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해석적인 기구학을 이용한 다물체계의 동력학해석

이돈용, 염영일, 정완균
포항공과대학교 기계공학과

Dynamics of Multibody Systems with Analytical Kinematics

Donyong Lee*, Youngil Youm and Wankyun Chung
School of Mechanical Engineering
POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Abstract

In this paper, the equations of motion are constructed systematically for multibody systems containing closed kinematic loops. For the displacement analysis of the closed loops, we introduce a new mixed coordinates by adding to the reference coordinates, relative coordinates corresponding to the degrees of freedom of the system. The mixed coordinates makes easy derive the explicit closed form solution. The explicit functional relationship expressed in closed form is of great advantages in system dimension reduction and no need of an iterative scheme for the displacement analysis. This forms of equation are built up in the general purpose computer program for the kinematic and dynamic analysis of multibody systems.

1 Introduction

During the last two decades a considerable effort has been focused on the development of methods and computational algorithms for mathematical modeling and simulation of multibody systems. These methods are mainly dealt with effective formulation and numerical solution of the dynamic equations of a general multibody system. The first problem encountered at the time of modeling the motion of a multibody system is that of finding an appropriate system of coordinates. A first choice is that of using a system of independent coordinates, whose number coincides with the number of degrees of freedom of motion of the multibody system and is thereby minimal. The most important types of coordinates currently used to define the motion of planar and three-dimensional multibody systems are relative coordinates, reference point coordinates(also called Cartesian coordinates), and natural coordinates(also called fully Cartesian coordinates).

Relative coordinates were the first ones used in the general purpose planar and three-dimensional analysis programs of Paul and Krajcinovic[7], Sheth and Uicker[9], and Smith et al.[10]. This coordinates define the position of each element in relation to the previous element in the kinematic chain by using the parameters or coordinates corresponding to the relative degrees of freedom allowed by the joint linking these elements. The advantages of relative coordinates can be summarized as, the reduced number of coordinates, hence good numerical efficiency, and suited for open-chain configurations of the system. However, the followings are considered to be the most important difficulties of the relative coordinates that the mathematical formulation can be more involved, because the absolute position of an element depends on the positions of the previous elements in the kinematic chain.

The reference point coordinates try to remedy the disadvantages of the relative coordinates by directly defining, using three coordinates or parameter, the absolute position of each one of the elements of the planar system. This is done by determining the position of a point of the element(the reference point, which often is the center of gravity) with two Cartesian coordinates, and by determining with an angle the orientation of the body in relation to a system of inertial axes. The advantages of reference point coordinates can be listed such that the position of each element is directly determined; hence the formulation is easier with less preprocessing and postprocessing requirements, and unlike relative coordinates, constraint equations are established at a local level.

Natural coordinates were originally introduced by García de Jalón et al. [1] and Serna et al.[8] for planar cases, and García de Jalón et al.[2] for spatial systems. In the case of planar multibody systems, natural coordinates can be considered as an evolution of the reference point coordinates in which the points are moved to the joints or to other important points of the elements, so that each element has at least two points. It is important to point out that since each body has at least two points, its position and angular orientation are determined by the Cartesian coordinates of these points, and the angular variables used by reference point coordinates are no longer necessary. It will be seen later on that this simplifies the formulation of the constraint equations along with the fact that points can be shared at the joints. Thus the natural coordinates in the case of planar multibody systems are made up of Cartesian coordinates of a series of points.

ACUBE(Advanced Analytical Analyzer for kinematics and dynamics of mechanical systems) software is a set of general purpose computer programs that can be used to model and predict the motion of a variety of real world mechanical systems using the closed-form solution modules. Based on a set of data that describes the machine to be modeled, the ACUBE builds a mathematical model of the real system that calculates positions, velocities, and accelerations of the various parts of the machine, as well as resultant forces that act in the system. By using the proposed software, the designer can simulate the behavior of a wide range of alternate designs prior to building and testing prototypes of mechanical systems.

2 A New Mixed Coordinates

It was mentioned previously that one of the advantages of the relative coordinates is the possibility of directly accounting for the relative degrees of freedom permitted by the joints. This type of coordinates allows the direct inclusion of motors or actuators at the joint with no further difficulties. On the other hand, neither natural coordinates nor reference point coordinates have this advantage. However, mixed coordinates can solve this problem.

Mixed coordinates are obtained by adding, to natural coordinates, angular or linear variables corresponding to the degrees of freedom of the system joints. When considering mixed coordinates, joint variables do not replace the other coordinates; rather they are simply added to them. When increasing the number of dependent coordinates without modifying the number of degrees of freedom, one should increase the number of constraint equations by the same amount. García de Jalón et al.[1] and Serna et al.[8] formulated the equation of motion of mechanical systems with this mixed coordinates.

Some authors as Jerkovsky [4] and Kim and Vanderploeg [5] use two different coordinate systems in two stages of the analysis. First, they describe the mechanism using reference point coordinates, and then they perform the analysis using relative coordinates, hoping this will be more effective. This successive use of two different types of coordinates is also called velocity transformations, and should be distinguished from the use of mixed coordinates.

In this paper, we introduce a new mixed coordinates by adding to the reference coordinates, the angular or linear variables corresponding to the degrees of freedom of the system joints and apply the coordinates to the dynamics of mechanical systems containing a compound linkage constraint allowing relative 4-DOF between two bodies i, j such as shown in Fig.(1). This coordinates has the advantages of easy formulation of relative degree of freedom permitted by the joint at local level. As an example, Fig.(2) shows the planar four bar mechanism with this mixed coordinates where the relative coordinate is ψ_4. Those 7 constraint equations with 7 coordinates(1 relative one, 6 generalized coordinates) notated by $\vec{r}$ are obtained as follows

$$\vec{\Phi} = \begin{pmatrix} (x_1 - x_0) - \frac{\ell_1}{2}\cos\psi_1 \\ (y_1 - y_0) - \frac{\ell_1}{2}\sin\psi_1 \\ (x_3 - x_1) - \frac{\ell_1}{2}\cos\psi_1 + \frac{\ell_3}{2}\cos\psi_3 - \ell_2\cos(\psi_1+\psi_4) \\ (y_3 - y_1) - \frac{\ell_1}{2}\sin\psi_1 + \frac{\ell_3}{2}\sin\psi_3 - \ell_2\sin(\psi_1+\psi_4) \\ (x_3 - x_D) - \frac{\ell_3}{2}\cos\psi_3 \\ (y_3 - y_D) - \frac{\ell_3}{2}\sin\psi_3 \\ \psi_1 - f(t) \end{pmatrix} = \vec{o} \quad (1)$$

which has 1 degree of freedom or 1 independent coordinates, The 7 constraint equations can be combined to be the single input-output relationship of the input angle, ψ_1 with the output angle, ψ_3 expressed as

$$C_1(\psi_1)\sin\psi_3 + C_2(\psi_1)\cos\psi_3 + C_3(\psi_1) = 0 \quad (2)$$

which equation is called closed form solution of the mechanism or analytical kinematic relationship. The another unknown coordinates, x_1, x_3, ψ_4 can be computed straightforwardly. As using the analytical relationship, we can avoid resorting to an iterative method for the displacement analysis of the mechanism.

3 Dynamic Analysis with the Independent Coordinates

The system equation of motion may be written in the following forms of constrained variational equations of motion:

$$\delta\vec{r}^T[M\ddot{\vec{r}} - \vec{Q}] = \vec{o} \quad (3)$$

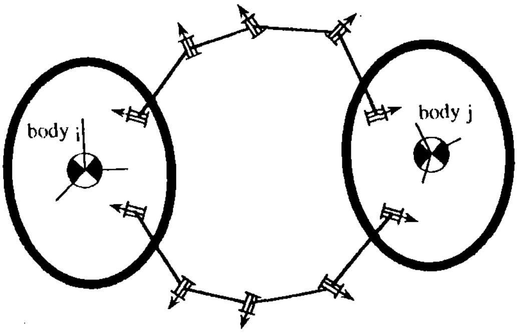

Figure 1: R^{10} linkage constraint

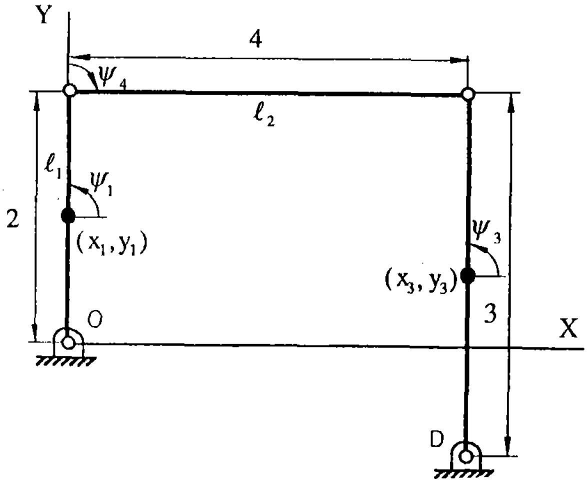

Figure 2: Four-bar mechanism with a new mixed coords

for all virtual displacements $\delta\vec{r}$ that are consistent with constraints that act on the system where M is inertia matrix and $\vec{Q}$ is generalized force vector associated with $\vec{r}$.

The combined set of kinematic and driving constraints is written in the form

$$\vec{\Phi}(\vec{r}, t) = \vec{0} \tag{4}$$

Since generalized coordinate variations(or virtual displacements) $\delta\vec{r}$ are considered to occur with time held fixed, the condition for a kinematically admissible virtual displacement $\delta\vec{r}$ is obtained by taking the differential of Eq.(4) with time held fixed; that is,

$$\vec{\Phi}_{\vec{r}}\delta\vec{r} = \vec{0} \tag{5}$$

where the Jacobian is evaluated at the state $\vec{r}$ that satisfies Eq.(4). Thus, the constrained variational equations of motion are that Eq.(3) hold for all virtual displacements $\delta\vec{r}$ that satisfy Eq.(5).

We can also derive the relationship as

$$\vec{r} = \vec{r}(\vec{q}) \tag{6}$$

where $\vec{q}$ is the independent coordinates. The condition for kinematically admissible virtual displacements of $\delta\vec{q}$ is

$$\delta\vec{r} = \vec{r}_{\vec{q}}\delta\vec{q} \tag{7}$$

Differentiating the Eq.(6) with time, we can get the velocity and acceleration relationship as

$$\dot{\vec{r}} = \vec{r}_{\vec{q}}\dot{\vec{q}} \tag{8}$$

$$\ddot{\vec{r}} = \vec{r}_{\vec{q}}\ddot{\vec{q}} + (\vec{r}_{\vec{q}}\dot{\vec{q}})_{\vec{q}}\dot{\vec{q}} \tag{9}$$

Substituting Eq.(7) into Eq.(5) yields

$$\vec{\Phi}_{\vec{r}}\vec{r}_{\vec{q}}\delta\vec{q} = \vec{0} \tag{10}$$

and into Eq.(3), we obtain the form of variational equation of motion about $\vec{q}$ as

$$\delta\vec{q}\left\{\vec{r}_{\vec{q}}^{T}M\vec{r}_{\vec{q}}\ddot{\vec{q}} + \vec{r}_{\vec{q}}^{T}M(\vec{r}_{\vec{q}}\dot{\vec{q}})_{\vec{q}}\dot{\vec{q}} - \vec{r}_{\vec{q}}^{T}\vec{Q}\right\} = \vec{0} \tag{11}$$

For arbitrary vector $\delta\vec{q}$, the coefficients matrix is equal to zero for satisfying Eq.(10) and (11) in the configuration of the nonsingular $\vec{\Phi}_{\vec{q}}$ and $\vec{r}_{\vec{q}}$ as

$$\vec{\Phi}_{\vec{q}}\vec{r}_{\vec{q}} = \vec{0} \tag{12}$$

$$\vec{r}_{\vec{q}}^{T}M\vec{r}_{\vec{q}}\ddot{\vec{q}} = \vec{r}_{\vec{q}}^{T}\vec{Q} - \vec{r}_{\vec{q}}^{T}M(\vec{r}_{\vec{q}}\dot{\vec{q}})_{\vec{q}}\dot{\vec{q}} \tag{13}$$

Therefore, using the independent coordinates we must not consider the constraint involved in the mechanical model with the Lagrangian multiplier because $\vec{q}$ is independent to the kinematic constraint.

4 Example

Consider the planar four bar mechanism as shown in Fig.(2) at initial state. For the mechanism, the inertia properties and initial conditions are set as: $m_1 = 22$, $i_1 = 11$, $m_2 = 22$, $i_2 = 11$, $m_3 = 22$, $i = 11$, $m_4 = 0$, $i_4 = 0$, and $\psi_1(0) = 90$(deg), $\dot{\psi}_1 = 2\pi$. The data input file named as fourbar.dat for ACUBE software is arranged as follows where the command is detected after ">" symbol.

```
ACUBE
August 22, 1994
filename fourbar.dat
This data file test the closed planar four bar constraint library.
>total_number_of_body
2
>planar_fourbar
0 0. 0. 4. -1.
1 -2. 0. 2. 0.
```

```
-1. 0. -1.5 0.
2. 3. 0. 0.
Ground link configuration, 4-th generalized coordinates
>absolute_x_position_constraint
0 0. 0. 0. 0. 0.
>absolute_y_position_constraint
0 0. 0. 0. 0. 0.
>absolute_angular_constraint
0 0. 0. 0.
Driver configuration, i-th generalized coordinates
>selected_angular_driver
0
1 1.5708 0. 0. 0. 0. 0. 0. 0. 0. 0.
1 0.10965 0. 0. 0. 0. 0. 0. 0. 0. 0.
0 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
At time = 0, the configuration of all generalized coordinates.
>initial_generalized_coordinates
0. 0. 0. 1.5708 1.5708 2. 2. 0.
mass and inertia of links
>inertia_properties
22. 22. 22. 0.
11. 11. 11. 0.
End of data sets
>END_OF_DATA
```

5 Discussion

The dynamic response of the follower link using the ACUBE is some different with the result of the commercial dynamics package ADAMS because of the selected integration algorithm as shown in Fig.(3). The CPU time of the preposed mixed coords. and closed form solution modules is faster than that of the reference coords. and the comparison of system coordinates dimension are shown in the table.

	mixed coords.	reference coords.
CPU time(sec)	10.923	43.791
dim. of coords.	1	12

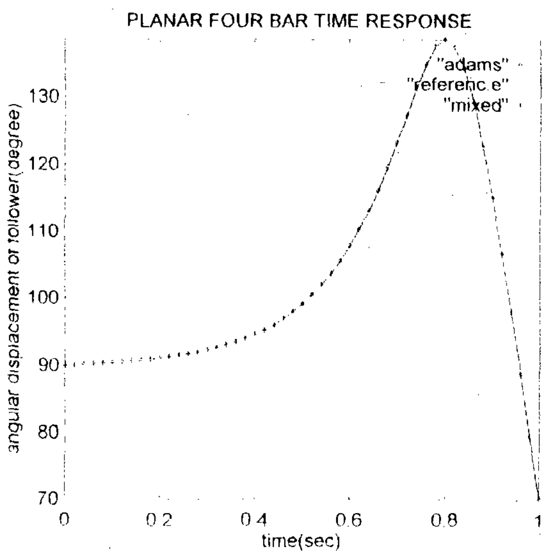

Figure 3: The follower response

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