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Fuzzy Modeling Using HPC-MEANS Algorithm and Genetic Algorithm

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Abstract

In this paper, we suggest new fuzzy modeling algorithm, which can be easily implemented, by combining HPC-MEANS Algorithm and Genetic Algorithm. HPC-MEANS is used to cluster the sample data in input-output space with hyper planes and to make structure identification roughly and Genetic Algorithm is used to tune the premise and consequent parameters. For the validity of suggested method, we model the system with I/O data from known system, and then compare two systems.

1. Introduction

Fuzzy model has the advantage that it is superior to conventional linear model in describing a system. Therefore various algorithms were developed[1][2][3][4]. As seen in [5], Takagi Sugeno's model is the best in descriptions of a system, and has the advantage that, since its consequence is linear equation, mathematical analysis is possible. But, this Takagi Sugeno's model algorithm is complex, and its realization is difficult and needs much time. In order to compensate for this shortness, this paper suggests the easier algorithm consisting clustering method based hyper plane and Genetic Algorithm searching optimal solution.
In order to verify the suggested algorithm, we construct fuzzy model using a known system's I/O data and then compare with other algorithms in terms of performance.

2. Fuzzy Modeling

This paper uses the fuzzy model suggested by Takagi and Sugeno[1]. Each fuzzy rule's type is as follows:

$$L^i: \text{ If } x_1 \text{ is } A_1^i,\ x_2 \text{ is } A_2^i,\ \cdots,\ x_m \text{ is } A_m^i \quad \text{then } y^i = c_0^i + c_1^i x_1 + c_2^i x_2 + \cdots + c_m^i x_m \tag{2-1}$$

$L^i (i=1,2,\cdots,n)$: i-th rule

$x_j (j=1,2,\cdots,m)$: j-th input.

y^i : i-th output

A_l^i : fuzzy variable.

Given $x_j^o (j=1,2,\cdots,n)$, y is as follows.

$$y = \frac{\sum_{i=1}^{n} \omega^i y^i}{\sum_{i=1}^{n} \omega^i},\quad \omega^i = \prod_{j=1}^{m} A_j^i (x_j^o) \tag{2-2}$$

From eq. 2-1 and 2-2, we know that Takagi and SugenoÆs model is acquired by dividing the input space into several regions according to piecewise linear equation.
But, we can find the shortness of the fuzzy modeling algorithm by the following remark.

Remark 2-1
The defuzzified output of the fuzzy model ,eq. 2-2, represents the small change of a system parameter as premise parameter change, $\Delta\omega_i$, and the large change of a system parameter as consequent parameter change, Δa_i^j.

Remark 2-1 means that, in the case of the same rate of change, the change of consequent parameter causes more change of output. This remark implies that, since the fuzzy model algorithm, in spite of the importance of consequent parameter, depends on premise parameter, it is inefficient. The following section suggests the algorithm getting over this inefficiency.

3. Suggested Algorithm

This algorithm consists of structure identification and parameter adjustment.

3-1. Structure Identification

The total structure of system is identified by not premise parameter but consequent parameter. Each implication of the fuzzy model is obtained by clustering input-output space. In another paper, C-MEANS algorithm is used[2]. But, since C-MEANS clustering makes spherical surfaces, it is difficult to apply to the piece-wise linear system. As a result, HPC-MEANS that makes cluster not spherical surface but hyper plane, is used[5]. HPC-MEANS algorithm gets some fitting planes from the given data, and then clusters them using it as the center of each cluster. Each clusters represent fuzzy-implications respectively.

By the way, since the structure identified in this way is rough, it is need to adjust parameters. Hence GA is used for tuning them.

The features of HPC-MEANS algorithm are the next follows.

(1) The conventional C-MEANS algorithm forms a cluster which is centralized to one point.

But HPC-MEANS algorithm makes a cluster which is centralized to hyper plane.

(2) The C-MEANS algorithm defines the new center which is found by the center of gravity.

However HPC-MEANS algorithm defines the new hyper plane that is found by the RLS(Recursive Least Square).

If the dimension of input space k, the center of ith cluster is the hyper plane and is expressed by eq. (3-1),(3-2)

$$y^i = p_0^i + p_1^i x_1 + p_2^i x_2 + \cdots + p_k^i x_k \quad (3\text{-}1)$$

$$y^i = X^T \cdot p^i \quad (3\text{-}2)$$

where $X = [\ 1\ x_1\ x_2\ x_3 \cdots x_k]^T$

$p^i = [\ p_0^i\ p_1^i\ p_2^i \cdots p_k^i]^T$

The hyper plane is obtained by following recursive eq. (3-3), (3-4), (3-5).

$$P_{j+1}^i = P_j^i + K_j[\ Y_{j+1} - x_{j+1}^T P_j^i] \quad (3\text{-}3)$$

$$K_j = S_{j+1} X_{j+1}(j+1) = S_j \frac{X_{j+1}}{1 + X_{j+1}^T S_j X_{j+1}} \quad (3\text{-}4)$$

$$S_{j+1} = [\ 1 - K_j X_{j+1}^T]\ S_j \quad (3\text{-}5)$$

HPC_MEANS algorithm

We have n sample data vectors (x_l, y_l), $1 \le l \le n$, and c is the number of cluster for division.

[STEP 1]

The initial hyper plane is determined by arbitrary value.

$$y^1 = X^T \cdot P^1(1)$$
$$\vdots$$
$$y^c = X^T \cdot P^c(1)$$

[STEP 2]

At iteration of kth, we allocate the sample data to the closest cluster on each sample data. That is

$$(x_l, y_l) \in S_j(k)$$
$$|\ y_l - X_l^T P^j(k)\ | \le |\ y_l - X_l^T P^i(k)\ |$$
$$\text{for } all\ i = 1, 2, \cdots, c$$

where $S_j(k)$ is a cluster which has the center to plane $y = X^T P^j(k)$.

[STEP 3]

Compute a clustering error of kth iteration D_k

$$D_k = \sum_{j=1}^{c} \frac{1}{N} \sum_{j(x_l, y_l) \in S_j(k)} |\ y_l - X_l^T P^j(k)\ |$$

This time if $\frac{D_k - D_{k-1}}{D_k} \le \delta$, then terminate else go to STEP 4.

[STEP 4]

From $S_1(k), S_2(k), \cdots, S_c(k)$, calculate new cluster center plane $y^i = X^T \cdot P^i(k+1)$. $P^i(k+1)$ is obtained from eq. (3-2),(3-3),(3-4).

[STEP 5]

Return to STEP 2 and k=k+1.

The structure by HPC-MEANS algorithm is coarse because it divide the input space crisply. Thus, we use the tuning method by genetic algorithm.

3 - 2 Parameter tuning

Genetic Algorithm is the latest method for searching global optimal solutions. It uses three major operators such as reproduction, crossover and mutation. A string is an array of many genes. Each parameter is encoded into gene: If membership function is triangle type, premise parameters consist of each membership function's width and center, and consequence parameters consist of each linear function's coefficients. A group of strings is called population and is changed according to its fitness. Of course, dominant strings are chosen, crossovered, and mutated. As a result, with generation being continued, all of strings in population become fittest.

[STEP 1]
Encode parameters obtained by HPC-MEANS

[STEP 2]
Construct population

[STEP 3]
Reproduce into mating pool, crossover and mutate in mating pool.

[STEP 4]
Evaluate the maximum fitness and average fitness of each string in mating pool. If the maximum fitness satisfies a criterion, then stop.

[STEP 5]
If both maximum fitness and average fitness is better than before, then use the mating pool as the next generation's population and go to [STEP 2]
Otherwise, go to [STEP 3]
In this paper, fitness function is as follows:

$$F = 100 * \exp\left(-\frac{1}{N}\sum_{j=0}^{N-1}(y_{dj}-y_j)^2\right)$$

4. Simulation

The suggested method is applied to SISO system. Therefore the possible rules is same as the number of division of input space. It is founded automatically by HPC-MEANS. In this simulation, input space is divided into three region.

As a result , the structure of fuzzy model is as follows:

Rule 1 : *If x is S, then* $y_1 = a_{10} + a_{11}x$

Rule 2 : *If x is M, then* $y_2 = a_{20} + a_{21}x$

Rule 3 : *If x is B, then* $y_3 = a_{30} + a_{31}x$

In order to tune all parameters, firstly all parameters are encoded in one string. Each parameter is encoded into 16bit binary code.

$$\textit{string}: \quad C_1 W_{10} W_{11} C_2 W_{20} W_{21} C_3 W_{30} W_{31} + a_{10} a_{11} a_{20} a_{21} a_{30} a_{31}$$

From simulation, fuzzy model is found as follows:

C_1 : 0.606 W_{10} : 2.515 W_{11} : 3.115
C_2 : 2.538 W_{20} : 0.243 W_{21} : 3.284
C_3 : 9.170 W_{30} : 3.981 W_{31} : 7.497
a_{10} : −1.004 a_{11} : 1.013
a_{20} : −1.453 a_{21} : 2.728
a_{30} : 10.168 a_{31} : −1.019

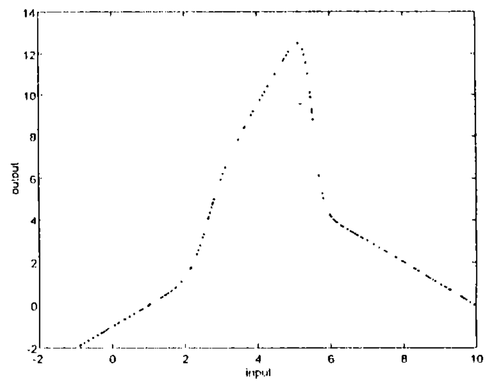

Fig. 4-1 Input/output data of the given system

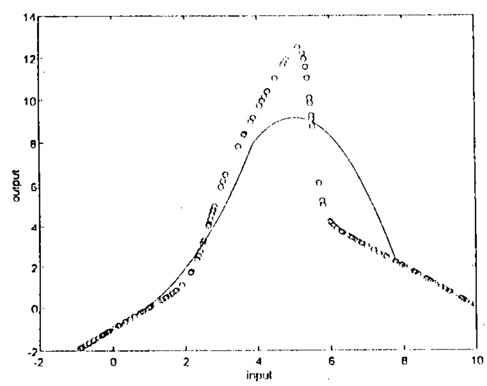

Fig. 4-2 Output of the fuzzy model after HPC-MEANS

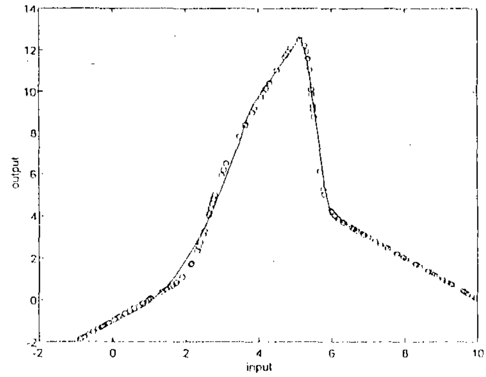

Fig. 4-3 Output of the fuzzy model after GA.

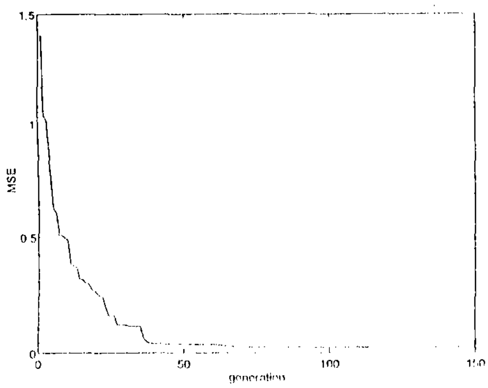

Fig. 4-4 Mean Square Errors during GA.

5. Conclusion

This paper shows by application that the suggested method can model a original system. When HPC-Means algorithm is used alone, a system can't be modeled accurately. Hence the fine tuning algorithm is needed. GA is one of the latest global optimization algorithms. So after the system is tuned by GA, the system can be modeled accurately. The global optimization of GA guarantees that the model can the globally optimal model of the system. For advanced modeling method, firstly, the algorithm should be extended to MIMO system. Secondly, more efficient clustering algorithm is needed. Thirdly, it is needed to compare various evaluation functions for GA.

Reference

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