'94 KACC (1994. 10. 17 ~ 20)

Vibration Control of Multi-Degree-of-Freedom Structure by Nonlinear H_∞ Control

†Kenta KUBOTA   Mitsuji SAMPEI

Department of Control and Systems Engineering,
Tokyo Institute of Technology
2-12-1 Oh-Okayama Meguro-ku Tokyo, JAPAN

Abstract

This study is concerned with H_∞ control theory of nonlinear systems. Recently H_∞ control theory has been developed to nonlinear systems, and especially nonlinear H_∞ control theory based on the Hamilton-Jacobi inequality has been proposed. This corresponds to linear H_∞ control theory based on the Riccati equation. In this paper, we apply it to a semi-active dynamic vibration absorber for multi-degree-of-freedom structure, and we design its state feedback controller via the Riccati equation. In the simulation, we show that it is effective for a vibration control.

1 Introduction

As the building technology is improved, high buildings are increasing in recent years. So importance of the vibration control of buildings is rising because it is necessary to improve living amenity and safety (i.e. earthquake-proof building) . Therefore, there are a lot of studies of the vibration control of the multi-degree-of-freedom structure using the DVA (dynamic vibration absorber). Since the vibration control is deeply concerned with natural frequency of the plant, H_∞ control theory, which is a design method of frequency domain, is effective in designing the controller of vibration control systems.

Usually the active DVA, of which control input is the force of actuator, is used for vibration control of building, while the semi-active DVA, of which control input is the coefficient of damping or spring, is treated in this study. The semi-active DVA is more efficient in terms of energy than the active DVA. However in the state equation of the plant, there is a product of the state and the control input. So the system becomes nonlinear, and there is no design method of it.

In this study, we apply nonlinear H_∞ control theory[1], which is based on the Hamilton-Jacobi inequality, to the semi-active DVA for four-degree-of-freedom structure. First, we tune up the system in passive control enough. Secondly, we vary the coefficient of damping by state feedback of nonlinear H_∞ control, and improve the performance of the system much more. We design its state feedback controller from the Riccati equation.

Finally, we show efficiency of nonlinear H_∞ control by simulation for the designed control system.

2 Modeling

We consider the model of the four-degree-of-freedom structure [Fig.1]. When the disturbance $\ddot{q}$ accelerates the earth, the DVA , which are installed above the top floor, controls the vibration of the building.

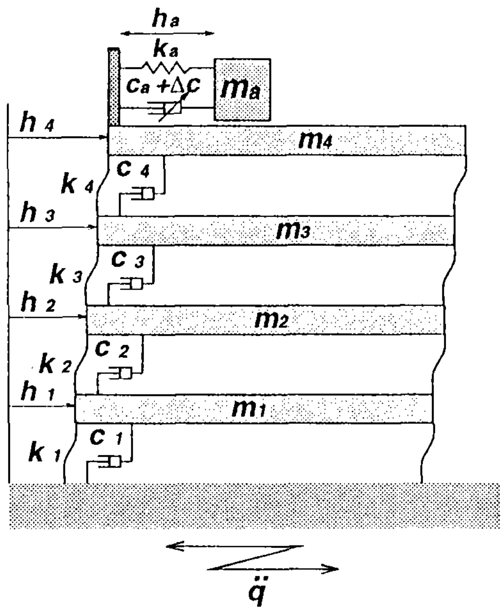

Fig. 1: Model of multi-degree-of-freedom structure

The parameter of the structure is shown in Table 1. While c_a and k_a of the DVA is constant, Δc is variable; Δc is the control input.

Table 1: Specification of model

	Main Structure	DVA
Mass	$m_1 = 1.62\ kg, m_2 = 1.48\ kg$ $m_3 = 1.48\ kg, m_4 = 2.20\ kg$	$m_a =$ $0.135\ kg$
Damping Constant	$c_i = 0.08\ N\ s/m$ $(i = 1,2,3,4)$	$c_a = 0.37$ $N\ s/m$
Spring Constant	$k_i = 2,600\ N/m$ $(i = 1,2,3,4)$	$k_a = 22.0$ N/m
Natural Frequency (Main Structure)		
$\omega_1 = 13.2\ rad/sec,$	$\omega_2 = 38.7\ rad/sec$	
$\omega_3 = 61.1\ rad/sec,$	$\omega_4 = 77.6\ rad/sec$	

2.1 Equation of motion

The equation of motion, which is concerned with the building of Fig.1, is shown as follows.

$$M_s\ddot{h}_s + C_s\dot{h}_s + K_s h_s + C_{12}\dot{h}_a + K_{12}h_a = E_s\ddot{q} + L_s\dot{h}_a\, u \quad (1)$$

$$\begin{aligned}
M_s &= diag[m_1, m_2, m_3, m_4] \\
C_s &= \begin{bmatrix} c_1+c_2 & -c_2 & 0 & 0 \\ -c_2 & c_2+c_3 & -c_3 & 0 \\ 0 & -c_3 & c_3+c_4 & -c_4 \\ 0 & 0 & -c_4 & c_4 \end{bmatrix} \\
K_s &= \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix} \\
E_s &= \begin{bmatrix} -m_1 & -m_2 & -m_3 & -m_4 \end{bmatrix}^T \\
L_s &= \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T \\
C_{12} &= \begin{bmatrix} 0 & 0 & 0 & -c_a \end{bmatrix}^T \\
K_{12} &= \begin{bmatrix} 0 & 0 & 0 & -k_a \end{bmatrix}^T \\
h_s &= \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \end{bmatrix}^T \\
u &= \Delta c
\end{aligned}$$

In addition, the equation of motion, which is concerned with the DVA, is shown as follows.

$$m_a\ddot{h}_a + c_a\dot{h}_a + k_a h_a = -m_a\ddot{q} - m_a\ddot{h}_4 - \dot{h}_a u$$

We eliminate $\ddot{h}_4$, then we have

$$\ddot{h}_a + C_{22}\dot{h}_a + K_{22}h_a + \frac{c_4}{m_4}\dot{h}_3 - \frac{c_4}{m_4}\dot{h}_4 + \frac{k_4}{m_4}h_3 - \frac{k_4}{m_4}h_4 = L_a\dot{h}_a\, u, \quad (2)$$

where

$$\begin{aligned}
C_{22} &= (\frac{1}{m_4} + \frac{1}{m_a})c_a \\
K_{22} &= (\frac{1}{m_4} + \frac{1}{m_a})k_a \\
L_a &= -(\frac{1}{m_4} + \frac{1}{m_a}).
\end{aligned}$$

We combine the equation (1) with (2), then

$$M\begin{bmatrix} \ddot{h}_s \\ \ddot{h}_a \end{bmatrix} + C\begin{bmatrix} \dot{h}_s \\ \dot{h}_a \end{bmatrix} + K\begin{bmatrix} h_s \\ h_a \end{bmatrix} = E\ddot{q} + L\dot{h}_a\, u, \quad (3)$$

where

$$M = \begin{bmatrix} M_s & O \\ O & 1 \end{bmatrix},\ E = \begin{bmatrix} E_s \\ 0 \end{bmatrix},\ L = \begin{bmatrix} L_s \\ L_a \end{bmatrix}$$

$$C = \begin{bmatrix} C_s & C_{12} \\ C_{21} & C_{22} \end{bmatrix}, C_{21} = \begin{bmatrix} 0 & 0 & \frac{c_4}{m_4} & -\frac{c_4}{m_4} \end{bmatrix}$$

$$K = \begin{bmatrix} K_s & K_{12} \\ K_{21} & K_{22} \end{bmatrix}, K_{21} = \begin{bmatrix} 0 & 0 & \frac{k_4}{m_4} & -\frac{k_4}{m_4} \end{bmatrix}.$$

We now use the modal analysis; we change coordinates as follows.

$$\begin{bmatrix} h_s \\ h_a \end{bmatrix} = \Phi\xi = \begin{bmatrix} \Phi_s & O \\ O & 1 \end{bmatrix}\begin{bmatrix} \xi_s \\ h_a \end{bmatrix}$$

Φ_s is the normalized modal matrix such that

$$\Phi_s{}^T M_s \Phi_s = I.$$

The equation (3) can now be written as

$$\ddot{\xi} + \Lambda\dot{\xi} + \Omega^2\xi = e\ddot{q} + l\dot{h}_a\, u, \quad (4)$$

where

$$\Lambda = \Phi^T C\Phi,\ \ \Omega^2 = \Phi^T K\Phi,\ \ e = \Phi^T E,\ \ l = \Phi^T L.$$

2.2 State Equation

We define the state x_f and the output y such that

$$x_f = \begin{bmatrix} \xi \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \xi_s \\ h_a \\ \dot{\xi}_s \\ \dot{h}_a \end{bmatrix},\ \ y = \begin{bmatrix} h_1 \\ h_4 \\ h_a \end{bmatrix}.$$

Then we have, from the equation (4),

$$\begin{aligned}
\dot{x}_f &= A_f x_f + B_f(x_f)u + D_f\ddot{q} \\
y &= C_f x_f,
\end{aligned} \quad (5)$$

where

$$A_f = \begin{bmatrix} O & I \\ -\Omega^2 & -\Lambda \end{bmatrix}, B_f(x_f) = \begin{bmatrix} o \\ l\,\dot{h}_a \end{bmatrix}, D_f = \begin{bmatrix} o \\ e \end{bmatrix}.$$

In addition, we construct the reduced order model, which is concerned with the first and second mode and then

$$\begin{aligned}
\dot{x}_r &= A_r x_r + B_r(x_r)u + D_r\ddot{q} \\
y &= C_r x_r.
\end{aligned} \quad (6)$$

In the equation (6),$B_r(x_r)$ includes the state x_r; The systems of (6) are nonlinear. In addition, if $x_r = o$, then $B_r(x_r) = o$, and the control input u has no effect on the systems (6).

3 Construction of Control system by nonlinear H_∞ control

3.1 Generalized plant

Using the nonlinear H_∞ state feedback, we construct the control system; that is shown in Fig.2. Then $w = \ddot{q}$, and W is the weighting function on the output y.

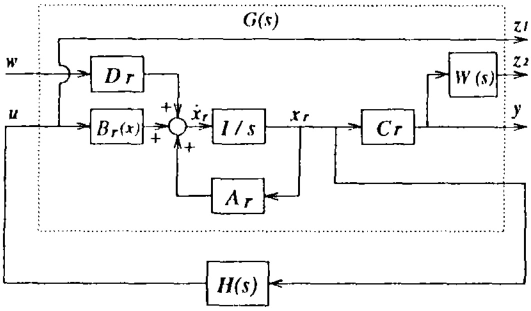

Fig. 2: Block diagram of generalized plant

3.2 Weighting function and controlled output

The weighting function W, whose input is y, is shown as follows.

$$z_2 = W(s)y$$

$$W(s) = diag[W_s, W_s, W_a] = \left[\begin{array}{c|c} A_W & B_W \\ \hline C_W & D_W \end{array}\right]$$

We choose W_s and W_a as

$$W_s = \frac{512.0}{s+40.0}, \quad W_a = \frac{36.0}{s+40.0}$$

so that we can control the vibration at the lower frequency than the second mode. In addition, z_1 is equal to the control input u.

3.3 State equation of $G(s)$

In this subsection, we get the state equation of $G(s)$ in Fig.2. We define the state x such that

$$x = \begin{bmatrix} x_r \\ x_W \end{bmatrix}.$$

Then we have

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2(x)u \\ z &= C_1 x + D_{12} u \\ y &= C_2 x, \end{aligned} \tag{7}$$

where

$$A = \begin{bmatrix} A_r & O \\ B_W C_r & A_W \end{bmatrix}, \quad B_1 = \begin{bmatrix} D_r \\ O \end{bmatrix}$$

$$B_2(x) = \begin{bmatrix} B_r(x) \\ O \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 1 \\ o \end{bmatrix}$$

$$C_1 = \begin{bmatrix} O & O \\ D_W C_r & C_W \end{bmatrix}, \quad C_2 = [\ C_r \quad O\ ].$$

3.4 Nonlinear H_∞ Control

We consider following nonlinear systems:

$$\begin{aligned} \dot{x} &= f(x) + g_1(x)w + g_2(x)u \\ z &= h_1(x) + j_{12}(x)u \end{aligned} \tag{8}$$

where $x \in R^n$ is the state, $w \in R^p$ is the disturbance, $u \in R^m$ is control input, and $z \in R^q$ is the controlled output. For simplicity, we assume that following condition holds.

$$h_1^T j_{12} = o, \ j_{12}^T j_{12} = I$$

Nonlinear H_∞ control problem For the system by (8), find the state feedback controller $u = k(x)$ which satisfies the following; The closed loop system S_{zw}, which is given by (8) and $u = k(x)$, is internally stable.In addition,

$$\|S_{zw}\|_{L2e} = \sup_{w \in L_2/\{o\}} \frac{\|S_{zw}w\|_2}{\|w\|_2} = \sup_{w \in L_2/\{o\}} \frac{\|z\|_2}{\|w\|_2} \le \gamma$$

Solvability of Nonlinear H_∞ Control Problem The nonlinear H_∞ control problem is solvable, if and only if there exsist two positive definite function, $\phi(x)$ and $\rho(x)$; these satisfy following two conditions [1].

$$\begin{aligned} &\frac{\partial \phi}{\partial x^T} f + \frac{1}{4\gamma^2}\frac{\partial \phi}{\partial x^T} g_1 g_1^T \frac{\partial \phi}{\partial x} + h_1^T h_1 + \rho \\ &\qquad - \frac{1}{4}\frac{\partial \phi}{\partial x^T} g_2 g_2^T \frac{\partial \phi}{\partial x} \le 0 \end{aligned} \tag{9}$$

$$\lim_{\|x\| \to 0} \|g_1^T \frac{\partial \phi}{\partial x}\|^2 / \rho < \infty \tag{10}$$

When there exist $\phi(x)$ and $\rho(x)$, one of nonlinear state feedback controllers can be given by

$$k(x) = -\frac{1}{2} g_2^T \frac{\partial \phi}{\partial x}. \tag{11}$$

3.5 State feedback controller for $G(s)$

Comparing (7) with (8), we get $f(x) = Ax, g_1(x) = B_1, g_2(x) = B_2(x), h_1(x) = C_1 x$ and $j_{12}(x) = D_{12}$. Then for all x,

$$\begin{aligned} h_1^T j_{12} &= x^T C_1^T D_{12} = 0 \\ j_{12}^T j_{12} &= D_{12}^T D_{12} = 1. \end{aligned}$$

The equation (9), one of conditions for solvability of the nonlinear H_∞ problem, can be written as follows.

$$\frac{\partial\phi}{\partial x^T}Ax + \frac{1}{4\gamma^2}\frac{\partial\phi}{\partial x^T}B_1B_1^T\frac{\partial\phi}{\partial x} + x^TC_1^TC_1x + \rho$$
$$-\frac{1}{4}\frac{\partial\phi}{\partial x^T}B_2(x)B_2^T(x)\frac{\partial\phi}{\partial x} \leq 0 \qquad (12)$$

We now choose the positive definite function $\phi(x)$ and $\rho(x)$ such that

$$\phi = x^TPx$$
$$\rho = \varepsilon x^Tx,$$

where P is a positive definite symmetric matrix, and ε is an sufficiently small positive number. Then we have

$$\frac{\partial\phi}{\partial x} = Px + P^Tx, \quad \frac{\partial\phi}{\partial x^T} = x^TP + x^TP^T.$$

The inequality (12) can be written as

$$x^T(PA + A^TP + \frac{1}{\gamma^2}PB_1B_1^TP + C_1^TC_1 + \varepsilon I)x$$
$$-x^TPB_2(x)B_2^T(x)Px \leq 0. \qquad (13)$$

If we choose the positive definite symmetric matrix P as the solution of the Riccati equation such that

$$PA + A^TP + \frac{1}{\gamma^2}PB_1B_1^TP + C_1^TC_1 + \varepsilon I = O, \quad (14)$$

then the left side member of the inequality (13) is

$$-x^TPB_2(x)B_2^T(x)Px.$$

This is zero or negative for all x. As a result, the condition (9) has been satisfied. In addition, the condition (10) is obviously satisfied by ϕ and ρ .So the condition for solvability of nonlinear H_∞ control problem has been satisfied. Then from the equation (11), the state feedback controller is

$$u = k(x) = -B_2^T(x)Px$$

Remark If $x = o$, then $B_2(x) = o$ and $u = o$; the state feedback controller has no effect, when the state is nearly equal to zero, but it has an effect when the amplitude of the vibration is large ($x \gg 0$).

Remark Finding the positive definite symmetric matrix P, which satisfies the Riccati equation (14), is equivalent to the following; we consider the linear systems:

$$\dot{x} = Ax + B_1w$$
$$z = C_1x$$

We find the necessary and sufficient condition for solvability of the linear H_∞ problem as follows.

- A is stable
- $\|C_1(sI - A)^{-1}B_1\|_\infty < \gamma$

In addition, this means the following; when $u = c_a = 0$ (passive control), the generalized plant (7) is stabilized, and the ∞-norm of the transfunction, from the disturbance w to the controlled output z, is less than γ.

3.6 Design method of control system

We design the control system by the following method.

- We choose c_a and k_a, so that in $u = \Delta c = 0$ (passive control), the control system can be stabilized and the DVA can control the first mode of vibration.
- By solving the Ricatti equation, we get the state feedback controller. Then the input $u = \Delta c$ can be defined, and the performance is improved much more.

4 Simulation

Now we get the feedback controller for the reduced order plant, and then we execute the simulation with the controller and the full order plant.

We choose $\gamma = 1.0$, $\varepsilon = 1.0 \times 10^{-4}$ in the Riccati equation (14). In the simulation, we limit the input $u = \Delta c$ so that $c + \Delta c$ may not be negative.

4.1 Result and consideration

We show results of simulation from Fig.3 to Fig.6.

First, when the small sine wave disturbance of first mode frequency inputs, the response of h_1 (the displacement of first floor of building) is shown in Fig.3. There is no difference between the passive control ($u = 0$) and the nonlinear state feedback control.

Secondly, when the large sine wave (the 100 times the former) disturbance inputs, the response of h_1 is shown in Fig.4. The response in the state feedback control is less amplitude of vibration than that in passive control. The amplitude of the disturbance make a difference, because $B_f(x_f)$ is nearly equal to zero in the small vibration ($\dot{h}_a \approx 0$), and then u is not effective.

Finally, when the small and large sine disturbance of second mode frequency inputs, the response of h_1 is shown in Fig.5 and Fig.6. In the case of small disturbance, the state feedback controller has no effect, but in the case of large disturbance, the state feedback controller has considerable effect.

In the case of large disturbance, the state feedback controller is more effective in the case of the second

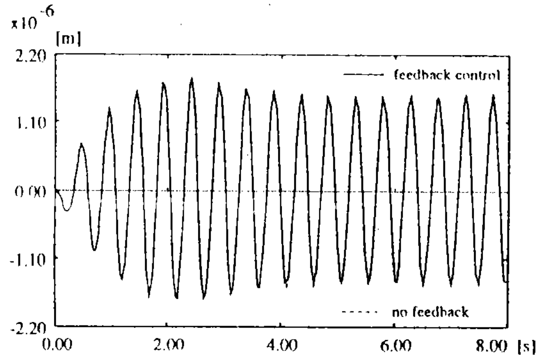

Fig. 3: Time responses of h_1 : $\ddot{q} = 1.0 \sin 13.0t \times 10^{-4}$

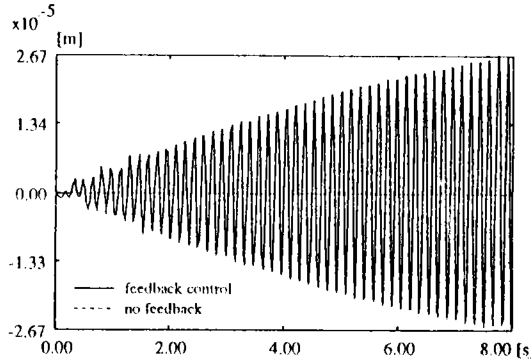

Fig. 5: Time responses of h_1 : $\ddot{q} = 1.0 \sin 39.0t \times 10^{-3}$

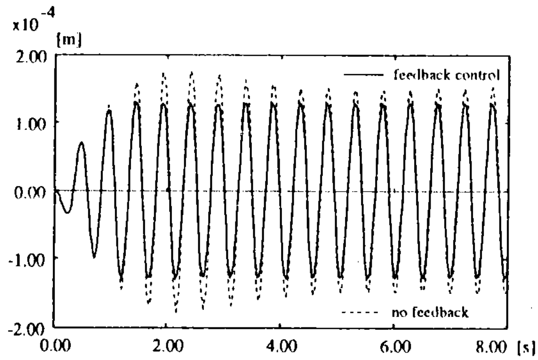

Fig. 4: Time responses of h_1 : $\ddot{q} = 1.0 \sin 13.0t \times 10^{-2}$

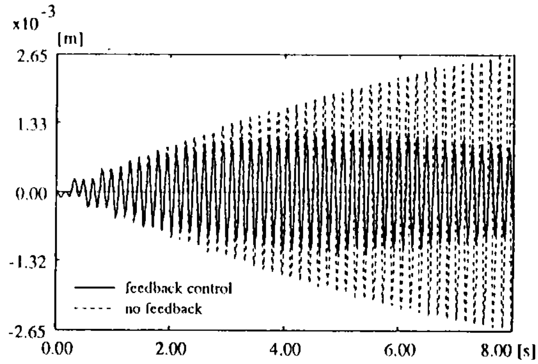

Fig. 6: Time responses of h_1 : $\ddot{q} = 0.1 \sin 39.0t$

mode than of the first mode. This means the followings; We tuned up k_a and c_a to control the first mode oscillation, and so in the first mode the feedback controller doesn't improve the performance very much, but in the second mode, the feedback controller is effective.

5 Conclusion

We sum up the results of application of nonlinear H_∞ control as follows.

- We determine the nonlinear H_∞ state feedback controller by solving the Riccati equation.
- While the nonlinear H_∞ state feedback controller is not effective against small disturbance very much, it is effective against large one.
- The nonlinear H_∞ state feedback controller improves the performance in the second mode vibration.

References

[1] J.Imura T.Sugie T.Yoshikawa. H_∞ control of nonlinear systems based on the hamilton-jacobi inequalities. The 36th Japan Joint Automatic Control Conference, 1993.

[2] T.Sugie J.Imura. H_∞ control for nonlinear systems. SICE Symposium on Robust Control, 1994.

[3] M.Sampei K.Kubota. Possibility of application of nonlinear H_∞control to vibration control of multidegree-of-freedom structure. 23rd SICE Symposium on Control Theory, 1994.