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The Construction of a Robust Model following System for an Unkown Plant

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Abstract

In this paper the system called the inverse model compensation system is proposed as a system whose input-output transfer function can be regarded as that of a model with uncertainty in spite of including an unknown plant. And then to construct the robust model following system, which is of low sensitivity and robust stability, in order to control the inverse model compensation system is proposed. The simulation experiments show that the robust model following system including the inverse model compensation system is practical and useful as a system which controls unknown plants.

1 Introduction

When we design a control system, we first require the dynamics of the controlled plant. That is, we have to begin by identifying the plant. The many robust system design techniques enabled us to roughly identify the plant. But there are plants, such as a robot arm, of which identification is very difficult. In this paper we propose a control design technique for the single input and output plant whose transfer function is given as a ratio of a polynomial in s, but order and paramaters are quite unknown. The proposed design procedures are made up of the following two steps: The first step is to construct the inverse model compensation system [IMCS] in which an unknown plant is included and whose input-output transfer function can be regarded as the model with uncertainty. In the second step, we apply a robust control system design technique to the IMCS.

2 Basic inverse model compensation system[BIMCS]

We consider an SISO plant whose input-output can be expressed as a linear differential equation with unknown order and coefficients. We call the plant like this the unknown plant. We construct the system shown in Fig.1, which is called a basic inverse model compensation system, where $P(s)$ is the input-output transfer function of the plant, $M(s)$ is of the model and $M^{-1}(s)$ is of the inverse model.

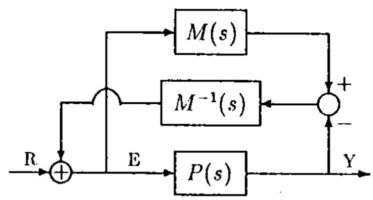

Fig.1 Basic inverse model compensation system

In the BIMCS we obtain

$$E = M^{-1}(s)(M(s) - P(s))E + R \qquad (1)$$

and then

$$E = \frac{M(s)}{P(s)}R \qquad (2)$$

Therefore we can derive the relation

$$Y = M(s)R \qquad (3)$$

independent of $P(s)$. But the BIMCS has two problems awaiting solution. The first problem is stability.

The system in Fig.2 is equivalent to the BIMCS.

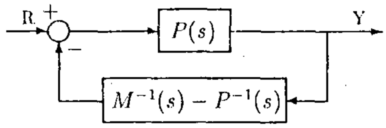

Fig.2 Equivalent system to the BIMCS

From the system in Fig.2 we can obtain the open loop transfer function as $G(s) = M(s)^{-1}P(s) - 1$ and the return difference as $1 + G(s) = M(s)^{-1}P(s)$. Therefore, drawing the graph of $1 + G(j\omega) = M(j\omega)^{-1}P(j\omega)$, for $-\infty < \omega < \infty$, stability of the BIMCS can be decided from Nyquist stability criterion. The transfer function $M(s)$ of the model must be strictly proper because the inverse model $M^{-1}(s)$ can be realized. This means that $1 + G(j\omega)$ may become 0 at high frequency and on that occasion the BIMCS may become unstable, because the graph of $1+G(j\omega)$, for $-\infty < \omega < \infty$, may encircle the origin. The second problem is that infinite gain appears in the BIMCS because of the positive feedback through the inverse model. In the next section we show a means of setting these problems.

3 Inverse model compensation system [IMCS]

We express explicitly the gain of the model as

$$M(s) = k\tilde{M}(s) \tag{4}$$

and the approximate inverse model as

$$\frac{1}{k}(\tilde{M}^{-1}(s) - \delta) \tag{5}$$

It is obvious that the infinite gain does not appear in the system when the approximate inverse model of eq.(5) is used instead of the exact inverse model. On the other hand, in this case the return difference can be expressed as

$$1 + G(s) = \frac{1}{k}(\tilde{M}^{-1}(s) - \delta)P(s) + \delta\tilde{M}(s) \tag{6}$$

The fact of

$$1+G(\infty) = \frac{1}{k}(\tilde{M}^{-1}(\infty)-\delta)P(\infty)+\delta\tilde{M}(\infty) = \delta\tilde{M}(\infty) \tag{7}$$

means that the graph of $1+G(j\omega)$, for $-\infty < \omega < \infty$, does not encircle the origin when δ is an appropriate value. Therefore, by using the approximate inverse model of eq.(5) instead of the exact inverse model we can solve the both problems of stabilizing the BIMCS and disappearing the infinite gain. We call the BIMCS with the approximate inverse model the IMCS.

The relation of

$$Y = M(s)(1 + \Delta(s))R \tag{8}$$

where

$$\Delta(s) = \frac{\frac{\delta}{k}M(s)(P(s) - M(s))}{P(s) - \frac{\delta}{k}M(s)(P(s) - M(s))} \tag{9}$$

can be derived from the IMCS.

Eq.(8) means that we can regard the input-output transfer function of the IMCS as M(s) with uncertainty $\Delta(s)$.

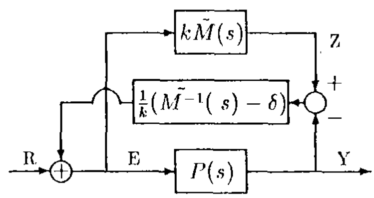

Fig.3 The inverse model compensation system

The reason why the form of eq.(5) is used as the approximate inverse model is that the large k yields the small uncertainty for the large δ which is necessary to obtain stability.

4 Robust model following system [1]

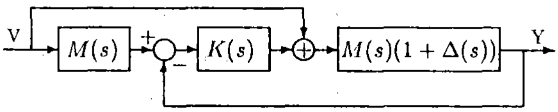

Fig.4 A Basic robust model following system

It is assumed that the input-output transfer function of the plant is expressed as

$$M(s)(1+\Delta(s)) \tag{10}$$

where $\Delta(s)$ represents uncertainty.

We consider the system shown in Fig.4 which is called a basic robust model following system. In this system the sensitivity and complementary sensitivity functions are obtained as

$$S(s) = \frac{1}{1+M(s)K(s)} \tag{11}$$

and

$$T(s) = 1 - S(s) = \frac{M(s)K(s)}{1+M(s)K(s)} \tag{12}$$

respectively.

Therefore, when we obtain the compensator K(s) as a solution of the mixed sensitivity problem in H infinity control theory under the performance index

$$\left\| \begin{matrix} W_S(s) & S(s) \\ W_T(s) & T(s) \end{matrix} \right\|_\infty \tag{13}$$

where $W_S(s)$ and $W_T(s)$ are appropriate weighting functions, low sensitivity and robust stability are guaranteed in the basic robust model following system. This means that in spite of existence of uncertainty $\Delta(s)$, we can regard the transfer function between V and Y as $M(s)$ because of low sensitivity and stability can be held because of robust stability.

It is assumed that the desired input-output properties can be realized by the system shown in Fig.5 which is called a reference model.

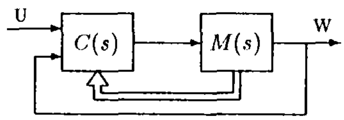

Fig.5 A reference model

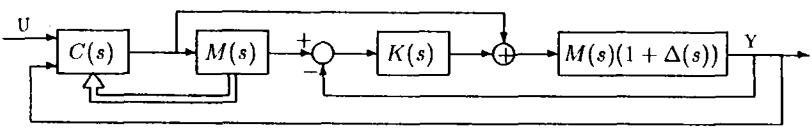

Fig.6 The robust model following system

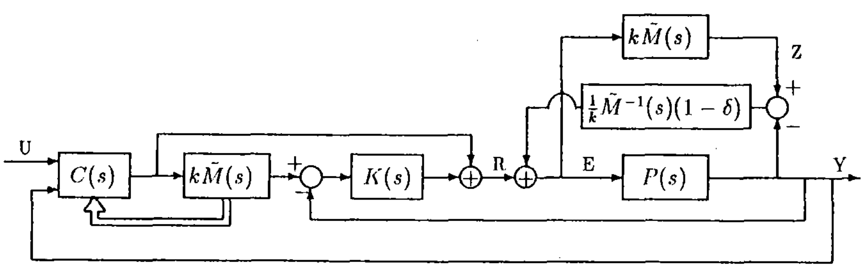

Fig.7 The robust model following system for an unknown plant

It is the only condition for the reference model that it must be constructed by using the nominal transfer function $M(s)$ of the plant. Consequently, we can construct the system shown in Fig.6 by uniting the basic robust model following system in Fig.4 and the reference model in Fig.5, which is called a robust model folloeing system.

It follows easily that the robust model following system is of low sensitivity and robust stability, and then has the same input-output properties as the reference model.

From above discussion, we can construct the robust model following system for an unknown plant such as shown in Fig.7.

5 Simulation

We show simulation results of the system shown in Fig.7, where

$$P(s) = \frac{1}{s^2 + 3s + 2} \tag{14}$$

$$k\tilde{M}(s) = \frac{s + 0.4}{s + 0.3} \tag{15}$$

$$\frac{1}{k}(\tilde{M}^{-1}(s) - \delta) = \frac{s + 0.3}{s + 0.4} - 0.1 \tag{16}$$

And we adopt the reference model as shown in Fig.8.

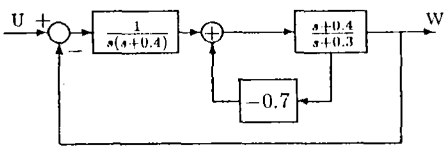

Fig.8 The reference model for simulation

We obtained

$$K(s) = \frac{0.0161s^3 + 140.7s^2 + 4152s + 1270}{s^3 + 1822s^2 + 2549s + 728.4} \tag{17}$$

by choosing eq(18) and eq(19) as the weighting functions.

$$W_S(s) = \frac{0.5(s + 3)}{s + 1} \tag{18}$$

$$W_T(s) = \frac{s + 100}{s + 10000} \tag{19}$$

Descretizing each element by using a sampler of which sampling period is 1msec and a zero-order hold device, we obtained the simulation results shown in Fig.9 and Fig.10. Fig.9 shows the step responses of the inverse model compensation system which was taken out from the system shown in Fig.7. The simulation result s in Fig.9 show that the IMCS is effective for unknown plants. Fig.10 shows the step responses of the robust model following system in Fig.7 and the reference model. The simulation results in Fig.10 show that the robust model following system including the IMCS is practical and useful as a system which controls unknown plants.

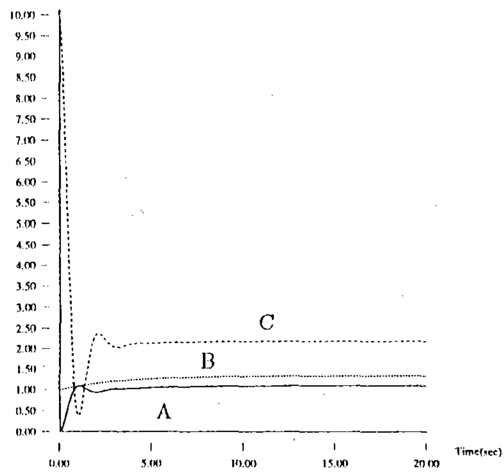

Fig.9 A, B and C show the step responses of Y, Z and E in the inverse model compensation system.

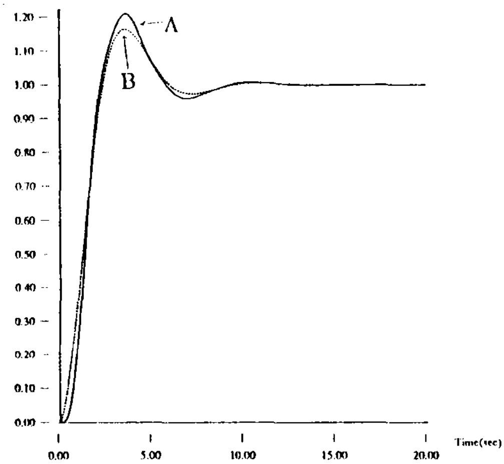

Fig.10 A and B show the step responses of the robust model following system in Fig.7 and the reference model, respectively.

6 Conclusion

We proposed the inverse model compensation system as a system of which input-output transfer function can be regarded as that of a model with uncertainty in spite of including an unknown plant. Moreover, we proposed to construct the robust model following system in order to control the inverse model compensation system.

7 Reference

[1] Hidekazu Hyogo, Yuji Kamiya and Koji Shibata: Construction of a robust compensation controller, '94 KACC