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$\mathcal{H}_\infty$ Control of Contact Position and Force of a Manipulator

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ABSTRACT

An application of $\mathcal{H}_\infty$ synthesis to contact control of a manipulator is suggested. Based on computed torque linearization of a manipulator, a target dynamics for contact motion control is defined and used as a reference model. The target dynamics relates position and force errors through free motion impedance and force error compensators. The $\mathcal{H}_\infty$ control synthesis is adopted to find an optimum the compensator for position and force control in various directions of the end-effector. The optimization is performed on the augmented criteria, which trades off the sensitivity function of the errors and the input load at the joints. A design example of the compensator is provided that meets the design specifications.

I. Introduction

Control of the end-effector in contact environment is difficult because the controller has to operate in two different states: free and contact motion. In free motion, a manipulator is controlled in a free workspace without contacting the environment. In contact motion, the end-effector of the manipulator interacts with the environment, which exerts external force on the manipulator. In the transition between free and contact motion, the reaction force changes from zero to a certain value or vice versa. This change makes it difficult to develop a reliable controller for contact motion of a manipulator.

During the last two decades, two main approaches in contact control have been developed: hybrid control (Raibert and Craig 1981) and impedance control (Hogan 1987). Hybrid control applies two control laws, one the direction of contact force and another along the remaining directions of unconstrained position control. The control torques obtained from each control law are incorporated to drive the actuators at the manipulator joint. Since the two kinds of control law are different, switching logic is required to transit between free motion and force control. Difficulties are that it is not easy to identify each direction for application of control law, and that inaccurate timing in transition between the two control laws can cause unpleasant vibration.

In impedance control, the relationship between the interaction force and the position of the end-effector is assigned so that, in a constrained environment, contact forces are appropriately maintained. In impedance control, transition between free motion and contact is smooth. It is, however, often difficult to select the impedance, especially when the environment is stiff (Lin 1992). The abrupt change of the stiffness causes high underdamped oscillatory response.

In the present paper, a control method is suggested that uses high-order target dynamics. The target dynamics assigns the impedance relationship between the position error and the compensated-force error of the end-effector in contact. The high-order compensator is selected so that it meets multi-specifications for both free motion and contact control.

As a design tool for the compensator, $\mathcal{H}_\infty$ control synthesis is adopted. Important consideration in optimal control is the choice of performance index to be minimized. The two typical indices are $\mathcal{H}_2$ and $\mathcal{H}_\infty$-norms (Grimble 1986). The former, $\mathcal{H}_2$ -norm minimization, minimizes the root-mean-square of errors and requires known covariance or power spectra of disturbance including unknown or unmodeled dynamics. The latter, $\mathcal{H}_\infty$-norm minimization, minimizes the worst possible case error and allows all signal having finite energy as system inputs. The $\mathcal{H}_\infty$-norm minimization requires less information about the disturbance while conservative design is reached. This research adopted the $\mathcal{H}_\infty$-control theory in the design of compensators that reject disturbance.

II. Target dynamics model

Since the dynamics of a manipulator is highly nonlinear, two stages control, inner and outer loop, can be effectively used in control of a manipulator. In the inner loop the system dynamics is linearized by cancelling the nonlinear effects of the manipulator using the computed torque method. An outer loop is then applied to modify the dynamics to follow a desired target dynamics. This desired dynamics can be selected to satisfy design specifications. A control scheme is described based on the target dynamics that guarantees by itself the smooth transition between free and contact control.

Assume that an articulated manipulator, as shown in Fig.1, is composed of n simply connected links, i.e., each joint which connects the links has one relative degree of freedom. A global coordinate system O-xyz is fixed on the ground. The position and orientation of the end-effector are $\mathbf{x} = [x, y, ...,]^T \in \mathbf{R}^{n\times 1}$ in the global coordinates. The joint coordinates are also used to express the relative translational or rotational motion as $\mathbf{q} = [q_1, q_2, ..., q_n]^T \in \mathbf{R}^{n\times 1}$ in vector form, and corresponding joint torque (including force for translational joint) $\boldsymbol{\tau} = [\tau_1, \tau_2, ..., \tau_n]^T \in \mathbf{R}^{n\times 1}$. The symbol $\mathbf{x}_0$ is the environmental position before contact, and $\mathbf{f}$ is the force applied to the environment by the manipulator. For simplicity, it is assumed that the degree of the manipulator, n, is same as the number of Cartesian coordinates used.

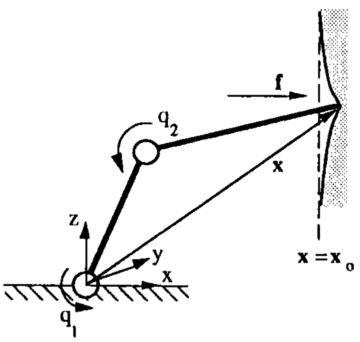

Figure 1 The robot configuration

Manipulator dynamics can be derived, using the Lagrangian or variational principles, in the joint coordinates (Chern and Yae 1991) as,

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{v}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau}_c + \mathbf{J}^T \mathbf{f}_{ext} \qquad (1)$$

where $\mathbf{v}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbf{R}^{nx1}$ is the gravitational, Coriolis, and centrifugal force; $\boldsymbol{\tau}_c \in \mathbf{R}^{nx1}$ is the control torque applied at joint actuators; $\mathbf{J}$ is the Jacobian of the Cartesian coordinates with respect to the joint coordinates; $\mathbf{f}_{ext} \in \mathbf{R}^{nx1}$ is the external force due to contact; and $\mathbf{M} \in \mathbf{R}^{nxn}$ is the generalized mass matrix.

In designing the target dynamics, two things are considered. Firstly the target dynamics for control of a robot can accommodate the free motion and contact motion and smooth transition between the two. Second, the target dynamics also allows tracking of contact position or force. Considering these two things, we suggest a model of the target dynamics that uses state error feedbacks and compensated force errors.

The simplest form of compensations for force tracking control is chosen in Cartesian formulation as,

$$\mathbf{G}(s)\,\mathbf{x}_e = \mathbf{H}(s)\,\mathbf{f}_e \qquad (2)$$

where $\mathbf{x}_e = \mathbf{x}_d - \mathbf{x}$, position error vector, $\mathbf{x}_d$ = desired trajectory $\in \mathbf{R}^{nx1}$, $\mathbf{x}$ = present Cartesian position and orientation vector $\in \mathbf{R}^{nx1}$ $\mathbf{f}_e = \mathbf{f}_s - \mathbf{f}_d$, force error vector $\mathbf{f}_d$ = the desired force or torque $\in \mathbf{R}^{nx1}$, $\mathbf{f}_s$ = the sensed force or torque ($\approx \mathbf{f}_{ext}$) $\in \mathbf{R}^{nx1}$, $\mathbf{G}(s) = (\mathbf{I}s^2 + \mathbf{K}_v s + \mathbf{K}_p)$ impedance for free motion control $\in \mathbf{R}^{nxn}$, $\mathbf{H}(s)$ = force compensator $\in \mathbf{R}^{nxn}$, and s is the Laplace transformation. The constants $\mathbf{K}_v$ and $\mathbf{K}_p$ are diagonal matrices of derivative and proportional position feedback gains, respectively.

For simplicity, $\mathbf{H}(s)$ and $\mathbf{G}(s)$ are chosen as diagonal matrices which decouple the control dynamics. When both the sensed force and the desired force are zero as in free motion, the right hand side of Eq.(2) vanishes, and it becomes a free motion controller as $\mathbf{G}(s)\,\mathbf{x}_e = \mathbf{0}$. It is noted that only position and velocity feedbacks of the contact point are used in free motion control.

When the end-effector of the robot begins to contact a surface, or when the desired force is activated with or without an actual contact, the force error is nonzero, and the nonzero force activate the right hand side of Eq.(2). Equation (2), then, becomes the contact controller. By properly chosen compensators, the contact control specifications as well as free motion specifications can be met. The target dynamics, then, simultaneously satisfies both the specifications of contact and free motion control.

The joint torque is derived from the control algorithm. Feedback and feedfowards of states and force are used according to the target dynamics. The joint driving torques based on the target dynamics Eq.(2) are obtained as,

$$\boldsymbol{\tau}_c(t) = \hat{\mathbf{v}}(\mathbf{q}, \dot{\mathbf{q}}) + \hat{\mathbf{M}}(\mathbf{q})\,\mathbf{u}(t) - \mathbf{J}^T \mathbf{f}_s \qquad (3)$$

where

$$\mathbf{u}(t) = \mathbf{J}^{-1}[\ddot{\mathbf{x}}_d + \mathbf{K}_v \dot{\mathbf{x}}_e + \mathbf{K}_p \mathbf{x}_e - \mathbf{H} * (\mathbf{f}_s - \mathbf{f}_d) - \dot{\mathbf{J}}\,\dot{\mathbf{q}}] \qquad (4)$$

and the symbol * is a time convolution and $\hat{\mathbf{v}}(\mathbf{q}, \dot{\mathbf{q}})$ and

$\hat{M}(q)$ are the estimates of nonlinear force **v** and inertia **M**, respectively. In practice, it is possible that the sensed force f_s may contain errors, and the estimates, $\hat{v}(q, \dot{q})$ and $\hat{M}(q)$, which results from on-line computation of the manipulator model dynamics, may have estimation errors. However, we assume, for simplicity in developing control algorithm, that the sensing and the estimations are so accurate that the measurement and estimation errors are negligible.

III. Parameterization of stabilizing compensators

Parameterization of controllers, in general, simplifies the design process. The parameterization is applied to the design of the compensators in the target dynamics. Since the contact force is regarded as the output of the control, and the position is used as the input to the controller, the plant model of the contact environment is improper. The parameterization of the improper plant is introduced. Sensitivity functions for position and force errors are then defined for controller design.

The major difference between the target dynamics and the model reference control is that the resulting dynamics is the referenced model in the latter, but not the target dynamics in the former. In the model reference control, a plant dynamics is cancelled, and the plant is controlled to follow a reference model. In the present control, the plant consists of the interaction between the end-effector and the environment, and only the dynamics of the manipulator, not the environment, is cancelled. As a result, the stability and performance depends on the interaction that is considered in the analysis and design.

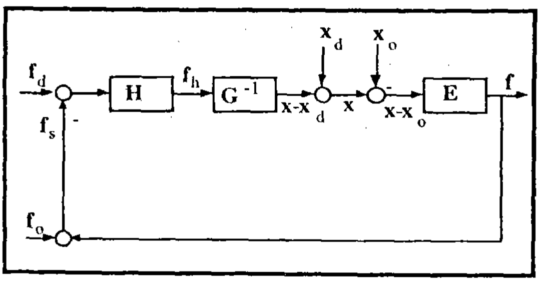

Figure 2. The control system

It is desirable to reduce design specifications by parameterizing the controllers. Since the plant, i.e., the contact environment in the present research, is stable, the closed-loop dynamics can be parameterized by a family of stable, proper, and rational polynomials (Zames and Francis 1983; Doyle et al. 1992). The design problem is then simplified to picking up a polynomial from that family, instead of checking broad potential compensators. In the result, the design problem of closed-loop control is transformed into an open-loop design problem in parameterized domain, which can easily satisfy the rest of the design specifications. Moreover, the sensitivity functions can be expressed as affine functions (linear plus constant) of the parameterization, and further derivation of $\mathcal{H}_\infty$ control problems is simplified.

It is assumed that the environment can be modelled as a simple passive mechanical system that consists of inertia, damping, and stiffness as,

$$f_s = E(s)(x - x_o) + f_o \approx f_{ext} \tag{5}$$

where

$$E(s) = M_E s^2 + C_E s + K_E \tag{6}$$

and the symbol f_o is the force disturbance from static load. The coefficients matrices; inertia M_E, damping C_E, and stiffness K_E of the environment; are diagonal matrices. These matrices as well as the environmental geometry x_o may vary according to contact position.

A simplified linear control loop can be drawn as in Fig.2 by substituting Eq.(5) to Eq.(2). The closed loop can be viewed as a system with two-input (x_d, f_d) and two-output (x, f_s) with position disturbance x_o and force disturbance f_o. Viewing the control loop in Fig.2 as a multi-input and multi-output system, each component of sensitivity functions is obtained. The force error $f_e = f_d - f_s$ is written as,

$$\begin{aligned} f_e &= -E(s)(x - x_o) + f_d - f_o \\ &= E(s)\, x_e - E(s)(x_d - x_o) + f_d - f_o \end{aligned} \tag{7}$$

From Eq.(2) and (7), the output errors are derived in terms of sensitivity functions as,

$$\begin{bmatrix} x_e \\ f_e \end{bmatrix} = \begin{bmatrix} S_{xx}(s) & S_{xf}(s) \\ S_{fx}(s) & S_{ff}(s) \end{bmatrix} \begin{bmatrix} x_d - x_o \\ f_d - f_o \end{bmatrix} \tag{8}$$

where the sensitivity functions are defined as,

$$S(s) := \begin{bmatrix} S_{xx}(s) & S_{xf}(s) \\ S_{fx}(s) & S_{ff}(s) \end{bmatrix}$$

$$= \begin{bmatrix} G^{-1}H(I+E\,G^{-1}H)^{-1}E & -G^{-1}H(I+E\,G^{-1}H)^{-1} \\ (I+E\,G^{-1}H)^{-1}E & (I+E\,G^{-1}H)^{-1} \end{bmatrix} \tag{9}$$

In Equation (9), $S_{xx}(s)$, $S_{xf}(s)$, $S_{fx}(s)$ and $S_{ff}(s)$ denote the sensitivity functions of position-position, position-force, force-position, and force-force, respectively. Equations (7) shows that the output force and position error are dependent. Since the output vector Eq.(8) has twice the dimension of the system output, either the position or force error can be exclusively chosen in design, depending on control task.

The family of all stable, proper, real-rational function matrices is denoted as $\mathcal{RH}_\infty$. In the closed loop of Fig.2, the inverse of the free motion controller, $\mathbf{G}^{-1}(s)$, is in $\mathcal{RH}_\infty$. The environmental model, E(s), is, however, order of two in s. It implies that the plant is improper or $E(s) \notin \mathcal{RH}_\infty$. The controller parameterizations are developed for proper plants (Cho 1990; Francis and Doyle 1987; Zames 1981). The theorem (Doyle et al. 1992) for proper plants is extended for improper plants in the present problem. Though $\mathbf{E}(s)$ is not in $\mathcal{RH}_\infty$, the combined function $\mathbf{E}(s)\mathbf{G}^{-1}(s)$ is in $\mathcal{RH}_\infty$. Using this fact, the following lemma is derived.

Lemma: Assume that $\mathbf{E}(s)\ \mathbf{G}^{-1}(s)$ stable and proper, and the position input $\mathbf{x}_d$ and disturbance $\mathbf{x}_o$ are bounded to their second order derivatives, i.e. $\| \mathbf{E}(s)\ (\mathbf{x}_d - \mathbf{x}_o)\|_2 < \infty$. The set of all compensators **H** for which the feedback system are bounded-input bounded-output (BIBO) stable equals

$$\left\{ H(s) = Q[\ I - E\ G^{-1}\ Q]^{-1} : Q(s) \in \mathcal{RH}_\infty \right\} \quad (10)$$

The proof is shown in Lee (1993).

The stabilization parameter Q(s) is the transfer function from $\mathbf{f}_d$ to $\mathbf{f}_h$ as,

$$Q(s) = \mathbf{H}\left[\ \mathbf{I} + \mathbf{E}\ \ \mathbf{G}^{-1}\ \mathbf{H}\ \right]^{-1} \quad (11)$$

The sensitivity functions Eq.(9) can be written in a multiplicative form of matrices from Eq.(11) as,

$$S(s) = \left[\begin{bmatrix} 0 & 0 \\ I & I \end{bmatrix} - \begin{bmatrix} -G^{-1} & G^{-1} \\ E\,G^{-1} & E\,G^{-1} \end{bmatrix} \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix}\right] \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix} \quad (12)$$

This parameterization is used through the subsequent development of control problems.

Optimization of the Compensators

An optimization problem is formulated by defining a performance index. The design problem for the compensator is formulated using $\mathcal{H}_\infty$-optimality criterion. In the problem of the optimization, the control efforts, i.e. motor torques, computed by the nonlinear dynamic model are approximated and incorporated. Based on the resulting augmented optimality, a compensator is determined by the model-matching technique.

The $\mathcal{H}_\infty$-optimization problem is to find a compensator that minimizes the worst errors due to exogenous disturbance or inputs. This optimality leads to the minimization of the $\mathcal{H}_\infty$- norm of sensitivity functions, which are the transfer functions from the disturbance inputs to the output errors.

In the present problem, the $\mathcal{L}_2$-norm is applied to both the position in $\mathcal{L}_2^n$ and the force output in $\mathcal{L}_2^n$. Therefore, the total disturbances to the system are in $\mathcal{L}_2^{2n}$. The resulting $\mathcal{H}_\infty$ minimization problem is still to find a stabilization parameter Q(s) in $\mathcal{H}_\infty^{n\times n}$.

From Eq. (8), a disturbance input vector that is any square-integrable vector function bounded with unit $\mathcal{L}_2^{2n}$ norm can be defined as,

$$\mathbf{v}_d := \begin{bmatrix} \mathbf{x}_d - \mathbf{x}_o \\ \mathbf{f}_d - \mathbf{f}_o \end{bmatrix} \in \mathcal{L}_2^{2n} \quad (13)$$

The possible disturbance $\mathbf{v}_d$ is modeled as

$$\{\ \mathbf{v}_d \mid \mathbf{v}_d = \mathbf{W}_2(s)\ \mathbf{d}\ ,\ \|\mathbf{d}\|_2 \le 1\ \} \quad (14)$$

where

$$\mathbf{W}_2(s) = \begin{bmatrix} \mathbf{W}_{2x}(s) & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_{2f}(s) \end{bmatrix} \quad (15)$$

The weighting functions $\mathbf{W}_{2x}(s) \in \mathcal{RH}_\infty^{n\times n}$ and $\mathbf{W}_{2f}(s) \in \mathcal{RH}_\infty^{n\times n}$ for position and force disturbance, respectively, are stable, proper, real-rational, and minimum phase. The weighting functions are chosen to reflect the disturbances to be attenuated.

It is noted that the control input is the torque that applies at the joints. For the torque is nonlinear with respect to the input and disturbance, the linearized torque is considered in the optimization. For the linearization, it is assumed that the manipulator is operating slowly, and the input force and gravity is relatively small. The joint torque is then written in terms of the stabilization parameter (Lee 1993) as

$$\tau_c(s) \approx (\ \mathbf{T}_{1\tau} - \mathbf{T}_{2\tau}\ \mathbf{Q}_{2n}\ \mathbf{T}_3\ )\ \mathbf{v}_d \quad (16)$$

where

$$\mathbf{T}_{1\tau} = [\mathbf{J}^T\ \ \mathbf{J}^T\ ] \quad (17)$$

$$\mathbf{T}_{2\tau} = [\mathbf{J}^T\{\mathbf{A}(2\mathbf{I}\text{-}s^2\mathbf{G}^{-1})\text{+}\mathbf{E}\mathbf{G}^{-1}\}\ \ \mathbf{J}^T(\mathbf{A}\ s^2\mathbf{G}^{-1}\text{+}\mathbf{E}\mathbf{G}^{-1})\ ] \quad (18)$$

$$T_3 = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix} W_2, \qquad Q_{2n} = \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix} \tag{19}$$

$$\Lambda(q) = J^{-T} M J^{-1} \tag{20}$$

The model-matching problem for the error minimization (Francis 1987) often results in improper controllers. This case is avoided by considering the control inputs in the criteria. By modifying the performance index, the $\mathcal{H}_\infty$ minimization yields a model-matching problem with augmented coefficient matrices.

Taking $\mathcal{L}_2$ norm to the Eqs.(8) and (16) yields an augmented optimization problem as

$$\underset{\|d\|_2 \le 1}{\text{minimize} \ \sup} \left(\left\| \begin{bmatrix} x_e \\ f_e \end{bmatrix} \right\|_2^2 + \| \tau_c \|_2^2 \right)^{1/2}$$

$$= \min \| T_1 - T_2 Q_{2n} T_3 \|_\infty \tag{21}$$

from Eq.(14), where the augmented matrices T_{1c} and T_{2c} are obtained from Eqs.(12), (13), (15), (17) and (18) as

$$T_1 = \begin{bmatrix} 0 & 0 \\ W_{1f} E W_{2x} & W_{1f} W_{2f} \\ W_{3\tau} J^T & W_{3\tau} J^T \end{bmatrix} \tag{22}$$

$$T_2 = \begin{bmatrix} -W_{1x} G^{-1} & W_{1x} G^{-1} \\ W_{1f} E G^{-1} & W_{1f} E G^{-1} \\ W_{3\tau} J^T \{\Lambda(2I - s^2 G^{-1}) + EG^{-1}\} & W_{3\tau} J^T (\Lambda s^2 G^{-1} + EG^{-1}) \end{bmatrix} \tag{23}$$

and $W_{3\tau} \in \mathcal{RH}_\infty^n$ is a weighting factor that assigns weight to joint each joint torque. The weighting functions $W_{1x}(s) \in \mathcal{RH}_\infty^{n\times n}$ and $W_{1f}(s) \in \mathcal{RH}_\infty^{n\times n}$, are stable, proper, real-rational, and minimum phase and select the outputs to be attenuated.

The weighted sensitivity-minimization problem Eq. (21) can be considered as a standard model-matching problem in $\mathcal{H}_\infty$ control theory (Francis 1987). The stabilization parameter $Q_{2n}(s) \in \mathcal{RH}_\infty^{n\times n}$ has, however, not 2n×2n independent function elements, but n×n. This situation occurs because the redundant inputs, i.e., position and force, are used in control of contact dynamics. The general solution method of Eq. (21) is not available so far. Nevertheless, the problem can be reduced to the general model-matching problem, by choosing the position or force disturbance exclusively.

The solution methods to the model-matching problem are found in many references (Lee 1993; Francis 1987; Doyle 1983). Once the parameter is found, the compensator is determined from Eq. (10).

V. A design example

In order to demonstrate the effectiveness of the described design tool, a design example is presented. For simplicity, a compensator is designed for a 2-degree of freedom manipulator. The position control specification is applied to the horizontal x-direction, and the force error specification to the vertical z-direction.

The contact environment has stiffness without damping or inertia. The free motion controller **G**(s) is designed to have zeros at -20. Then the environment and the free motion controller are written as,

$$E = \begin{bmatrix} 10^5 & 0 \\ 0 & 10^5 \end{bmatrix} \tag{24}$$

$$G = \begin{bmatrix} s^2+40s+400 & 0 \\ 0 & s^2+40s+400 \end{bmatrix} \tag{25}$$

The bandwidth for position control is 0.1 rad/sec, and for the force control 10 rad/sec. To simplify the problem, the force input weighting $W_{2f}(s)$ is taken as zero. The rest weighting functions are selected as,

$$W_{1x} = \begin{bmatrix} \frac{s+1}{10s+1} & 0 \\ 0 & 0 \end{bmatrix}, \quad W_{1f} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{0.01s+1}{0.1s+1} \end{bmatrix} \tag{26}$$

$$W_{2x} = E^{-1}, \qquad W_{2f}(s) = 0 \tag{27}$$

$$W_{3\tau} = \rho \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \rho = 10^{-6} \tag{28}$$

where the weighting for motors $W_{3\tau}$ are distributed equally to both motors. For the optimization, a configuration is chosen that forms a regular triangle with the ground. Jacobian and inertia matrices are evaluated at the configuration at

$$J = \begin{bmatrix} 0 & 1.7321 \\ 2 & 1 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 0.6805 & -0.5773 \\ -0.5773 & 1.0417 \end{bmatrix} \tag{29}$$

Substituting Eqs.(24)-(29) into Eq.(21), the standard model-matching problem is obtained. The design problem is solved, and the results are shown in the Figs. 3 and 4.

The position-position and force-force sensitivity functions corresponding to the position and force control directions are shown with the weighting functions in Figs.

3 and 4. The tracking errors in steady state are $S_{xx,11}(0) = 5.335\times10^{-6}$ for position control in the horizontal direction, and $S_{ff,22}(0) = 0.1064$ for force control in the vertical direction. The tracking error is reduced when the bandwidth of weighting function decreases.

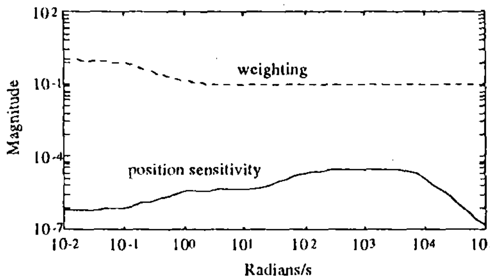

Figure 3. The position sensitivity

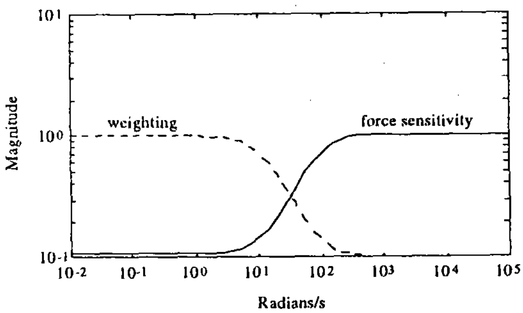

Figure 4. The force sensitivity

The performance of the force control in Fig.4 shows typical behavior such that the sensitivity is low at the frequencies where the weighting is large. The position-position sensitivity in Fig.3 is low at the low frequencies where the weighting is large. As the weighting increases, the position-position sensitivity increases at the middle frequencies. The sensitivity at the infinity yet approaches zero at the infinity, because the position-position sensitivity function with the stiffness environment is strictly proper.

The compensator can be computed from Eq. (10). The resulting compensator is, however, of high order. The high order compensator is impractical because of reliability and difficulty in hardware construction. The compensator needs reductions in order (Therapos 1992). This compensator reduction problem is out of the scope of the present research. Actually, the techniques for controller reduction are still under development. Anderson and Liu (1989) recently gave a review on controller reduction, and Chen et al. (1992) presented a system-order compensator for $\mathcal{H}_\infty$ optimization.

VI. Conclusion

A control method based on the target dynamics is suggested. The target dynamics uses a high order compensator for the control of the end-effector in contact. The compensator is designed applying $\mathcal{H}_\infty$ control theory.

The target dynamics can meet free motion and contact control specifications. Stabilizing parameterization is applied to the improper plant of the environment with relative order 2 high. Considering the linearized torque in the optimization, the design problem for the compensator is transformed into the model-matching problem, which can be solved theoretically and computationally. The $\mathcal{H}_\infty$ synthesis is adopted to determine the stabilizing parameter.

Finally, a design example shows that the present tool effectivley determines the compensator that meets the design specifications.

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