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Disturbance Suppression and Decoupling via Eigenstructure Assignment

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Abstract - An effective and disturbance suppressible controller can be obtained by assigning the left eigenstructure (eigenvalues/left eigenvectors) of a system. However, the disturbance decouplability is governed by the right eigenstructure (eigenvalues/right eigenvectors) of the system. In this paper, in order to obtain a disturbance decouplable as well as effective and disturbance suppressible controller, the concurrent assignment scheme of the left and right eigenstructure is proposed. The biorthogonality property between the left and right modal matrices of a system as well as the relations between the achievable right modal matrix and states selection matrices are used to develop the scheme. The proposed concurrent eigenstructure assignment scheme guarantees that the desired eigenvalues are achieved exactly and the desired left and right eigenvectors are assigned to the best possible(achievable) sets of eigenvectors in the least square sense, respectively. A numerical example is presented to illustrate the usefulness of the proposed scheme.

I. Introduction

The problem of eigenstructure assignment (simultaneous assignment of eigenvalues and eigenvectors) is of great importance in control theory and applications because the stability and dynamic behavior of a linear multivariable system are governed by the eigenstructure of the system[1]. In general, the speed of response is determined by the assigned eigenvalues whereas the shape of the response is furnished by the assigned eigenvectors.

The eigenstructure assignment algorithm can be divided into two groups, that is, the right eigenstructure (eigenvalues/right eigenvectors) assignment and the left eigenstructure (eigenvalues/left eigenvectors) assignment, and their roles in a system are different[2]. The right eigenstructure assignment is widely used to solve mode decoupling problems[3]-[6], and to design a controller for the vibration suppression of flexible structures[7]-[9], and can be applied to disturbance decoupling problems[10]. On the other hand, the left eigenstructure is used to define the controllability measure[11] and also can be used to design an effective and disturbance suppressible controller[12]-[16].

Thus, in order to obtain a disturbance decouplable as well as effective and disturbance suppressible controller, the appropriate assignment of a concurrent eigenstructure (that is, simultaneous assignment of the left and right eigenstructure) is required.

In this paper, a concurrent eigenstructure assignment scheme is proposed by using the biorthogonality property between the left and right modal matrices of a system as well as the relations between the achievable right modal matrix and states selection matrices. The whole procedure of the proposed scheme is attractively simple and provides more insight into the concurrent eigenstructure assignment. The presented scheme is illustrated by a numerical example.

II. Problem Formulation

Consider a linear time invariant multivariable controllable system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ef(t) \\ &= Ax(t) + \sum_{k=1}^{m} b_k u_k(t) + \sum_{l=1}^{n} e_l f_l(t), \qquad (1) \\ u(t) &= Kx(t), \qquad (2) \\ z_j(t) &= D_j x(t), \quad \text{for } j = 1,\, 2 \qquad (3) \end{aligned}$$

where (i) $x \in R^N$, $u \in R^m$, $f \in R^n$, $z_1 \in R^{r_1}$, and $z_2 \in R^{r_2}$, ($m \leq N$, and $r_1 + r_2 \leq N$) denote the state, control, disturbance which is assumed not to be directly measurable by the controller, and controlled output vectors, respectively. And b_k and e_l are the *k-th* and *l-th* column vectors of the control input matrix B and disturbance input matrix E, respectively; (ii) A, B, E, K, and D_j are real constant matrices of appropriate dimensions; and (iii) rank $B = m \neq 0$.

The responses of the state and the controlled output of the given system due to control input $u(t)$ and disturbance $f(t)$ with zero initial conditions are represented, respectively, using the modal matrices of the system by[17], [18]

$$\begin{aligned} x(t) &= \Phi \int_0^t e^{\Lambda(t-\tau)} \left\{ \Psi^{\mathrm{T}} Bu(\tau) + \Psi^{\mathrm{T}} Ef(\tau) \right\} d\tau \\ &= \sum_{i=1}^{N} \phi_i e^{\lambda_i t} \sum_{k=1}^{m} (\psi_i^{\mathrm{T}} b_k) \int_0^t e^{-\lambda_i \tau} u_k(\tau) d\tau \\ &\quad + \sum_{i=1}^{N} \phi_i e^{\lambda_i t} \sum_{l=1}^{n} (\psi_i^{\mathrm{T}} e_l) \int_0^t e^{-\lambda_i \tau} f_l(\tau) d\tau, \qquad (4) \end{aligned}$$

$$z_j(t) = D_j \Phi \int_0^t e^{\Lambda(t-\tau)} \{ \Psi^{\mathrm{T}} Bu(\tau) + \Psi^{\mathrm{T}} Ef(\tau) \} d\tau, j = 1,\, 2 \quad (5)$$

where (i) Φ and Ψ are the right and left modal matrices of the closed-loop system, respectively, and Λ is the diagonal matrix of the desired closed-loop eigenvalues; (ii) λ_i, ϕ_i and ψ_i are the *i-th* eigenvalue, right and left eigenvectors of the closed-loop system, respectively, and $u_k(t)$ is the *k-th* control input; and (iii) $D_1 \in R^{r_1 \times N}$ and $D_2 \in R^{r_2 \times N}$ matrices are chosen to the controlled outputs $z_1(t)$ and $z_2(t)$ be composed of disturbance decoupled states and composed of (at most) the other states, respectively, by a designer depending on the system considered, and are called the orthogonal and parallel states selection matrices, respectively.

Note, from *Eqs.*(4), (5), that the response to the disturbance $f(t)$ can be eliminated if the columns(ψ_i) of Ψ are orthogonal to the columns(e_l) of E. Thus, for suppressing undesired disturbances, it is required that the left eigenvectors of the system lie in the space orthogonal to the columns of E. Note also that the control efforts are effectively transferred (that is, the desired maneuver is achieved with small control efforts), if the left eigenvectors are parallel to the columns(b_k) of B. Therefore, for both effective control and disturbance suppression, it is required that the left eigenvectors of the system lie simultaneously in the space orthogonal to the columns of E and parallel to the columns of B, at least, in the least square sense. Then, the corresponding system can be manipulated with small control efforts without being disturbed by the disturbance input. Therefore, it can be said that the *left eigenstructure* of a system plays an important role in designing an *effective and disturbance suppressible* controller[2], [13]-[16].

On the other hand, the system is said to be *disturbance decoupled* relative to the pair $f(\cdot)$, $z_1(\cdot)$ if, for each initial state, the controlled output $z_1(t)$, $t \geq 0$, is the same for every $f(\cdot)$. Thus, disturbance decoupling simply means that the forced response

$$z_1(t) = D_1\Phi \int_0^t e^{\Lambda(t-\tau)}\{\Psi^T Bu(\tau) + \Psi^T Ef(\tau)\}d\tau = 0 \qquad (6)$$

for all $f(\cdot)$ and $t \geq 0$[10]. That is, from *Eq.*(6), if the right modal matrix Φ resides in the subspace of the kernel of the orthogonal states selection matrix D_1, the system (*Eqs.*(1), (6)) is disturbance decoupled. Thus, for solving *disturbance decoupling* problems, the appropriate assignment of the *right eigenstructure* of a system is required.

Meanwhile, for the r_2-states of $z_2(t)$ determined by D_2, the columns of the *right modal matrix* Φ are required to be parallel to the rows of the parallel states selection matrix D_2 in order *to preserve the control effectiveness and disturbance suppressibility* of the controller obtained by the appropriate assignment of the *left eigenstructure* of the system. Otherwise, that is, if the columns of Φ are designed not to be parallel to the rows of D_1, the control efforts may not be effectively transferred to the controlled output $z_1(t)$, even though the columns of Ψ are designed to be parallel to the columns of B for maximum control efforts transferring.

Each eigenstructure assignment scheme has been studied individually by many researchers. However, a concurrent eigenstructure assignment problem has not been solved because of its inherent conflicting nature of the left and right eigenstructure of a system. That is, if the left eigenstructure of the system is determined, the right one is also determined of itself, and vice versa.

In order to obtain a disturbance decouplable as well as effective and disturbance suppressible controller, the left and right eigenstructure should be assigned to the appropriate ones simultaneously. Thus, the objective is to find a concurrent eigenstructure assignment scheme to obtain such a controller.

III. Preliminaries on Eigenstructure Assignment by State Feedback

In this section, we briefly review the right and left eigenstructure assignment schemes individually for an understanding of what is to follow.

Consider *Eq.*(1) in section II. If state feedback(*Eq.*(2)) is applied to *Eq.*(1), the closed-loop system becomes

$$\dot{x}(t) = (A + BK)x(t) + Ef(t). \qquad (7)$$

Let $\Lambda = \{\lambda_1, \cdots, \lambda_N\}$ be a self-conjugate set of distinct complex numbers. Then, the right and left eigenstructure problems for the closed-loop system can be defined by[19]

$$(A + BK - \lambda_i I_N)\phi_i = 0, \qquad (8)$$

$$\psi_i^T(A + BK - \lambda_i I_N) = 0 \qquad (9)$$

where I_N is an $(N \times N)$ identity matrix. For the case that the system has repeated eigenvalues, the eigenvalue problem can be easily generalized[20].

Each problem of the right and left eigenstructure assignment is then to choose the feedback gain matrix K such that the required conditions for the eigenvalues and eigenvectors are satisfied, and therefore may be considered as inverse eigenvalue problem.

The right modal matrix Φ can be denoted as

$$\Phi = [\phi_1,\ \phi_2,\ \cdots,\ \phi_i,\ \cdots,\ \phi_N].$$

In the following matrices Ψ and W are defined similarly, and the superscript $(\cdot)^*$ denotes the conjugate of a given complex vector or scalar $(\cdot)$.

As mentioned in the previous sections, for solving disturbance or mode decoupling problems using eigenstructure assignment scheme, the appropriate assignment of the *right* eigenstructure of a system is required.

To present the previous result on *right* eigenstructure assignment, we define the following two matrices.

$$S_{\lambda_i} \equiv [\ \lambda_i I_N - A \ \mid\ B\ ], \qquad R_{\lambda_i} \equiv \begin{bmatrix} N_{\lambda_i} \\ -- \\ M_{\lambda_i} \end{bmatrix}$$

where the columns of the matrix R_{λ_i} form a basis for the null space of S_{λ_i}. For rank $B = m$, it can be shown that the columns of N_{λ_i} are linearly independent[3].

The following theorem gives necessary and sufficient conditions for the existence of K which yields the prescribed *right* eigenstructure.

Theorem 3.1[3]
Let $\{\lambda_1, \lambda_2, \cdots, \lambda_N\}$ be a self-conjugate set of distinct complex numbers. There exists a real $(m \times N)$ matrix K such that

$$(A + BK)\phi_i = \lambda_i\phi_i, \qquad i = 1,\ 2,\ \cdots,\ N$$

if and only if, for each i,

1) $\{\phi_1, \phi_2, \cdots, \phi_N\}$ are a linearly independent set in C^N, the space of complex N-vectors
2) $\phi_i = \phi_j^*$ when $\lambda_i = \lambda_j^*$
3) $\phi_i = \text{span}\ \{N_{\lambda_i}\}$.

Also, if K exists and rank $B = m$, then K is unique.

Remark:
In general, for a system which has repeated eigenvalues, the result of Theorem 3.1 can be easily extended and well described in Ref. [20].

The theorem indicates that the closed-loop right eigenstructure assignment by state feedback is constrained by the requirement that the right eigenvectors lie in certain subspaces. That is, the desired right eigenvectors are achieved in the least square sense, guaranteeing the exact assignment of the desired eigenvalues, by an appropriate linear combination of the column vectors of the null space of $[\lambda_i I_N - A \mid B]$. Note that the theorem provides only a *right* eigenstructure assignment scheme.

Now, we shall point out the emerging problem when we try to assign a left eigenstructure using Theorem 3.1. The matrix form of the equation(*Eq.*(9): the left eigenstructure problem) can be represented by

$$[\ \lambda_i I_N - A^T \ \mid\ I_N\ ] \begin{bmatrix} \psi_i \\ ----- \\ -K^T B^T \psi_i \end{bmatrix} = 0. \qquad (10)$$

where the superscript $(\cdot)^\dagger$ denotes the pseudo-inverse of a given matrix $(\cdot)$.

In this case(*Eq.*(10)), the feedback gain matrix K is given in the least square sense because the control input matrix B in the equation is not square in general, and therefore it is expected that the achieved closed-loop eigenvalues may not coincide with the desired eigenvalues[16]. Thus, the left eigenstructure assignment scheme by state feedback based only upon Theorem 3.1 is of little use. This fact can be explicitly verified by the following simple example.

Example 3.1
Consider a controllable system given by

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Let a set of the desired closed-loop eigenvalues be $\{\lambda_1, \lambda_2\} = \{-1, -2\}$. Then, the following matrices defined in this section are obtained.

$$N_{\lambda_1} = \begin{bmatrix} 0.4414 & 0.0353 \\ -0.1577 & 0.3046 \end{bmatrix}, M_{\lambda_1} = \begin{bmatrix} 0.8828 & 0.0707 \\ -0.0316 & 0.9492 \end{bmatrix},$$
$$N_{\lambda_2} = \begin{bmatrix} 0.3150 & 0.0129 \\ -0.0841 & 0.2393 \end{bmatrix}, M_{\lambda_2} = \begin{bmatrix} 0.9451 & 0.0388 \\ -0.0213 & 0.9701 \end{bmatrix}.$$

The first and second columns of the achievable left eigenvectors are given as a linear combination of the matrices N_{λ_1} and N_{λ_2} by Theorem 3.1, respectively. That is,

$$\psi_1^a = N_{\lambda_1} p_1, \quad \psi_2^a = N_{\lambda_2} p_2.$$

where the vectors p_1 and p_2 are linear combination coefficient vectors. If the vectors p_1 and p_2 are selected by $p_1 = [1 \;\; 0]^{\mathrm{T}}$ and $p_2 = [1 \;\; 1]^{\mathrm{T}}$, then the achievable left modal matrix Ψ^a is obtained by

$$\Psi^a = [\psi_1^a \;\; \psi_2^a] = \begin{bmatrix} 0.4414 & 0.3280 \\ -0.1577 & 0.1552 \end{bmatrix}.$$

The feedback gain matrix K is calculated as follows:

$$K^{\mathrm{T}} = [-M_{\lambda_1} p_1 \;\; -M_{\lambda_2} p_2] \left[B^{\mathrm{T}} \psi_1^a \;\; B^{\mathrm{T}} \psi_2^a\right]^{\dagger}$$
$$= [-0.2766 \;\; -3.1104]^{\mathrm{T}}$$

and therefore the eigenvalues of the resulting closed-loop system $(A + BK)$ are given as 0.8596, −0.9701, which are completely different from the desired one (*i.e.*, −1, −2) since the feedback gain K is obtained in the least square sense. It is due to the singularity of the matrix $\left[B^{\mathrm{T}} \psi_1^a \;\; B^{\mathrm{T}} \psi_2^a\right]$. This verifies the inconsistency of the desired eigenvalues with the achieved ones.

The left eigenstructure assignment scheme plays an important role in designing an effective and disturbance suppressible controller. Choi *et al.*[14]-[16] found that a left eigenstructure assignment scheme by state feedback using Theorem 3.1 cannot be directly applied to get the desired left eigenstructure, and thus proposed a novel left eigenstructure assignment scheme, that does not depend on the theorem, using only the biorthogonality property between the right and left modal matrices of a system.

The left eigenstructure assignment scheme makes it possible to achieve the desired closed-loop left eigenstructure exactly, provided the desired left eigenvectors reside in the achievable subspace. In case the desired left eigenvectors do not reside in the achievable subspace, the closed-loop eigenvalues are achieved exactly and the left eigenvectors are assigned to the best possible set of eigenvectors in the least square sense.

The details of the scheme are reported in Ref. [16] but are briefly summarized here for completeness. These details form the basis for an understanding of what is to follow. The derivation procedures are the following.

According to Theorem 3.1, the achievable right eigenvector ϕ_i^a should lie in the span of $\{N_{\lambda_i}\}$. An achievable right modal matrix Φ^a is defined as follows:

$$\Phi^a = [\phi_1^a, \; \phi_2^a, \; \cdots, \phi_i^a, \; \cdots, \phi_N^a] \tag{11}$$

and ϕ_i^a is given as a linear combination of the columns of N_{λ_i}, that is,

$$\phi_i^a = N_{\lambda_i} p_i. \tag{12}$$

In *Eq.*(12), the $(m \times 1)$ coefficient vector p_i is chosen to minimize the following performance index

$$J = \|(\Psi^d)^{\mathrm{T}} \Phi_{\mathrm{aug}}^a P - I_N\| \tag{13}$$

where the $(mN \times N)$ coefficient matrix P is formed as follows:

$$P = \text{block diag}\,[p_1, \; p_2, \; \cdots, \; p_i, \; \cdots, \; p_N]. \tag{14}$$

The $(N \times N)$ matrix Ψ^d is determined appropriately according to the guideline described in Ref. [16] to reflect the specified modal controllability and disturbance suppressibility weightings, and the $(N \times mN)$ augmented achievable right modal matrix Φ_{aug}^a is formed as follows:

$$\Phi_{\mathrm{aug}}^a = [N_{\lambda_1}, \; N_{\lambda_2}, \; \cdots, \; N_{\lambda_i}, \; \cdots, \; N_{\lambda_N}]. \tag{15}$$

The vector p_i minimizing the performance index J is given by the following equation

$$p_i = (\Omega_i)^{\dagger} n_k \tag{16}$$

where the $(N \times m)$ submatrix Ω_i is a component of the following matrix $\{(\Psi^d)^{\mathrm{T}} \Phi_{\mathrm{aug}}^a\}$ of dimension $(N \times mN)$

$$[\Omega_1, \; \Omega_2, \; \cdots, \; \Omega_i, \; \cdots, \; \Omega_N] = \{(\Psi^d)^{\mathrm{T}} \Phi_{\mathrm{aug}}^a\}, \tag{17}$$

and the vector n_k is the *k-th* column of an $(N \times N)$ identity matrix corresponding to the *k-th* submatrix of $\{(\Psi^d)^{\mathrm{T}} \Phi_{\mathrm{aug}}^a\}$. Then, the achievable right modal matrix Φ^a is given by the following equation

$$\Phi^a = \Phi_{\mathrm{aug}}^a P. \tag{18}$$

Considering the biorthogonality property between modal matrices, the achievable left modal matrix Ψ^a, satisfying the design specifications in the least square sense (in case the desired left modal matrix Ψ^d does not reside in the achievable subspace), can be represented by

$$\Psi^a = (\Phi^a)^{-\mathrm{T}}. \tag{19}$$

The feedback gain matrix K for obtaining the achievable left modal matrix Ψ^a is calculated by using the null space of S_{λ_i} and the obtained achievable right modal matrix Φ^a.

IV. Concurrent Eigenstructure Assignment

In this section, a simultaneous assignment scheme of the left and right eigenstructure of a system is proposed. The proposed assignment scheme can be used to obtain a disturbance decouplable as well as effective and disturbance suppressible controller. However, the concurrent eigenstructure assignment (simultaneous assignment of the left and right eigenstructure) problem has not been solved because of its inherent conflicting nature of the left and right eigenstructure of the system. That is, if the left eigenstructure of the system is determined in advance, the right one is also determined of itself, and vice versa.

Thus, the objective of this section is to find a solution for the concurrent eigenstructure assignment in the least square sense to overcome the inherent conflicting nature of each eigenstructure. That is, if the following three conditions are satisfied simultaneously, the desired left and right modal matrices are achieved in the least square sense guaranteeing the exact assignment of the desired eigenvalues.

$$q_1 \left[(\Psi^d)^{\mathrm{T}} \cdot \Phi_{\mathrm{aug}}^a P - I_N\right] = 0, \tag{20}$$

$$q_2 D_1 \cdot \Phi_{\mathrm{aug}}^a P = 0, \tag{21}$$

$$q_3 (D_2^{\mathrm{T}} \widehat{K} - \Phi_{\mathrm{aug}}^a P \cdot S) = 0 \tag{22}$$

where $D_1 \in R^{r_1 \times N}$ is the orthogonal states selection matrix, $D_2 \in R^{r_2 \times N}$ is the parallel states selection matrix, q_i are weighting factors corresponding to each condition and $0 \leq q_i \leq 1$

$(i=1,2,3)$, $\sum_{i=1}^{3} q_i = 1$, and $P \in C^{mN\times N}$ is the coefficient matrix to be determined.

The first condition($Eq.(20)$) denotes the biorthogonality property between the desired left modal matrix($(\Psi^d)^{\mathrm{T}}$) and the achievable right modal matrix($\Phi^a_{\mathrm{aug}}P$), and can be used to design the left eigenstructure of a system. The second condition ($Eq.(21)$) denotes the orthogonality property between D_1 and $\Phi^a_{\mathrm{aug}}P$. By adding the second condition to the first one, the corrupted disturbances in the states selected by D_1 ($i.e.$, $z_1(t)$) are decoupled in the least square sense. The third condition($Eq.(22)$) denotes the parallel condition between D_2 and $\Phi^a_{\mathrm{aug}}P$, where the matrices $\widehat{K}$, and $S = [s_1, s_2, \cdots, s_{r_2}]$ are a linear combination coefficient matrix and a state selection matrix, respectively. The element vector s_i, which is equal to the transpose of the i-th row vector of the parallel states selection matrix D_2, of the matrix S denotes a column selection vector of the matrix $\Phi^a_{\mathrm{aug}}P$.

Now, our objective is to find the feedback gain matrix K which yields Ψ^a and Φ^a with exact desired eigenvalues, satisfying the imposed three conditions simultaneously in the least square sense, through choosing P and $\hat{k}_{ij}$, which are the elements of the matrix $\widehat{K}$, using the given D_1, D_2, Ψ^d, Φ^a_{aug}, and q_i.

For convenience, we vectorize the elements of the coefficient matrix P in $Eq.(14)$ as

$$\widehat{p} = [p_1^{\mathrm{T}},\ p_2^{\mathrm{T}},\ \cdots,\ p_i^{\mathrm{T}},\ \cdots,\ p_N^{\mathrm{T}}]^{\mathrm{T}}. \tag{23}$$

If we assume, as an illustrative example, that all the three conditions are imposed only on the i-th achievable right eigenvector simultaneously and for the other achievable right eigenvectors, only the first two conditions are imposed, then the stacked augmented coefficient vector $\widehat{p}_{\mathrm{aug}}$ including the linear combination coefficients $\hat{k}_{ij}$ is formed as

$$\widehat{p}_{\mathrm{aug}} = [p_1^{\mathrm{T}},\ p_2^{\mathrm{T}}, \cdots, \underbrace{p_i^{\mathrm{T}},\ \hat{k}_{i1},\ \hat{k}_{i2}, \cdots, \hat{k}_{ir_2}}_{\equiv\ \widehat{p}^{\,i}_{\mathrm{aug}}},\ p_{i+1}^{\mathrm{T}}, \cdots, p_N^{\mathrm{T}}]^{\mathrm{T}}. \tag{24}$$

The elements of the i-th augmented coefficient vector $\widehat{p}^{\,i}_{\mathrm{aug}}$ are used for determining the i-th achievable right eigenvector satisfying all the imposed conditions. The dimension and elements of the vector $\widehat{p}_{\mathrm{aug}}$ are determined by the imposed conditions on each achievable right eigenvector.

Then, the imposed three conditions ($Eqs.(20)$-(22)) can be represented by the following compact form.

$$T\ \widehat{p}_{\mathrm{aug}} = \eta. \tag{25}$$

where the vectors $\widehat{p}_{\mathrm{aug}} \in C^{mN+r_2}$, $\eta \in R^{N(N+r_1)+r_2}$, and $T \in C^{(N(N+r_1)+r_2)\times(mN+r_2)}$ are given in this special case, respectively, by

From $Eq.(25)$, the stacked augmented coefficient vector $\widehat{p}_{\mathrm{aug}}$ is given by

$$\widehat{p}_{\mathrm{aug}} = T^{\dagger}\eta, \tag{29}$$

and from $Eqs.(20)$-(22), the achievable right and left eigenvectors are given in the least square sense.

In general cases, that is, all the three conditions are imposed on the achievable (N-1) right eigenvectors of a system with repeated eigenvalues, the dimensions of the vectors $\widehat{p}_{\mathrm{aug}}$, η, and the matrix T are extended appropriately.

Remember that the objective of this study is to find the state feedback gain matrix K satisfying the imposed three conditions described in this section in the least square sense. The following algorithm gives such a gain matrix. The algorithm guarantees that the desired eigenvalues are achieved exactly and the desired right and left eigenvectors are achieved in the least square sense.

Algorithm:

- *Step 1:* Determine the desired eigenvalues (λ_i), corresponding desired left eigenvectors (ψ_i^d), the orthogonal states selection matrix D_1, the parallel states selection matrix D_2, and weighting factors q_i (i=1,2,3).
- *Step 2:* Find the following matrices
$$S_{\lambda_i} \equiv [\ \lambda_i I_N - A \ \ | \ \ B\ ],\quad R_{\lambda_i} \equiv \begin{bmatrix} N_{\lambda_i} \\ -- \\ M_{\lambda_i} \end{bmatrix}$$
where the columns of the matrix R_{λ_i} form a basis for the null space of S_{λ_i}.
- *Step 3:* Construct the augmented achievable right modal matrix Φ^a_{aug} given by $Eq.(15)$.
- *Step 4:* Calculate the stacked augmented coefficient vector $\widehat{p}_{\mathrm{aug}}$ satisfying the three conditions described in this section in the least square sense.
- *Step 5:* Form the achievable right eigenvectors
$$\phi_i^a = N_{\lambda_i} p_i,$$
and construct the achievable right modal matrix Φ^a.
- *Step 6:* Construct the achievable left modal matrix Ψ^a using the biorthogonality condition ($(\Psi^a)^{\mathrm{T}}\Phi^a = I_N$) between the left and right modal matrices of the given system.
- *Step 7:* Calculate vector chains and construct the matrix W as follows:
$$w_i = -M_{\lambda_i} p_i, \qquad W = [w_1,\ w_2,\ \cdots,\ w_i,\ \cdots,\ w_N].$$

$$\widehat{p}_{\mathrm{aug}} = [p_1^{\mathrm{T}},\ p_2^{\mathrm{T}},\ \cdots,\ p_i^{\mathrm{T}},\ \hat{k}_{i1},\ \hat{k}_{i2},\ \cdots,\ \hat{k}_{ir_2},\ p_{i+1}^{\mathrm{T}},\ \cdots,\ p_N^{\mathrm{T}}]^{\mathrm{T}}, \tag{26}$$

$$\eta = [\underbrace{\overbrace{q_1\ 0 \cdots 0}^{N+r_1}}_{1st}\ \underbrace{\overbrace{0\ q_1\ 0 \cdots 0}^{N+r_1}}_{2nd} \cdots \underbrace{\overbrace{0 \cdots 0\ q_1\ 0 \cdots 0}^{N+r_1+r_2}}_{ith}\ \underbrace{\overbrace{0 \cdots 0\ q_1}^{N+r_1}}_{Nth}]^{\mathrm{T}}, \tag{27}$$

$$T = \begin{pmatrix} \begin{bmatrix} q_1\Omega_1 \\ q_2 D_1 N_{\lambda_1} \end{bmatrix} & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \begin{bmatrix} q_1\Omega_2 \\ q_2 D_1 N_{\lambda_2} \end{bmatrix} & 0 & \cdots & \cdots & 0 \\ 0 & 0 & \ddots & 0 & \ddots & 0 \\ \vdots & \vdots & 0 & \begin{bmatrix} q_1\Omega_i & | & 0_{N\times r_2} \\ q_2 D_1 N_{\lambda_i} & | & 0_{r_1\times r_2} \\ q_3 N_{\lambda_i} & | & X \end{bmatrix} & 0 & \vdots \\ 0 & 0 & \ddots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \cdots & \begin{bmatrix} q_1\Omega_N \\ q_2 D_1 N_{\lambda_N} \end{bmatrix} \end{pmatrix} \tag{28}$$

where X denotes the following matrix

$$\begin{bmatrix} -q_3 I_{r_2} \\ 0_{(N-r_2)\times r_2} \end{bmatrix}_{N\times r_2}.$$

- *Step 8:* Calculate the state feedback gain matrix which yields the achievable right(Φ^a) and left(Ψ^a) matrices with exact desired eigenvalues satisfying the imposed three conditions simultaneously in the least square sense

$$K = W(\Phi^a)^{-1}.$$

Remark:
In *Step 1* of the algorithm, the desired left modal matrix Ψ_d can be determined to have the specified modal controllability and disturbance suppressibility[11], [13]-[17], [21] in order to obtain an effective and disturbance suppressible controller.

V. An Illustrative Example

In this section, a numerical example is presented to illustrate the proposed design scheme.

Consider a third-order two-input continuous controllable system with a disturbance,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ef(t) \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 1 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f(t). \end{aligned}$$

The open-loop spectrum of A is given by $\Lambda^{open} = \{-1, 1, 2\}$. Let a set of the desired real distinct eigenvalues be $\Lambda^d = \{\lambda_1, \lambda_2, \lambda_3\} = \{-1, -2, -3\}$. From the given system, N, m, and n are 3, 2, and 1, respectively. Assume that the desired left modal matrix Ψ^d and its normalized matrix Ψ^d_{nor} using $(\psi_i^T \psi_j = \delta_{ij})$ are determined, to have the specified modal disturbance suppressibility according to the guideline described in Ref. [16], as follows:

$$\begin{aligned} \Psi^d &= \begin{bmatrix} 0.6 & 0.7 & 0.65 \\ 0.2 & 0.2 & 0.2 \\ 0.2 & 0.1 & 0.15 \end{bmatrix}, \\ \Psi^d_{nor} &= \begin{bmatrix} 0.9045 & 0.9526 & 0.9333 \\ 0.3015 & 0.2722 & 0.2872 \\ 0.3015 & 0.1361 & 0.2154 \end{bmatrix}, \end{aligned}$$

and the orthogonal(D_1) and parallel(D_2) states selection matrices are given, respectively, as follows:

$$D_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

That is, it is assumed that the third state (*i.e.*, $x_3(t) = z_1(t)$) is chosen by D_1 as an orthogonal (to the right modal matrix) state, and the other two states (*i.e.*, $[x_1(t)\ x_2(t)]^T = z_2(t)$) are selected by D_2 as parallel (to the right modal matrix) states.

Consider the following two cases for demonstration.

Case 1: $q_1 = 1$, $q_2 = 0$, $q_3 = 0$

In this case, the biorthogonality condition ($Eq.$(20)), among the three conditions described in the previous section, between the desired left modal matrix and the achievable right modal matrix is considered only, and the other two conditions corresponding to the weighting factors q_2 and q_3 are neglected in this case. In other words, the left eigenstructure assignment which used to design an effective and disturbance suppressible controller is considered, and the right eigenstructure assignment that used to decouple the disturbance of the system is not considered here.

According to the design procedure described in the previous section, the normalized achievable left modal matrix Ψ^a_{nor} is given in the least square sense by

$$\Psi^a_{nor} = \begin{bmatrix} 0.9177 & 0.9250 & -0.9153 \\ 0.3611 & 0.3542 & -0.3815 \\ 0.1655 & 0.1378 & -0.1288 \end{bmatrix}.$$

The feedback gain matrix $K_{Case\ 1}$ is given by

$$K_{Case\ 1} = \begin{bmatrix} 40.6 & 14.4 & 2.6 \\ -20.2333 & -10.6 & -3.5667 \end{bmatrix}.$$

Case 2: $q_1 = 0.9$, $q_2 = 0.05$, $q_3 = 0.05$

In this case, all the three conditions ($Eqs.$(20)-(22)) are considered simultaneously. Thus, it is possible to assign the concurrent eigenstructure of the given system in the least square sense

Similar to Case 1, the normalized achievable left and right modal matrices are given in the least square sense as follows:

$$\begin{aligned} \Psi^a_{nor} &= \begin{bmatrix} 0.9150 & 0.9308 & -0.9108 \\ 0.3585 & 0.3451 & -0.3991 \\ 0.1852 & 0.1206 & -0.1054 \end{bmatrix}, \\ \Phi^a_{nor} &= \begin{bmatrix} -0.1977 & 0.4035 & 0.3050 \\ 0.1977 & -0.8069 & -0.9150 \\ 0.9601 & -0.4314 & 0.2642 \end{bmatrix}. \end{aligned}$$

The feedback gain matrix $K_{Case\ 2}$ is obtained by

$$K_{Case\ 2} = \begin{bmatrix} 19.0019 & 5.8485 & -0.9094 \\ -11.3361 & -7.0906 & -2.0801 \end{bmatrix}.$$

We assume that the disturbance with magnitude one is corrupted for a sufficiently small time interval for investigating the impulse responses of the states of the closed-loop system for the two cases. That is, the following impulsive disturbances($\delta(t)$) is applied:

$$f(t) = \delta(t),$$

and all initial conditions are assumed to be zero in the two cases.

Figure 1 shows the designed closed-loop system responses for each state due to the disturbance. In the figure, Case 2 shows better regulation performance in spite of the corrupted disturbance in all states compared with Case 1 since the orthogonality (the $2nd$ condition) and parallel (the $3rd$ condition) conditions are considered additionally. That is, the left eigenstructure of the system is assigned to have the desired disturbance suppressibility and modal controllability, and then for the first two states, the right eigenstructure is assigned to preserve the control effectiveness and disturbance suppressibility of the controller designed by the appropriate assignment of the left eigenstructure of the system, and at the same time, to decouple the suppressed disturbance for the third state in the least square sense. The results are consistent with our intent in this paper.

VI. Conclusions

In order to obtain an effective, disturbance suppressible and decouplable controller using eigenstructure assignment technique, a simultaneous assignment scheme of the left and right eigenstructure (concurrent eigenstructure assignment scheme) is required. In this paper, the concurrent eigenstructure assignment scheme has been proposed by using the biorthogonality property between the left and right modal matrices of a system as well as the relations between the achievable right modal matrix and states selection matrices. The proposed concurrent eigenstructure assignment scheme guarantees that the desired eigenvalues are achieved exactly and the desired left and right eigenvectors are assigned to the best possible(achievable) set of eigenvectors in the least square sense. A numerical example has been presented to confirm the usefulness of the proposed scheme.

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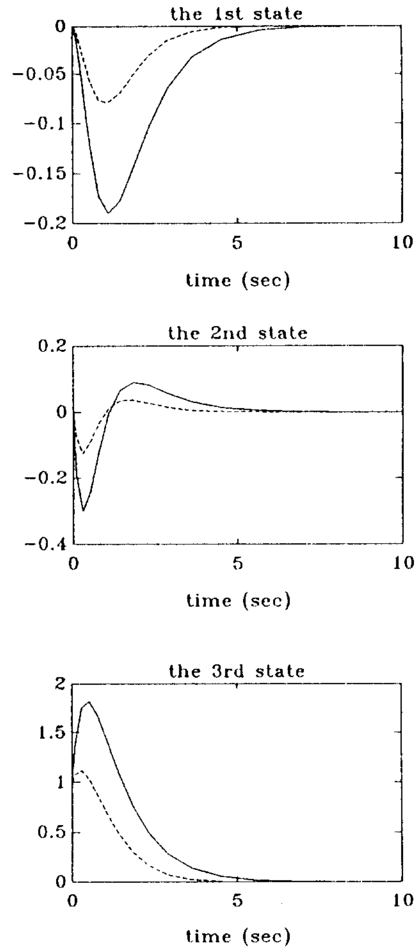

Figure 1: Impulse responses for the two cases

(solid line: Case 1, dashed line: Case 2)