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A Stochastic Model Based Tracking Control Scheme for Flexible Robot Manipulators

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Abstract– The presence of joint elasticity or the arm flexibility causes low damped oscillatory position error along a desired trajectory. We utilize a stochastic model for describing the fast dynamics and the approximation error. A second order shaping filter is synthesized such that its spectrum matches that of the fast dynamics. Augmenting the state vector of slow part with that of shaping filter, we obtain a nonlinear dynamics to which a Gaussian white noise is injected. This modeling approach leads us to the design of an extended Kalman filter(KEF) and a linear quadratic Gaussian (LQG) control scheme. We present the simulation results of this control method. The simulation results show us that our Kalman filtering approach is one of prospective methods in controlling the flexible arms.

I. Introduction

Recently several advanced control algorithms for the flexible joint manipulators have been proposed. Marino and Nicosia [6] reformulated the dynamic model of an elastic joint as a singularly perturbed nonlinear system, where the "slow" variables are the joint position and velocity and the "fast" variables are the joint forces and their time derivatives. A method based on the concept of integral manifold was suggested by Khorasani *et al.* [3], [4] and Spong *et al.* [6].

But, recent works have shown that the control algorithms based on a rigid model are limited in their applicability to real robots because the effect of joint elasticity is not so small enough to be neglected. The small angular deviation due to the joint compliance will produce a significant error in the end-effector position through the amplification effect of the link. As can be seen from [1], the flexible modes will not be damped out as far as the fast subsystem contains poles on the imaginary axis of the complex plane. Indeed the flexible arm under consideration exhibits low damped oscillatory poles, non-minimum phase behavior, time varying dynamics due to inertia changes and nonlinear characteristics due to the friction.

To find a slow control is related to finding a integral submanifold, thus it involves solving a set of partial differential equations. In obtaining the slow control we utilize the approximation method[6], thereby it leaves an approximation error. Further, the fast dynamics is coupled with slow dynamic variables and very sensitive to parameter uncertainty and/or variation.

In such a situation, a statistical approach may yield a good solution. We describe the fast dynamics as a stochastic model. Since the fast dynamics generates a low damped oscillatory term along a mean trajectory, we describe it by a second order shaping filter driven by a white Gaussian noise. Obviously, the poles of shaping filter locate on the imaginary axis and its oscillation frequency is tuned to that of arm vibration. Also then variance of white noise is selected so that the spectrum of its output is best suited for that of the real oscillatory motion. Augmenting the state of the rigid arm dynamics with that of the stochastic model yield a nonlinear stochastic system. That necessitates the use of the EKF and LQG control. Another feature of this control scheme is that it decouple and linearizes the highly coupled dynamics of a series flexible arm based on the state estimate. Further, with this control law, we can include the effects of the measurement noise, thus it helps us to make a more realistics solution. Finally we will show through computer simulation results that our control scheme outperforms the existing control schemes especially in the presence of parameter mismatch.

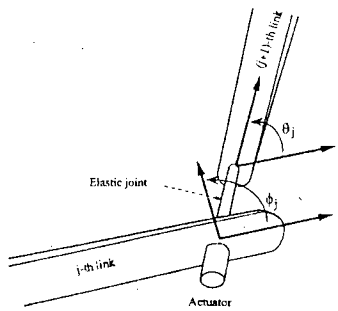


Fig. 1. j^{th} joint of an n−link manipulator with joint flexibility

II. The Singularly Perturbed Model

We are considering an n-link robot manipulator that has n elastic revolute joints. But we assume that the links are rigid. The jth elastic revolute joint is depicted in Figure 1. To describe the joint elasticity, we use the two variables, ϕ_j and θ_j. Here, ϕ_j denotes the joint angle at the jth actuator side and θ_j the joint angle at the $(j+1)$th link side.

The dynamic equations for this system are given by

$$J_l(\theta)\ddot{\theta} + B_l(\theta,\dot{\theta})\dot{\theta} + G_l(\theta) = K(\phi - \theta) \tag{1}$$

$$J_m\ddot{\phi} + B_m\dot{\phi} = K(\theta - \phi) + \tau_m, \tag{2}$$

where $J_l(\theta) \in \mathbb{R}^{n\times n}$, $B_l(\theta,\dot{\theta})\dot{\theta} \in \mathbb{R}^n$, $G_l(\theta) \in \mathbb{R}^n$ represent the inertial matrix, the Coriolis's force plus the centrifugal force, and the gravity vector, respectively. The matrix $K = diag\{k_1,\cdots,k_n\}$ is the stiffness matrix of the flexible joints. $J_m, B_m \in \mathbb{R}^{n\times n}$ are the inertial and the friction matrices, respectively, and τ_m is the torque vector generated at the motor sides. Note that J_m, B_m are diagonal. The torque to the links is delivered through the elastic force given by

$$\psi = K(\phi - \theta). \tag{3}$$

Noting that J_l is invertible, we can rewrite (1) as

$$\ddot{\theta} = -J_l^{-1}B_l\dot{\theta} - J_l^{-1}G_l + J_l^{-1}\psi. \tag{4}$$

The elasticity matrix E is defined by $E = diag\{\epsilon_1,\cdots,\epsilon_n\} = K^{-1}$. Substituting $\phi = \theta + K^{-1}\psi = \theta + E\psi$ in (2), we obtain that

$$E\ddot{\psi} = -(J_l^{-1} + J_m^{-1})\psi - J_m^{-1}B_mE\dot{\psi} + (J_l^{-1}B_l - J_m^{-1}B_m)\dot{\theta} + J_l^{-1}G_l + J_m^{-1}\tau_m. \tag{5}$$

For $E = 0$ one can find a submanifold which is characterized by $\psi = (J_l^{-1} + J_m^{-1})^{-1}\{(J_l^{-1}B_l - J_m^{-1}B_m)\dot{\theta} + J_l^{-1}G_l + J_m^{-1}\tau_m$ in the $3n$-dimensional space. If each component $\epsilon_i > 0$ of E is sufficiently small, there exists an invariant submanifold $\psi = \psi_s(\theta, \dot{\theta}, \tau_m, E)$ in a neighborhood of $\psi = \psi_s(\theta, \dot{\theta}, \tau_m, 0)$[2].

The system (4),(5) represent a singularly perturbed system where the slow variables are the joint angle θ and the fast variables are the elastic force ψ. The full order model (4),(5) represent a highly complex nonlinear system that, in general, is difficult to handle for the design of a controller.

In general ψ can be expressed as

$$\psi = \psi_s + \psi_f, \tag{6}$$

where ψ_f represents the transient behavior of ψ, i.e., it is the deviation of ψ from the integral manifold $\psi = \psi_s(\theta, \dot{\theta}, \tau_m, E)$. We approximate the invariant manifold up to first order as follows:

$$\psi_s = \psi_{s0} + E\psi_{s1} + O(E^2), \tag{7}$$

where ψ_{s0} is the zeroth order approximation of ψ_s, and $E\psi_{s1}$ represents the first order correction to ψ_{s0}. The control input τ_m can be decomposed into two parts:

$$\tau_m = \tau_s + \tau_f, \tag{8}$$

where τ_s and τ_f are called the slow control and the fast control, respectively. We let

$$\tau_s = \tau_{s0} + E\tau_{s1} + O(E^2), \tag{9}$$

where τ_{s0} is called the reduced order slow control and τ_{s1} is the first order approximation of the corrective control which will be used for compensating the effects of ψ_{s1}. Substituting (6),(8) for ψ, τ_m in (5) respectively, we can decompose (5) such that

$$E\ddot{\psi}_s = -(J_l^{-1} + J_m^{-1})\psi_s - J_m^{-1}B_m E\dot{\psi}_s + (J_l^{-1}B_l - J_m^{-1}B_m)\dot{\theta} + J_l^{-1}G_l + J_m^{-1}\tau_s \tag{10}$$

$$E\ddot{\psi}_f = -(J_l^{-1} + J_m^{-1})\psi_f - J_m^{-1}B_m E\dot{\psi}_f + J_m^{-1}\tau_f. \tag{11}$$

In order to obtain slow control, we further substitute (7),(9) for ψ_s, τ_s in (10), respectively. Then equating the terms of the same powers of E, we obtain

$$0 = -(J_l^{-1} + J_m^{-1})\psi_{s0} + (J_l^{-1}B_l - J_m^{-1}B_m)\dot{\theta} + J_l^{-1}G_l + J_m^{-1}\tau_{s0} \tag{12}$$

$$E\ddot{\psi}_{s0} = -(J_l^{-1} + J_m^{-1})E\psi_{s1} - J_m^{-1}B_m E\dot{\psi}_{s0} + J_m^{-1}E\tau_{s1}. \tag{13}$$

Since J_m, B_m are diagonal, it follows that $EJ_m = J_m E$ and $EB_m = B_m E$. Thus, we obtain from (12), (13) that

$$\psi_{s0} = (J_l^{-1} + J_m^{-1})^{-1}\{(J_l^{-1}B_l - J_m^{-1}B_m)\dot{\theta} + J_l^{-1}G_l + J_m^{-1}\tau_{s0}\} \tag{14}$$

$$\psi_{s1} = (J_m E^{-1} J_l^{-1} E + I)^{-1}(\tau_{s1} - J_m\ddot{\psi}_{s0} - B_m\dot{\psi}_{s0}), \tag{15}$$

where $I \in \mathbb{R}^{n\times n}$ is the identity matrix. It can be seen from (14), (15) that the rigid control τ_{s0} is related to ψ_{s0}, and τ_{s1} to ψ_{s1}.

Applying (6),(7),(14),(15) to (4), we obtain

$$\ddot{\theta} = F(\theta, \dot{\theta}) + G(\theta)\tau_{s0} + J_l^{-1}EH(\theta)(\tau_{s1} - J_m\ddot{\psi}_{s0} - B_m\dot{\psi}_{s0}) + J_l^{-1}\psi_f + O(E^2) \tag{16}$$

$$F(\theta, \dot{\theta}) = -J_l^{-1}B_l\dot{\theta} - J_l^{-1}G_l + J_l^{-1}(J_l^{-1} + J_m^{-1})^{-1}\{(J_l^{-1}B_l - J_m^{-1}B_m)\dot{\theta} + J_l^{-1}G_l\},$$

$$G(\theta) = J_l^{-1}(J_l^{-1} + J_m^{-1})^{-1}J_m^{-1},$$

$$H(\theta) = (J_m E^{-1} J_l^{-1} E + I)^{-1}.$$

In (16), $O(E^2)$ is caused by the error between the invariant manifold ψ_s and its first order approximation, $\psi_{s0} + E\psi_{s1}$.

We denote by $\theta_d, \dot{\theta}_d, \ddot{\theta}_d$ the desired path and its derivatives, and assume that $\ddot{\theta}$ is further differentiable. Choose matrices $L_p, L_v \in \mathbb{R}^{n\times n}$ such that the roots of $det(s^2 I + L_v s + L_p) = 0$ lie in the open left half plane of $\mathbb{C}$. Now we choose the slow control such that

$$\tau_{s0} = -G^{-1}(\theta)\big(F(\theta, \dot{\theta}) - \ddot{\theta}_d + L_p(\theta - \theta_d) + L_v(\dot{\theta} - \dot{\theta}_d)\big). \tag{17}$$

Then, defining the trajectory error by $e = \theta - \theta_d$, we obtain the error dynamics such that

$$\ddot{e} + Lv\dot{e} + L_p e = J_l^{-1}EH(\theta)(\tau_{s1} - B_m\dot{\psi}_{s0} - J_m\ddot{\psi}_{s0}) + J_l^{-1}\psi_f + O(E^2). \tag{18}$$

In order to let the trajectory error vanish, the corrective control, ψ_{s1} needs to be chosen such that $\tau_{s1} = B_m\dot{\psi}_{s0} + J_m\ddot{\psi}_{s0}$. In obtaining $\dot{\psi}_{s0}$, $\ddot{\psi}_{s0}$, we must use the chain rule:

$$\dot{\psi}_{s0} = \sum_{i=1}^{n}\frac{\partial\psi_{s0}}{\partial\theta}\dot{\theta} + \sum_{i=1}^{n}\frac{\partial\psi_{s0}}{\partial\dot{\theta}}\ddot{\theta} + \sum_{i=1}^{n}\frac{\partial\psi_{s0}}{\partial\tau_{s0}}\dot{\tau}_{s0} \tag{19}$$

$$\ddot{\psi}_{s0} = \sum_{i=1}^{n}\frac{\partial\dot{\psi}_{s0}}{\partial\theta}\dot{\theta} + \sum_{i=1}^{n}\frac{\partial\dot{\psi}_{s0}}{\partial\dot{\theta}}\ddot{\theta} + \sum_{i=1}^{n}\frac{\partial\dot{\psi}_{s0}}{\partial\tau_{s0}}\dot{\tau}_{s0} + \sum_{i=1}^{n}\frac{\partial\dot{\psi}_{s0}}{\partial\tau_{s1}}\dot{\tau}_{s1}. \tag{20}$$

But instead of the exact values, we use, following the method in [6], their approximation $\dot{\bar{\psi}}_{s0}$, $\ddot{\bar{\psi}}_{s0}$ which are obtained by neglecting the terms containing E in (19), (20). Hence if we choose

$$\tau_{s1} = B_m\dot{\bar{\psi}}_{s0} + J_m\ddot{\bar{\psi}}_{s0} \tag{21}$$

along with (17), the system (18) yields

$$\ddot{e} + L_v\dot{e} + L_p e = J_l^{-1}\psi_f + O(E^2). \tag{22}$$

The error dynamics (22) has the form in which the stable linear dynamics is perturbed by the fast dynamics ψ_f and the terms of higher order in E than one.

III. A Statistical Approach to the Fast Dynamic Model

Even after applying the slow control $\tau_s = \tau_{s0} + E\tau_{s1}$ with the use of (17) and (21), the error system (22) is under the subject to the perturbation caused by the fast dynamics, $J_l^{-1}\psi_f$ and the high order error term, $O(E^2)$ in E. We let $\xi = J_l^{-1}\psi_f + O(E^2)$. The presence of such a perturbation, ξ, causes the flexible arms to exhibit low damped oscillatory mode when ϵ_i grows. That is, it yields an oscillatory error along a desired trajectory, θ_d.

It is almost impossible to describe the exact dynamics of ξ, since it involves solving a set of partial differential equations to obtain exactly the submanifold of the slow dynamics. Correspondingly, it is not easy to find a fast control τ_f based on it which suppresses the oscillatory error. Aside from the modeling, the parameter uncertainty is another great problem.

A fast dynamic model representing the deviation of the flexible joint dynamics from the rigid dynamics will be derived based on the response of the manipulator in frequency domain. It would be desirable to be able to generate empirical autocorrelation or power spectral density data, and then to develop a mathematical model that would produce an output with duplicate characteristics. Since the perturbation causes a low damped oscillatory error, we model ξ by an output of a second order system with an acceleration input which has a white Gaussian noise characteristics. That is, we use the second order shaping filter approach for modeling of the unavailable quantity, ξ, i.e.,

$$\ddot{\xi} + \Lambda_a \dot{\xi} + \Lambda_b \xi = W(t) + B\tau_f, \qquad (23)$$

where $\Lambda_a = diag\{2\zeta_1\omega_{N1}, \cdots, 2\zeta_1\omega_{Nn}\}$, $\Lambda_b = diag\{\omega_{N1}^2, \cdots, \omega_{Nn}^2\}$, $W(t) = diag\{w_1(t), \cdots, w_n(t)\}$. $B = diag\{b_1, \cdots, b_n\}$. Here, $w_i(t)$ is a zero-mean white Gaussian noise process with $E[\omega_i(t)\omega_i(t+\tau)] = q_i\delta(\tau)$, where $\delta(\cdot)$ denotes the delta function. Generally, a second order model provides a good model of an oscillatory random phenomena, such as vibration, bending, and fuel slosh in aerospace vehicles[5].

We let $x_a = \theta$, $x_b = \dot{\theta}$, $x_c = \xi$, $x_d = \dot{\xi}$, and define an augmented vector $x \in \mathbb{R}^n$ by $x = [x_a^T, x_b^T, x_c^T, x_d^T]^T$. Then, we obtain from (16), (23) that

$$\dot{x} = \begin{bmatrix} x_b \\ F(x_a, x_b) + x_c \\ x_d \\ \Lambda_a x_d + \Lambda_b x_c \end{bmatrix} + \begin{bmatrix} 0 \\ G(x_a) \\ 0 \\ 0 \end{bmatrix} \tau_{s0} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ W(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ B \end{bmatrix} \tau_f$$

$$+ \begin{bmatrix} 0 \\ J_l^{-1} E H(x_a) \\ 0 \\ 0 \end{bmatrix} (\tau_{s1} - J_m \ddot{\psi}_{s0} - B_m \dot{\psi}_{s0}) \qquad (24)$$

$$y = [I\ 0\ 0\ 0]x + V(t), . \qquad (25)$$

where $V(t)$ is also Gaussian white noise process which represents the measurement noise such that $E[V(t)V^T(\tau)] = R\delta(t-\tau)$ with R positive definite.

In order to validate the above model, we have to choose properly the coefficients of the shaping filter and the statistics of white noises. The oscillation frequency w_{Ni} and the covariance $q_i > 0$ of w_i are determined by experimentally. Figure 2 shows an example of the power spectrum of a flexible arm in which one can estimate the frequency of the side band that is caused by joint flexibility. The damping coefficients ζ_i's and b_i's of the shaping filter are obtained in such a way that the power spectrum of the system (1),(2) matches that of (24) for a given slow control τ_s. Figure 3 shows the power spectrum of a flexible arm and the power spectrum of the system (24),(25) after tunning the shaping filter parameters, respectively.

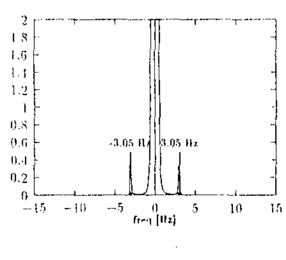


Fig. 2. Power spectrum of a single link manipulator with elastic joint under hammer impact.

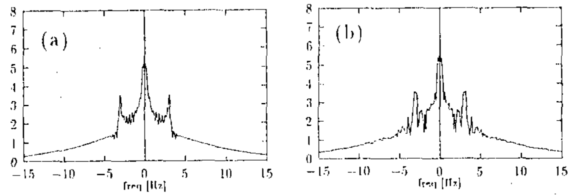


Fig. 3. Power spectrum(a) of a single link manipulator with elastic joint and the power spectrum(b) of its model in which fast dynamics is replaced by a stochastic model. The shaping filter parameters were tunned to match. ($\epsilon = 0.01$).

IV. A Construction of a Fast Controller Based on EKF

In order to construct a fast controller for the stochastic model (24),(25) we need to estimate the state, and the extended Kalman filter is best suited for obtaining the state estimates of such a nonlinear system. Since the construction of EKF is rather routine, we omit the equations for EKF here.

The fast control, τ_f needs to be chosen so that the error $x_c = \xi$ is minimized. Based on the state estimate, $\hat{x}$ we choose the fast control input such that $\tau_f = K_v\hat{x}_d + K_p\hat{x}_c$ where $K_v, K_p \in \mathbb{R}^{n\times n}$. Applying the separation principle, one can obtain K_v, K_p by solving the algebraic Riccati equation for a subsystem of x_c, x_d. In any case, the feedback gain K_v, K_p must be chosen such that ξ damps out with a reasonable speed. Hence, it follows from (17),(21) that the total control law becomes

$$\tau_m = -G^{-1}(x_a)\big(F(x_a, x_b) - \ddot{\theta}_d + L_p(x_a - \theta_d) + L_v(x_b - \dot{\theta}_d)\big)$$

$$+ E\big(B_m\dot{\psi}_{s0} + J_m\ddot{\psi}_{s0}\big) + K_v\hat{x}_d + K_p\hat{x}_c\,. \qquad (26)$$

V. A Simulation Study: a Single Link Manipulator with Joint Flexibility

The dynamics of the single link are depicted in Figure 1 is given by

$$\ddot{\theta} = a_1\dot{\theta}_l + a_2\sin\theta + a_3\psi$$

$$\epsilon\ddot{\psi} = a_4\dot{\theta} + a_2\sin\theta + a_5\epsilon\dot{\psi} + a_6\psi + a_7\tau_m$$

where $\psi = k(\phi - \theta) = (\phi - \theta)/\epsilon$, $a_1 = -3B_l/ml^2$, $a_2 = -3g/2l$, $a_3 = -3/ml^2$, $a_4 = 3B_l/ml^2 - B_m/J_m$, $a_5 = -B_m/J_m$, $a_6 = a_3 - 1/J_m$, $a_7 = 1/J_m$. Utilizing (14),(15), we obtain

$$\psi_{s0} = a_6^{-1}\left(-a_4\dot{\theta} - a_2\sin\theta - a_7\tau_{s0}\right)$$

$$\psi_{s1} = (1 + 3J_m/ml^2)^{-1}\left(\tau_{s1} - J_m\ddot{\psi}_{s0} - B_m\dot{\psi}_{s0}\right)$$

Then with the stochastic model for the fast dynamics we obtain

$$\ddot{\theta} = \alpha_0\dot{\theta} + \alpha_1\sin\theta + \beta_0\tau_{s0} + \xi - \epsilon a_6^{-1}(\tau_{s1} - J_m\ddot{\psi}_{s0} - B_m\dot{\psi}_{s0})$$

$$\ddot{\xi} = -\omega_N^2\xi - 2\zeta\omega_N\dot{\xi} + w + b\tau_f,$$

where $\alpha_0 = a_1 - a_3a_6^{-1}a_1$, $\alpha_1 = a_2 - a_3a_6^{-1}a_2$, and $\beta_0 = -a_3a_6^{-1}b$. Letting $x = [x_1\ x_2\ x_3\ x_4]^T = [\theta\ \dot{\theta}\ \xi\ \dot{\xi}\ ]^T$ we obtain

$$\dot{x} = f(x) + g(x, \tau_{s0}, \tau_{s1}) + hw(t)$$

$$y = c^T x + v(t),$$

$$f = \begin{bmatrix} x_2 \\ \alpha_0 x_2 + \alpha_1 \sin x_1 + x_3 \\ x_4 \\ -\omega_N^2 x_3 - 2\zeta\omega_N x_4 \end{bmatrix}, \quad g = \begin{bmatrix} 0 \\ \beta_0\tau_{s0} - \\ \epsilon a_6^{-1}(\tau_{s1} - J_m \ddot{\hat{\psi}}_{s0} - B_m \dot{\hat{\psi}}_{s0}) \\ 0 \\ b\tau_f \end{bmatrix},$$

$h = [0\ 0\ 0\ 1]^T$, and $c = [1\ 0\ 0\ 0]^T$.

We construct an an extended Kalman filter for it and choose a feedback such that

$$\tau_m = \tau_{s0} + \epsilon\tau_{s1} + \tau_f = \frac{1}{\beta_0}\Big(-\alpha_0 x_2 - \alpha \sin x_1 - \ddot{\theta}_d$$

$$+ l_p(x_1 - \theta_d) + \ l_v(x_2 - \dot{\theta}_d)\Big) + \epsilon\big(B_m \dot{\hat{\psi}}_{s0} + J_m \ddot{\hat{\psi}}_{s0}\big) + k_v \hat{x}_4 + k_p \hat{x}_3 .$$

The desired trajectory is defined by the following fifth order equation:

$$\theta_d(t) = \theta_{d0} + d\left(6\left(\frac{t}{t_m}\right)^5 - 15\left(\frac{t}{t_m}\right)^4 + 10\left(\frac{t}{t_m}\right)^3\right),$$

where θ_{d0} is the desired initial position, d denotes the displacement in position, and t_m is the expected duration of the displacement. In our example, we let $a_1 = 1.2$, $a_2 = -4.9$, $a_3 = 0.033$, $a_4 = 0.825$, $a_5 = -0.375$, $a_6 = -3.583$, and $a_7 = 0.25$.

We compared the performances of our control method with the existing two flexible arm control methods: The first one is the Spong et al.'s method[6] that utilizes just the slow control, $\tau_s = \tau_{s0} + \epsilon\tau_{s1}$ which is the summation of the rigid control and the the corrective term. The second one is the method of Al-Ashoor et al.[1], namely, full composite control that utilizes the fast control based on the state estimates of the fast variables along with the slow control. The third one is our stochastic model based control scheme. Figure 4 shows the real angular position trajectory that fails to follow the desired one in the case of $\epsilon = 0.01$ when only the slow control is applied. Figures 5 shows the real trajectory and the desired one under the same condition with the full composite controller(slow plus fast control) and the stochastic model based controller respectively, but in both cases they are not distinguished. Figures 6 shows the performances of the full composite control method and the stochastic model based control method in the case where there is a mismatch in ϵ, respectively. In both cases, we assumed $\epsilon = 0.015$, while the true value is $\epsilon = 0.02$. In this example, one can see that our control scheme is more robust than the Al-Ashoor et al.'s method.

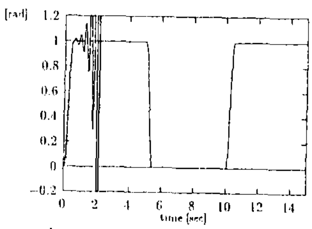
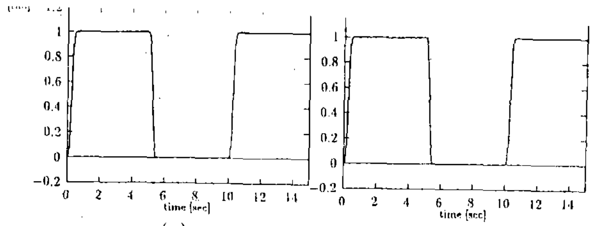
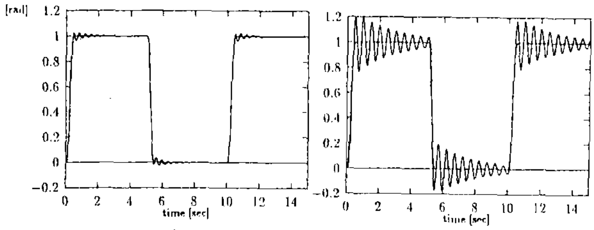


Fig. 4. A real angular position trajectory that fails to follow the desired one when only the slow control is applied ($\epsilon = 0.01$).



(a) (b)

Fig. 5. A real trajectory and the desired one with the full composite controller(a) and with the stochastic model based controller(b).



(a) (b)

Fig. 6. A real trajectory and the desired one with the full composite controller(a) with the stochastic model based controller(b) when $\epsilon = 0.015$ is assumed, while the true value is $\epsilon = 0.02$.

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