Improved Transient Response Design of MRACS

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Abstract

The global stability of model reference adaptive control system (MRACS) in the ideal case was resolved in the 1980's. However the improvement of the transient behaviour of MRACS has not been discussed sufficiently even in the ideal case. Only a few attempts have so far been made at the application of MRACS to the practical systems in contrast to the theoretical systematization. Therefore, when we consider the practical usage of MRACS it is necessary to develop an improved design scheme with respect to transient behaviour. In this paper, we propose two design schemes improving transient behaviour of MRACS by modifying the input synthesis in the conventional design scheme of MRACS. We present a design scheme of MRACS in which we utilize the design approach of variable structure system(VSS). After describing the above design scheme, we also propose the improved design scheme in which we introduce the dead-zone decided by the magnitude of the output-error between the plant and the reference model. The effectiveness of the proposed two design schemes are shown through computer simulations. As the results, by using these methods, the convergence of the transient response is greatly improved in comparison with the conventional one.

1 Introduction

When we apply an adaptive control to the practical system, its transient response is very important. A model reference adaptive control system(MRACS) is one of the main stream of adaptive control systems. Under ideal conditions, the global stability of MRACS was resolved. It was reported that the persistent excitation(PE) signals help to enhance parameter identification, and that the transient behaviour is improved in the conventional MRACS [1],[2]. However, it is difficult to make the signals PE continually in MRACS in practice.

In an approach to modify the MRACS aimed at the improving transient response, the input synthesis is modified with an additional feedback signal which is introduced to counteract the error caused by parameter uncertainty and the inaccuracy of parameter identification [3].

In this paper, we propose two design schemes of MRACS to improve the transient response. One is the design scheme in which the input synthesis is modified with an additional switching function to counteract the output error. This modification is motivated from the concept of variable structure(VS) method. As the results, the convergence of the output error becomes very rapid in this design scheme.

However, it is possible that the chattering phenomenon occurs because of the nature of VS method in this design scheme. Although the convergence of the output error is improved, the convergence rate of the parameter error becomes smaller compared with the conventional one. In order to overcome these problems, we propose another improved design scheme, in which we introduce the dead-zone decided by the magnitude of the output error. That is, the above mentioned input synthesis is used outside the dead-zone, and the conventional one is used inside the dead-zone. In this design scheme, not only the chattering problem but also the convergence of the parameter error will be resolved by using the dead-zone.

This paper is organized as follows. In Section 2, we explain the problem. In Section 3, we describe the conventional design scheme of MRACS. In Section 4, we propose two design schemes which improve the transient response. In Section 5, we investigate the convergence of these design schemes. In Section 6, we present the simulation results to illustrate the effectiveness of these design schemes.

2 Problem Formulation

We consider a single-input single-output linear time-invariant plant, which can be described by the following

$$y(t) = \frac{B(s)}{A(s)}u(t) \tag{1}$$

where

$$A(s) = s^n + \sum_{i=1}^{n} a_i s^{n-i}, \tag{2}$$

$$B(s) = \sum_{i=0}^{n-1} b_i s^{n-1-i}. \tag{3}$$

Assuming that the plant satisfies the following conditions,

A1) Polynomial $A(s)$ and $B(s)$ are coprime.

A2) The degree n is known.

A3) The coefficients a_i and b_i are unknown constants, where $b_0 > 0$.

A4) Polynomial $B(s)$ is a Hurwitz polynomial.

In (1), $u(t)$ and $y(t)$ are the input and the output of the plant. We consider the reference model as

$$y_M(t) = \frac{B_M(s)}{A_M(s)}r(t) \tag{4}$$

where

$$A_M(s) = s^n + \sum_{i=1}^{n} a_{M_i} s^{n-i}, \tag{5}$$

$$B_M(s) = \sum_{i=0}^{n-1} b_{M_i} s^{n-1-i}. \tag{6}$$

In (4), $r(t)$ is the bounded reference input, $y_M(t)$ is the output of the reference model and $A_M(s)$ is a Hurwitz polynomial.

The problem considered here is to contract MRACS in which the output $y(t)$ of the plant in (1) tracks the output $y_M(t)$ of the reference model in (4) and to improve the transient behaviour of MRACS.

3 Conventional Design Scheme

In this section, we describe the conventional design scheme of MRACS.

First, we introduce the following Hurwitz polynomials.

$$F(s) = s^{n-1} + \sum_{i=1}^{n-1} f_i s^{n-1-i}. \tag{7}$$

By using the above equation, (1) can be written as

$$y(t) = \frac{b_0}{s+\lambda}\{u(t) + \boldsymbol{\theta}^T \boldsymbol{\xi}(t)\}\ ,\ \lambda > 0 \tag{8}$$

where

$$\boldsymbol{\xi}(t) \triangleq [\xi_1(t), \cdots, \xi_{2n-1}(t)]^T$$

$$\left.\begin{aligned} \xi_i(t) &= \tfrac{s^{n-1-i}}{F(s)}u(t) \ : i = 1 \sim n-1 \\ \xi_n(t) &= y(t) \\ \xi_{n+i}(t) &= \tfrac{s^{n-1-i}}{F(s)}y(t) \ : i = 1 \sim n-1 \end{aligned}\right\} \tag{9}$$

$$\boldsymbol{\theta} \triangleq [\theta_1, \cdots, \theta_{2n-1}]^T \tag{10}$$

: unknown parameter vector corresponding to $\boldsymbol{\xi}$(t).

Defining the output error as

$$e(t) \triangleq y(t) - y_M(t), \tag{11}$$

then, the following relationship is obtained from (4) and (8).

$$(s+\lambda)e(t) = b_0\{u(t) + \bar{\theta}^T\zeta(t)\} \tag{12}$$

where

$$\bar{\theta} \triangleq [\theta^T, \theta_{2n}]^T \tag{13}$$

$$\begin{aligned}\zeta(t) &\triangleq [\zeta_1(t), \cdots, \zeta_{2n}(t)]^T \\ &= [\boldsymbol{\xi}^T(t), \tilde{y}_M(t)]^T\end{aligned} \tag{14}$$

$$\tilde{y}_M(t) = (s+\lambda)y_M(t). \tag{15}$$

By introducing the adjustable parameter vector

$$\hat{\theta}(t) \triangleq [\hat{\theta}_1(t), \cdots, \hat{\theta}_{2n}(t)]^T, \tag{16}$$

the input is synthesized as

$$u(t) = -\hat{\theta}^T(t)\zeta(t). \tag{17}$$

By substituting (17) into (12), we obtain the following error equation.

$$(s+\lambda)e(t) = b_0\psi^T(t)\zeta(t) \tag{18}$$

where

$$\psi(t) \triangleq \bar{\theta} - \hat{\theta}(t). \tag{19}$$

In order to achieve $e(t) \to 0$ as $t \to \infty$, we use the following adaptive law

$$\dot{\hat{\theta}}(t) = \Gamma\zeta(t)e(t), \tag{20}$$

where

$$\Gamma = \Gamma^T > 0.$$

4 Improved Design Scheme

In this section, we propose two design schemes for improving the transient response.

4.1 Design scheme I

We modify the input synthesis of the conventional design scheme as follows

$$u(t) = -\hat{\theta}^T(t)\zeta(t) - \alpha\,\mathbf{sgn}(e(t)) \tag{21}$$

where

$$\mathbf{sgn}(e(t)) \triangleq \begin{cases} 1 \;;\; e(t) > 0 \\ 0 \;;\; e(t) = 0 \\ -1 \;;\; e(t) < 0. \end{cases} \tag{22}$$

In (21), α is a positive constant decided by the designer. In this design scheme, the adaptive law is the same as the conventional one. Hereafter, we refer to this scheme as the design scheme I. It was pointed out that the above mentioned design scheme is robust in the presence of modelling errors [4]. In this paper, we clarify that the same design scheme is effective to improve the transient response of MRACS. By introducing the switching function (22), transient behaviour of MRACS is improved. However, there is a possibility that the switching function makes the input occur the so-called chattering phenomenon. Another problem is that the convergence rate of the parameter estimation value which is adjusted by (20) becomes slow when the output error $e(t)$ decrease rapidly. Therefore, it is desirable to improve the convergence of the parameter error $\psi(t)$ as well as the output error $e(t)$.

4.2 Design scheme II

For the above mentioned purpose, we modify a constant α in (21) of design scheme I such as

$$\alpha = \begin{cases} \alpha_0 \;;\; |e(t)| > e^* \\ 0 \;;\; |e(t)| \le e^*, \end{cases} \tag{23}$$

where α_0 and e^* are positive constants chosen by the designer. Henceforth, we refer to this design scheme as the design scheme II.

5 Investigation of Convergence

We analyze the convergence of the above mentioned design schemes. At first, we obtain the following error equations from (18) and (20) in the case of the conventional design scheme.

$$\dot{e}(t) = -\lambda e(t) + b_0 \boldsymbol{\psi}^T \boldsymbol{\zeta}(t). \tag{24}$$

$$\dot{\boldsymbol{\psi}}(t) = -\boldsymbol{\Gamma}\boldsymbol{\zeta}(t)e(t). \tag{25}$$

Here, we introduce a positive quadratic function defined as

$$V(t) \triangleq \frac{1}{2}\{e^2(t) + b_0 \boldsymbol{\psi}^T(t)\boldsymbol{\Gamma}^{-1}\boldsymbol{\psi}(t)\}. \tag{26}$$

We obtain the time derivative of $V(t)$ along the trajectories of (24) and (25) as

$$\dot{V}(t) = -\lambda e^2(t). \tag{27}$$

In both design schemes, i.e., the design scheme I and II, substituting (21) into (12), we obtain the following expression.

$$\begin{aligned} \dot{e}(t) &= -\lambda e(t) + b_0 \boldsymbol{\psi}^T(t)\boldsymbol{\zeta}(t) \\ &\quad -\alpha \mathbf{sgn}(e(t)). \end{aligned} \tag{28}$$

The quadratic function $V(t)$ defined in (26) yields a time derivative $\dot{V}(t)$, and it can be evaluated along the trajectries of (25) and (28) as

$$\dot{V}(t) = -\lambda e^2(t) - \alpha b_0 |e(t)|. \tag{29}$$

Comparing (27) with (29), it is reasonable to conclude that the convergence of the output error $e(t)$ in the new design schemes is improved. In the design scheme II, since $\alpha = 0$ within the range of $|e(t)| \leq e^*$, (27) must be adopted as $\dot{V}(t)$. That is, it is prevented that the output error $e(t)$ decreases too rapidly. As the result, it is guaranteed that the parameter error $\boldsymbol{\psi}(t)$ as well as the output error $e(t)$ decreases simultaneously.

6 Simulation Results

In this Section, in order to confirm the effectiveness of the proposed design schemes, the computer simulations were carried out.

The plant is described as

$$y(t) = \frac{0.8}{(s+1.5)}u(t).$$

The reference model is adopted as the following,

$$y_M(t) = \frac{1}{(s+1)}r(t).$$

The following adaptive law is chosen.

$$\dot{\hat{\boldsymbol{\theta}}}(t) = 20\boldsymbol{I}\boldsymbol{\zeta}(t)e(t).$$

The input synthesis of each design scheme is like as follows.

Conventional design scheme:

$$u(t) = -\hat{\boldsymbol{\theta}}^T(t)\boldsymbol{\zeta}(t).$$

Design scheme I:

$$u(t) = -\hat{\boldsymbol{\theta}}^T(t)\boldsymbol{\zeta}(t) - 0.8\mathbf{sgn}(e(t)).$$

Design scheme II:

$$u(t) = -\hat{\boldsymbol{\theta}}^T(t)\boldsymbol{\zeta}(t) - \alpha\mathbf{sgn}(e(t))$$

$$\alpha = \begin{cases} 0.3 \ ; & |e(t)| > 0.05 \\ 0 \ ; & |e(t)| \leq 0.05. \end{cases}$$

The sine-wave with the amplitude of ± 1 and the periodic time of 30 sec. is used as the reference input $r(t)$.

Fig.1, Fig.2 and Fig.3 show the simulation results of the conventional design scheme, the design scheme I and the design scheme II, respectively. Comparing Fig.1(a) with Fig.2(a), it can be seen that the convergence of the output error $e(t)$ is improved in the design scheme I. However, from Fig.1(b) and Fig.2(b), the covergence rate of the parameter error norm $\|\boldsymbol{\psi}(t)\|$ in the design scheme II decreases compared with the conventional one. Moreover, the chattering phenomenon is seen in the input signal $u(t)$ in the design scheme I, i.e., Fig.2(c). Comparing Fig.3 with Fig.1, we can see that the convergence rate of the output error $e(t)$ as well as one of the parameter error norm $\|\psi(t)\|$ in the design scheme II is improved. Furthermore, the chattering phenomenon is not present in the input signal $u(t)$ in the design scheme II, i.e., Fig.3(c). These results show the effectiveness of the proposed design schemes.

7 Conclusion

In this paper, we proposed the two design schemes of MRACS for the purpose of improving the transient behaviour. We also analize the convergence properties of these design schemes. And we confirmed the usefulness of the proposed design schemes by using computer simulations.

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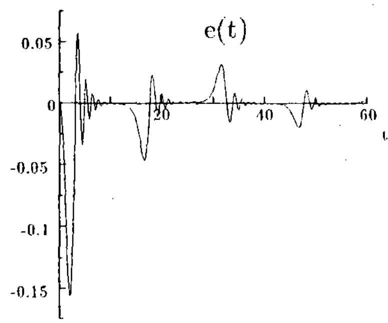
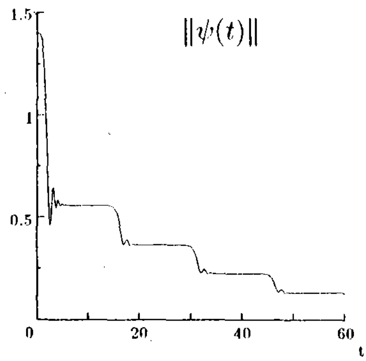
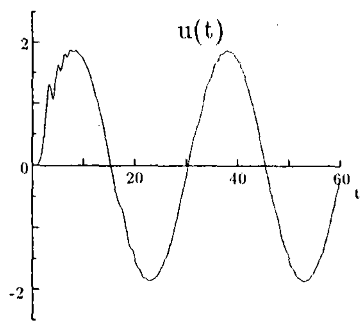


(a) The output error.

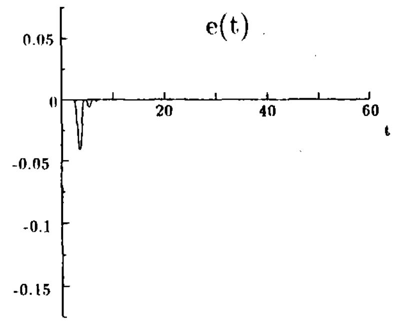


(b) The parameter error norm.

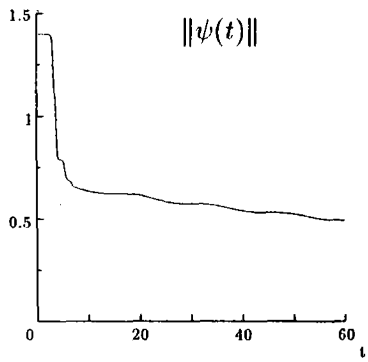


(c) The input signal.

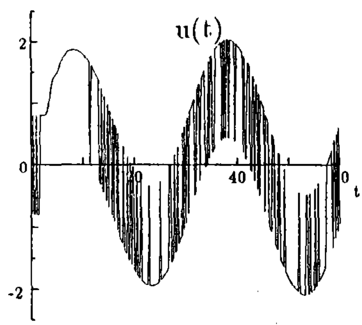
Fig. 1. The conventional design scheme.



(a) The output error.

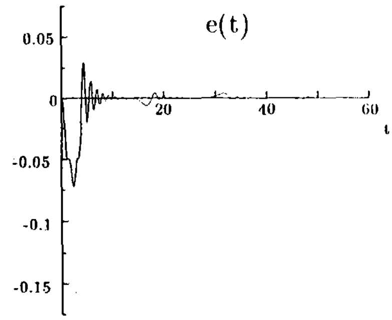


(b) The parameter error norm.

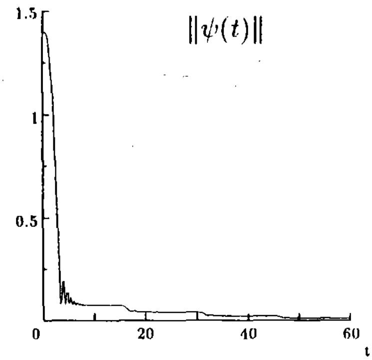


(c) The input signal.

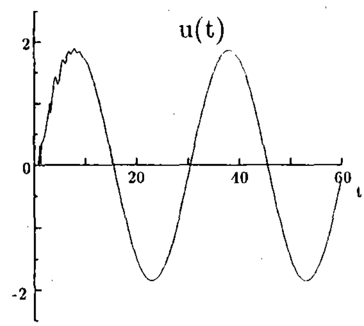
Fig. 2. The design scheme I.



(a) The output error.



(b) The parameter error norm.



(c) The input signal.

Fig. 3. The design scheme II.