Satellite's orbit tracking with Batch estimation

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Abstract

This paper deals with a Batch processor application to determine orbit trajectories from satellite tracking data. The purpose of this paper is to find the initial state vectors. In order to determine the better estimation process, several different cases are compared. Here we adapt a minimum variance concept to develop estimation and prediction techniques. These results are compared with by SEP, Spherical Error Probable, values.

1 Introduction

The purpose of this paper is to find a desirable algorithm (method) to estimate the initial state of the satellite orbit by modern estimation and predication theory. There are nominal initial states, but observation data of the actual orbit differ from data of preliminary orbit using dynamic equations of orbit from nominal initial states. It is desirable to predict a preliminary orbit of satellite with a prescribed (known) accuracy. The Batch estimation make initial states accurate according to selected dynamic equation of orbit. Here, we use minimum least-square error criterion.

When several perturbation terms are added to basic two-body dynamic equation, the results in initial states are more accurate with the expense of the extra processing time. Our main goal is to have a high accuracy of results with minimum processing time. Thus we compare the accuracy and the processing time in several different cases of Batch processor.

2 Dynamic mathematical model description

To develop a computer program to set up the trajectory of an Earth-orbiting satellite, we choose a nearly circular orbit (an eccentricity of which is about 0.077) at about 665 kilometer altitude with an inclination of 51.0 degree. For this orbit an effect of gravity potential can not be neglected according to range, longitude, latitude. In gravity potential equation, terms without longitude are called zonal harmonics. Terms with all, range,logitude, and latitude, are called tesseral harmonics. The atmospheric drag effects a major contribution.

The dynamic equation of orbit,

$$\dot{R} = V \tag{1}$$

$$\dot{V} = \nabla U - \beta\rho(R) \mid V_{rel} \mid V_{rel} \tag{2}$$

1. Earth Gravitational Model (by gravity potential,U) : We have adapted the following two gravitational model.

- Model 1 : Jonal harmonics terms only are included.

$$U = \frac{GM}{R}[1 - J_2(\frac{R_e}{R})^2 P_2(\sin\phi) - J_3(\frac{R_e}{R})^3 P_3(\sin\phi) - J_4(\frac{R_e}{R})^4 P_4(\sin\phi)] \tag{3}$$

- Model 2 : Jonal and tesseral harmonics are included.

$$\begin{aligned} U = \frac{GM}{R}[1 &- J_2(\frac{R_e}{R})^2 P_2(\sin\phi) \\ &- (\frac{R_e}{R})^2 (C_{22}\cos 2A + S_{22}\sin 2A) P_{22}(\sin\phi) \\ &- J_3(\frac{R_e}{R})^3 P_3(\sin\phi) \\ &- (\frac{R_e}{R})^3 (C_{31}\cos 1A + S_{31}\sin 1A) P_{31}(\sin\phi) \\ &- (\frac{R_e}{R})^3 (C_{32}\cos 2A + S_{32}\sin 2A) P_{32}(\sin\phi) \\ &- (\frac{R_e}{R})^3 (C_{33}\cos 3A + S_{33}\sin 3A) P_{33}(\sin\phi)] \end{aligned} \tag{4}$$

where

λ : longitude of satellite position in ECEF frame

ϕ : latitude of satellite position in ECEF frame

2. Atmospheric Drag Model : Ballistic Coefficient, β, is 0.01. The model of atmosphere is given by,

$$\rho(R) = \rho_0 exp[-k(R - R_0)] \quad (5)$$

$$V_{rel} = V - \omega_e \times R \quad (6)$$

where,

$\rho_0 = 4.36 \times 10^{-14}[kg/m^3]$
$R_0 = 7278000.0[m]$
$k = 5.381 \times 10^{-6}[1/m]$

3. Earth tracking station locations : For the tracking of satellite, three tracking stations around the world are assumed to take instantaneous range, azimuth, elevation, range rate data. The satellite station locations in ECEF frame are;

$$\begin{cases} station1(481819.913, -5507226.811, 3170377.915) \\ station2(-4440926.694, 784483.444, -4487417.923) \\ station4(-1353749.543, -5052262.074, 3637908.977) \end{cases}$$

4. Epoch time :
1 Jan 1990, 42300.0 (sec) since 0000 GMT

5. The initial satellite orbital state vector in Inertial reference system : Nominal inertial orbital state vector for the satellite at the epoch time are;

$$\begin{cases} x = -5701600.0(m) \\ y = 2892300.0(m) \\ z = -2061500.0(m) \\ \dot{x} = -3590.0(m/s) \\ \dot{y} = -4024.0(m/s) \\ \dot{z} = 5771.0(m/s) \end{cases}$$

6. Greenwich sidereal time at 1 JAN 1990, 0.0 (sec) : 1.75202 (rad)

7. Observation data of the satellite tracking : Required data set in the trajectory Batch Estimation are following ;
   - Data set : number of tracking site, time of observation, azimuth angle, elevation angle, range, range rate.
   - Number of sets of observation, m : 484 sets

3 Batch processor algorithm

The system governing equations are :

$$\dot{x} = f[x(t), t] \quad (7)$$

The measurement vector equations are :

$$z(t) = h[x(t), t] + v(t) \quad (8)$$

where $v(t)$ is observation error as a zero mean and normal process with covariance matrix $R(t)$.

The sensitivity matrix, H of the measurement vector to changes in the state vector at epoch time, t_0 is defined as

$$H_i = \frac{\partial h[x(t), t]}{\partial x}\Phi(t, t_0) \quad (9)$$

where $\Phi(t, t_0)$ is state transition matrix and is calculated in the following relationship,

$$\dot{\Phi}(t, t_0) = A(t)\Phi(t, t_0) : \Phi(t_0, t_0) = I_n \quad (10)$$

where

$$A(t) = \frac{\partial f[x(t), t]}{\partial x} \quad (11)$$

The weight-least-squares estimate of the state may then be found using the following process :

1. Select a batch data size, m for processing the data
2. Read in m sets of observations (measurements), $z(t_i)$
3. Propagate the state from the epoch to each observation time t_i,

$$\dot{x} = f[x(t), t] \quad (12)$$

with initial condition $x(t_0)$. Here we use MATLAB to integrate systems of ODEs.

   (a) Calculate the predicted observations.

$$\bar{z}(t_i) = h[x(t_i), t_i] \quad (13)$$

   (b) Calculate the sensitivities $H_i[x(t_i), t_i]$.

   (c) Calculate

$$L = \sum_{i=1}^{m} H_i^T R^{-1} H_i \quad (14)$$

$$M = H_i^T R^{-1}(z - \bar{z}) \quad (15)$$

Here R is an observation error covariance and it should be nonsingular matrix.

4. Calculate $\hat{x}$, the best estimate of the state, using

$$\begin{aligned} \hat{x} &= \bar{x} + (H^T R^{-1} H)^{-1} H^T R^{-1}(z - \bar{z}) \quad (16) \\ &= \bar{x} + L^{-1}M \end{aligned}$$

where $\bar{x}$ is the previous best estimate of the state at epoch.

5. The vector $\hat{x}$ is the new best estimate of the state at epoch. Now go to step 3, using $\hat{x}$ as the new initial condition and continue steps 3 - 5 until $|\hat{x} - \bar{x}|$ converges to a predetermined tolerance.

4 Conditions and numerical results of cases

The error is defined as a difference between preliminary calculated orbit data and observation data of actual orbit.

Simulation results show that typical values for the mean error on measuring azimuth and elevation have shown 0.002 0.005°(3.5E-5 - 8.7E-5 radians). We adapt SEP(Spherical Error Probable) as an error index. SEP is an integral of the trivariation (three-variable) Gaussian probability density function over a sphere, which is centered at the mean.

$$SEP \approx [\sigma_T^2(1 - \frac{V}{9})^3]^{\frac{1}{2}} \tag{17}$$

where

$$\sigma_T^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 \tag{18}$$

$$V = \frac{2(\sigma_x^4 + \sigma_y^4 + \sigma_z^4)}{\sigma_T^4} \tag{19}$$

1. Case I : The nominal initial state before batch estimation

2. Case II : The initial state after batch estimation using gravitational Model 1; Gravitational potential model is Model 1, and only range data are included in estimation process as observation data. For the calculation of $A(t)$ matrix, we use symbolic method (Mathematica), the explicit differentiation of symbolic equation $f[x(t), t]$.

3. Case III : The initial state after batch estimation using symbolic method ; Gravitational potential model is Model 2, and in estimation process only range data are included as observation data. We use symbolic method to calculate $A(t)$, the explicit differentiation of symbolic equation $f[x(t), t]$.

4. Case IV : The initial state after batch estimation using numerical method ; Gravitational potential model is Model 2, and in estimation process only range data are included as observation data. We use numerical method to calculate $A(t)$, as the derivative values of symbolic equation $f[x(t), t]$, using the following equation.

$$\frac{\partial f[x(t), t]}{\partial x} \cong \frac{f[x + \Delta x, t] - f[x, t]}{\Delta x} \tag{20}$$

5. Case V : The initial state after batch estimation using Range, Azimuth, Elevation ; Gravitational potential model is Model 2, and range,azimuth,elevation data are included in estimation process as observation data. We use numerical method to calculate $A(t)$, as the derivative values of symbolic equation $f[x(t), t]$. The observation error covariances, R of range, azimuth, elevation are 0.01, 7.569e-9, 7.569e-9.

6. Case VI : The initial state after batch estimation using Range, Azimuth, Elevation ; Gravitational potential model is Model 2, and range,azimuth,elevation data are included in estimation process as observation data. We use numerical method to calculate $A(t)$, as the derivative values of symbolic equation $f[x(t), t]$. The observation error covariances, R of range, azimuth, elevation are 0.01, 1.225e-9, 1.225e-9.

We show the numerical results on the table.

5 Conclusion

The accuracy of results using dynamic equation with the zonal and tesseral harmonics of gravity potential and atmospheric drag is shown better than that of using dynamic equation only including the zonal harmonics and atmospheric drag. This symbolic method take more processing time. Using the same dynamic equation, the algorithm of the numerical method takes far less time, and keeps the same accuracy as that of the symbolic method. With range, azimuth, and elevation as the observation data, the accuracy is better than any other combination. With the comparison of the accuracy of results and processing time, it is better to use the numerical method with more perturbation terms and range, azimuth, elevation as observation data.

6 References

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The numerical results of cases

	axis	error average	standard deviation	SEP
case I	x-axis	-8919.467533	9819.081307	1.2394e+4
	y-axis	-7988.170847	6211.623286	
	z-axis	8438.421901	8030.786532	
case II	x-axis	995.176347	3356.567142	6.2699e+3
	y-axis	517.314835	4536.381094	
	z-axis	1915.002479	4283.908935	
case III	x-axis	1001.527339	3318.700200	6.1863e+3
	y-axis	554.837603	4417.254886	
	z-axis	1904.076653	4249.751612	
case IV	x-axis	1001.750893	3343.010790	6.1829e+3
	y-axis	516.333264	4420.097381	
	z-axis	1916.545455	4245.982527	
case V	x-axis	1001.141510	3341.951488	6.1755e+3
	y-axis	543.026260	4411.048105	
	z-axis	1913.494628	4241.698764	
case VI	x-axis	998.021971	3336.455541	6.1369e+3
	y-axis	525.866260	4364.318261	
	z-axis	1897.627479	4219.717414	

The illustrative results of case VI