: The tangent velocity is decelerated from $V$ to $V_m$ ($V_m = \sqrt{\alpha_{max} r}$: the tangent velocity when the acceleration of the circle $r$ is $\alpha_{max}$) by the maximum deceleration $-\alpha_{max}$. Then, the trajectory draws the circle and the tangent velocity is accelerated form $V_m$ to $V$ by the maximum acceleration $\alpha_{max}$.

According to the above mentioned procedure, the trajectory $(w_x(t), w_y(t))$ is generated within the torque constraint and the locus $(w_x, w_y)$ could coincide with the objective locus $(r_x, r_y)$ within the working accuracy $(\epsilon)$.

We derive the trajectory when the objective locus is two lines and one corner as shown in Fig. 3. The radius of the circle, which is satisfied with the working accuracy, is calculated as $r = \epsilon\cos(\theta_2 - \theta_1)/(1 - \cos(\theta_2 - \theta_1))$ by a geometric relationship. The position, velocity and acceleration of the generated trajectory are derived as

$$w_x(t) = \begin{cases} V t \cos\theta_1 & (t \leq t_1) \\ w_x(t_1) + \left(V(t - t_1) - \dfrac{\alpha_{max}(t - t_1)^2}{2}\right)\cos\theta_1 & (t_1 < t \leq t_2) \\ w_x(t_2) + r\left(\sin\left(\theta_1 + \dfrac{V_m(t - t_2)}{r}\right) - \sin\theta_1\right) & (t_2 < t \leq t_3) \\ w_x(t_3) + \left(V(t - t_3) + \dfrac{\alpha_{max}(t - t_3)^2}{2}\right)\cos\theta_2 & (t_3 < t \leq t_4) \\ w_x(t_4) + V t\cos\theta_2 & (t_4 < t) \end{cases} \tag{4a}$$

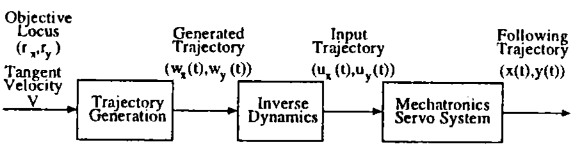


Figure 2 Structure of the contour control of the mechatronics servo system contained with the torque saturation

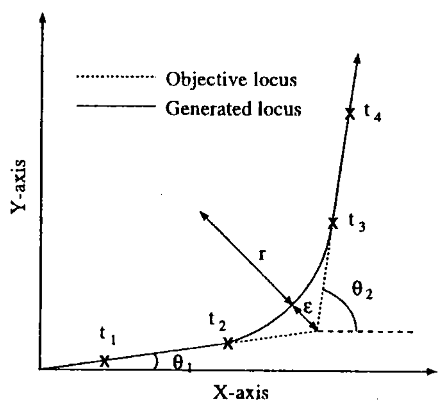


Figure 3 Trajectory generation when the objective locus is two lines and one corner

$$w_y(t) = \begin{cases} Vt\sin\theta_1 & (t \le t_1) \\ w_y(t_1) + \left(V(t-t_1) - \frac{\alpha_{max}(t-t_1)^2}{2}\right)\sin\theta_1 & (t_1 < t \le t_2) \\ w_y(t_2) + r\left(\cos\left(\theta_1 + \frac{V_m(t-t_2)}{r}\right) - \cos\theta_1\right) & (t_2 < t \le t_3) \\ w_y(t_3) + \left(V(t-t_3) + \frac{\alpha_{max}(t-t_3)^2}{2}\right)\sin\theta_2 & (t_3 < t \le t_4) \\ w_y(t_4) + Vt\sin\theta_2 & (t_4 < t) \end{cases} \tag{4b}$$

$$\dot{w}_x(t) = \begin{cases} V\cos\theta_1 & (t \le t_1) \\ (V - \alpha_{max}(t-t_1))\cos\theta_1 & (t_1 < t \le t_2) \\ V_m\cos\left(\theta_1 + \frac{V_m(t-t_2)}{r}\right) & (t_2 < t \le t_3) \\ (V + \alpha_{max}(t-t_3))\cos\theta_2 & (t_3 < t \le t_4) \\ V\cos\theta_2 & (t_4 < t) \end{cases} \tag{5a}$$

$$\dot{w}_y(t) = \begin{cases} V\sin\theta_1 & (t \le t_1) \\ (V - \alpha_{max}(t-t_1))\sin\theta_1 & (t_1 < t \le t_2) \\ V_m\sin\left(\theta_1 + \frac{V_m(t-t_2)}{r}\right) & (t_2 < t \le t_3) \\ (V + \alpha_{max}(t-t_3))\sin\theta_2 & (t_3 < t \le t_4) \\ V\sin\theta_2 & (t_4 < t) \end{cases} \tag{5b}$$

$$\ddot{w}_x(t) = \begin{cases} 0 & (t \le t_1) \\ -\alpha_{max}\cos\theta_1 & (t_1 < t \le t_2) \\ -\alpha_{max}\sin\left(\theta_1 + \frac{V_m(t-t_2)}{r}\right) & (t_2 < t \le t_3) \\ \alpha_{max}\cos\theta_2 & (t_3 < t \le t_4) \\ 0 & (t_4 < t) \end{cases} \tag{6a}$$

$$\ddot{w}_y(t) = \begin{cases} 0 & (t \le t_1) \\ -\alpha_{max}\sin\theta_1 & (t_1 < t \le t_2) \\ \alpha_{max}\cos\left(\theta_1 + \frac{V_m(t-t_2)}{r}\right) & (t_2 < t \le t_3) \\ \alpha_{max}\sin\theta_2 & (t_3 < t \le t_4) \\ 0 & (t_4 < t) \end{cases} \tag{6b}$$

where the deceleration and the acceleration time interval is $t_4 - t_3 = t_2 - t_1 = (V - V_m)/\alpha_{max}$ and the time interval of the circle drawing is calculated as $t_3 - t_2 = r(\theta_2 - \theta_1)/V_m$. The acceleration of the generated trajectory (6) is within the torque constraint and the generated locus $(w_x, w_y)$ is satisfied with the working accuracy as shown in Fig. 3.

### 3.3 Compensation Method by Using the Inverse Dynamics

The following trajectory $(x(t), y(t))$ of the mechatronics servo system follows the generated trajectory $(w_x(t), w_y(t))$ by using the inverse dynamics compensation. The generated trajectory is 2-times differentiable and the acceleration is within the saturation. Hence, the compensation of the inverse dynamics can be used. The inverse dynamics of the system (3) within the torque constraint is derived as

$$F(s) = \frac{s^2 + K_v s + K_p K_v}{K_p K_v}. \tag{7}$$

The input trajectory $(u_x(t), u_y(t))$ is calculated from the generated trajectory $(w_x(t), w_y(t))$ by modifying the inverse dynamics (7) as

$$u_x(t) = w_x(t) + \frac{1}{K_p}\dot{w}_x(t) + \frac{1}{K_p K_v}\ddot{w}_x(t) \tag{8a}$$

$$u_y(t) = w_y(t) + \frac{1}{K_p}\dot{w}_y(t) + \frac{1}{K_p K_v}\ddot{w}_y(t). \tag{8b}$$

By using the input trajectory $(u_x(t), u_y(t))$ for the input of the servo system, the following trajectory $(x(t), y(t))$ could coincide with the generated trajectory $(w_x(t), w_y(t))$

## 4. Examples

We derive the trajectory when the objective locus is the orthogonal two lines of the length $L$ for each axis as shown in Fig. 4. The objective trajectory is given as

$$\dot{r}_x(t) = \begin{cases} V & (0 \le t \le T) \\ 0 & (T < t \le 2T) \end{cases} \tag{9a}$$

$$\dot{r}_y(t) = \begin{cases} 0 & (0 \le t \le T) \\ V & (T < t \le 2T) \end{cases} \tag{9b}$$

where $T = L/V$.

**Case 1:** $r < r_{min}$

The first example is the general case, i.e., the corner is approximated by the circle and the tangent velocity is

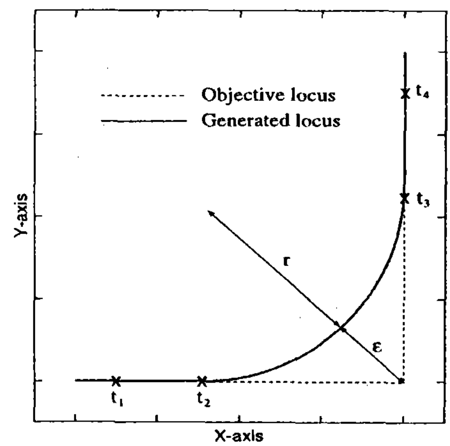


Figure 4 Generated objective locus for orthogonal two lines

decelerated. The generated trajectory is derived as

$$w_x(t) = \begin{cases} Vt & (0 \le t \le t_1) \\ Vt - \dfrac{\alpha_{max}(t-t_1)^2}{2} & (t_1 < t \le t_2) \\ w_x(t_2) + r\sin\left(\dfrac{V_m(t-t_2)}{r}\right) & (t_2 < t \le t_3) \\ L & (t_3 < t \le t_5) \end{cases} \quad (10a)$$

$$w_y(t) = \begin{cases} 0 & (0 \le t \le t_2) \\ r\left(1-\cos\left(\dfrac{V_m(t-t_2)}{r}\right)\right) & (t_2 < t \le t_3) \\ r + V_m(t-t_3) + \dfrac{\alpha_{max}(t-t_3)^2}{2} & (t_3 < t \le t_4) \\ w_y(t_4) + V(t-t_4) & (t_4 < t \le t_5) \end{cases} \quad (10b)$$

$$\dot{w}_x(t) = \begin{cases} V & (0 \le t \le t_1) \\ V - \alpha_{max}(t-t_1) & (t_1 < t \le t_2) \\ V_m\cos\left(\dfrac{V_m(t-t_2)}{r}\right) & (t_2 < t \le t_3) \\ 0 & (t_3 < t \le t_5) \end{cases} \quad (11a)$$

$$\dot{w}_y(t) = \begin{cases} 0 & (0 \le t \le t_2) \\ V_m\sin\left(\dfrac{V_m(t-t_2)}{r}\right) & (t_2 < t \le t_3) \\ V_m + \alpha_{max}(t-t_3) & (t_3 < t \le t_4) \\ V & (t_4 < t \le t_5) \end{cases} \quad (11b)$$

$$\ddot{w}_x(t) = \begin{cases} 0 & (0 \le t \le t_1) \\ -\alpha_{max} & (t_1 < t \le t_2) \\ -\alpha_{max}\sin\left(\dfrac{V_m(t-t_2)}{r}\right) & (t_2 < t \le t_3) \\ 0 & (t_3 < t \le t_5) \end{cases} \quad (12a)$$

$$\ddot{w}_y(t) = \begin{cases} 0 & (0 \le t \le t_2) \\ \alpha_{max}\cos\left(\dfrac{V_m(t-t_2)}{r}\right) & (t_2 < t \le t_3) \\ \alpha_{max} & (t_3 < t \le t_4) \\ 0 & (t_4 < t \le t_5) \end{cases} \quad (12b)$$

where $t_1, \cdots, t_5$ are derived as $t_1 = (L - r - (V^2 - V_m^2)/2\alpha_{max})/V$, $t_2 = t_1 + (V - V_m)/\alpha_{max}$, $t_3 = t_2 + \pi r/2V_m$, $t_4 = t_3 + (V - V_m)/\alpha_{max}$, $t_5 = t_1 + t_4$. The generated trajectory is shown in Fig. 5

**Case 2:** $r \ge r_{min}$

The second example is that the working accuracy is not a strict condition and the tangent velocity of the generated trajectory is fixed at the tangent velocity $V$. The velocity of the generated trajectory is derived as

$$w_x(t) = \begin{cases} Vt & (0 \le t \le t_1) \\ Vt_1 + r\sin\left(\dfrac{V(t-t_1)}{r}\right) & (t_1 < t \le t_2) \\ L & (t_2 < t \le t_3) \end{cases} \quad (13a)$$

$$w_y(t) = \begin{cases} 0 & (0 \le t \le t_1) \\ r\left(1-\cos\left(\dfrac{V(t-t_1)}{r}\right)\right) & (t_1 < t \le t_2) \\ r + V(t-t_2) & (t_2 < t \le t_3) \end{cases} \quad (13b)$$

$$\dot{w}_x(t) = \begin{cases} V & (0 \le t \le t_1) \\ V\cos\left(\dfrac{V(t-t_1)}{r}\right) & (t_1 < t \le t_2) \\ 0 & (t_2 < t \le t_3) \end{cases} \quad (14a)$$

$$\dot{w}_y(t) = \begin{cases} 0 & (0 \le t \le t_1) \\ V\sin\left(\dfrac{V(t-t_1)}{r}\right) & (t_1 < t \le t_2) \\ V & (t_2 < t \le t_3) \end{cases} \quad (14b)$$

$$\ddot{w}_x(t) = \begin{cases} 0 & (0 \le t \le t_1) \\ -\alpha_{max}\sin\left(\dfrac{V(t-t_1)}{r}\right) & (t_1 < t \le t_2) \\ 0 & (t_2 < t \le t_3) \end{cases} \quad (15a)$$

$$\ddot{w}_y(t) = \begin{cases} 0 & (0 \le t \le t_2) \\ \alpha_{max}\cos\left(\dfrac{V(t-t_1)}{r}\right) & (t_1 < t \le t_2) \\ 0 & (t_2 < t \le t_3) \end{cases} \quad (15b)$$

where $t_1$, $t_2$ and $t_3$ are derived as $t_1 = (L - r)/V$, $t_2 = t_1 + \pi r/2V$, $t_3 = t_1 + t_2$.

**Case 3:** $\epsilon = 0$

The last example is the most accurate, i.e., the objective locus and the generated locus is identical. The trajectory is stopped at the corner. The trajectory is derived as

$$w_x(t) = \begin{cases} Vt & (0 \le t \le t_1) \\ Vt - \dfrac{\alpha_{max}(t-t_1)^2}{2} & (t_1 < t \le t_2) \\ L & (t_2 < t \le t_4) \end{cases} \quad (16a)$$

$$w_y(t) = \begin{cases} 0 & (0 \le t \le t_2) \\ \dfrac{\alpha_{max}(t-t_2)^2}{2} & (t_2 < t \le t_3) \\ V(t-t_3) + \dfrac{V^2}{2\alpha_{max}} & (t_3 < t \le t_4) \end{cases} \quad (16b)$$

$$\dot{w}_x(t) = \begin{cases} V & (0 \le t \le t_1) \\ V - \alpha_{max}(t-t_1) & (t_1 < t \le t_2) \\ 0 & (t_2 < t \le t_4) \end{cases} \quad (17a)$$

$$\dot{w}_y(t) = \begin{cases} 0 & (0 \le t \le t_2) \\ \alpha_{max}(t-t_2) & (t_2 < t \le t_3) \\ V & (t_3 < t \le t_4) \end{cases} \quad (17b)$$

$$\ddot{w}_x(t) = \begin{cases} 0 & (0 \le t \le t_1) \\ -\alpha_{max} & (t_1 < t \le t_2) \\ 0 & (t_2 < t \le t_4) \end{cases} \quad (18a)$$

$$\ddot{w}_y(t) = \begin{cases} 0 & (0 \le t \le t_2) \\ a_{max} & (t_2 < t \le t_3) \\ 0 & (t_3 < t \le t_4) \end{cases} \quad (18b)$$

where $t_1, \cdots, t_4$ are derived so that the length of the generated trajectory is $L$ as $t_1 = (L - V^2/2\alpha_{max})/V$, $t_2 = t_1 + V/\alpha_{max}$, $t_3 = t_2 + V/\alpha_{max}$, $t_4 = t_1 + t_3$.

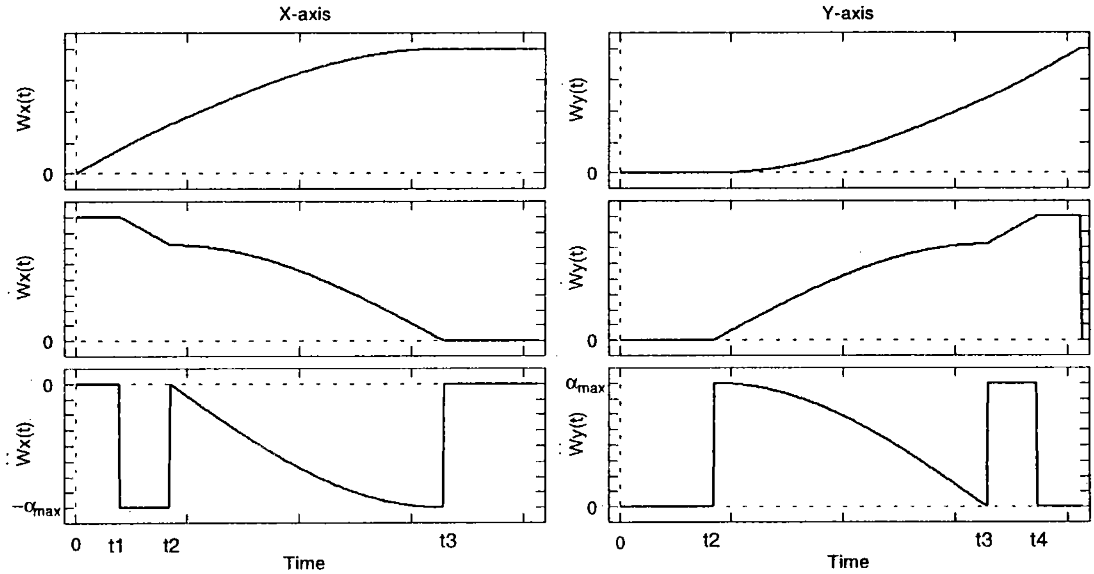


Figure 5 Genarated objective trajectory for orthogonal two lines (Case 1)

The generated trajectory $(w_x(t), w_y(t))$ is modified to the input trajectory $(u_x(t), u_y(t))$ by using equation (8). When the input trajectory $(u_x(t), u_y(t))$ is used, the following trajectory $(x(t), y(t))$ is identical to the generated trajectory $(w_x(t), w_y(t))$.

## 5. Conclusion

A method of trajectory generation for contour control of mechatronics servo systems was proposed. The method generated the trajectory within the torque constraint and it was satisfied with the assigned working accuracy. According to the inverse dynamics compensation, the following trajectory of the servo systems could coincide with the generated trajectory.

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