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Memoryless H_∞ Controller for State and Input Delayed Systems

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Abstract

In this paper, a memoryless H_∞ controller for linear systems with state and input delays is presented. The proposed controller is a delay independent stabilizer which reduces the H_∞ norm of the closed loop transfer function, from the disturbance to the controlled output, to a precribed level. The controller is obtained by solving a minimization problem involving linear matrix inequalities.

1 Introduction

The state space approach to the H_∞ control problem has attracted attentions of many researchers owing to its simplicity [1], [2]. Recently, the H_∞ controller design methods have been extended to deal with delayed systems[3]. In [3], a modified Riccati equation was presented to obtain H_∞ controller for systems with delays in state. On the other hand, a convex optimization method was proposed to design a stabilizing delayed state feedback control for similar systems [4].

In this paper, we consider linear systems with multiple delays in both state and input, and then present a memoryless stabilizing state feedback controller which reduces the H_∞ norm of the closed loop transfer function, from the disturbance to the controlled output, to a precribed level. The proposed controller is obtained by solving a minimization problem involving LMI conditions. Several recently developed convex optimization algorithms can be applied here[5].

2 Main Result

Let us consider a linear system with state and input delays

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + \sum_{i=1}^{p} A_i x(t-h_{1i}) + B_0u(t) \\ &\quad + \sum_{j=1}^{q} B_j u(t-h_{2j}) + Dw(t) \\ z(t) &= Ex(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control, $w(t) \in R^k$ is the disturbance, $z \in R^l$ the controlled output, and h_{1i}, $h_{2j} \geq 0$ are delays in the system. In addition, A_0, A_i, B_0, B_j, D, and E are constant matrices with appropriate dimensions. In this paper, i and j denote indexes $i = 1, \ldots, p$ and $j = 1, \ldots, q$, respectively.

We shall design a memoryless linear state feedback control

$$u(t) = Fx(t) \tag{2}$$

where $F \in R^{m\times n}$ is a constant matrix. The closed loop transfer function T_{zw} from the disturbance w to the output z is given by

$$T_{zw}(s) := E\{sI - (A_0 + B_0F) - \sum_{i=1}^{p} A_i e^{-sh_{1i}} - \sum_{j=1}^{q} B_jFe^{-sh_{2j}}\}^{-1}D. \tag{3}$$

Our aim is to find the H_∞ controller which stabilizes the delayed system (1) and guarantees the H_∞ norm bound γ of the transfer function T_{zw}, namely, $\|T_{zw}\|_\infty < \gamma$, where γ is a positive constant.

The following lemma gives a sufficient condition for the stability of the closed loop system with the control (2).

LEMMA 1 *If there exist positive definite matrices* P_0, P_{1i}, P_{2j} *such that*

$$(A_0 + B_0F)'P_0 + P_0(A_0 + B_0F) + \sum_{i=1}^{p} P_0A_iP_{1i}^{-1}A_i'P_0$$
$$+\sum_{j=1}^{q} P_0B_jFP_{2j}^{-1}F'B_j'P_0 + \sum_{i=1}^{p} P_{1i} + \sum_{j=1}^{q} P_{2j} < 0 \quad (4)$$

then the delayed system (1) with the control (2) is stable for all $h_{1i}, h_{2j} \geq 0$.

Proof: Let's assume that $w(t)$ is equal to zero for all t. Define a Lyapunov functional $V(x_t)$ as follows:

$$V(x_t) := x'(t)P_0x(t) + \sum_{i=1}^{p}\int_{t-h_{1i}}^{t} x'(s)P_{1i}x(s)ds$$
$$+\sum_{j=1}^{q}\int_{t-h_{2j}}^{t} x'(\tau)P_{2j}x(\tau)d\tau.$$

The corresponding Lyapunov derivative is given by

$$\frac{dV(x_t)}{dt} = y'Wy \quad (5)$$

where

$$y = [x(t)\ x(t-h_{11})\dots x(t-h_{1p})\ x(t-h_{21})\dots x(t-h_{2q})]'$$

and

$$W = \begin{bmatrix} N & P_0A_1 & \dots & P_0A_p & P_0B_1F & \dots & P_0B_qF \\ A_1'P_0 & -P_{11} & & & \dots & & 0 \\ \vdots & & \ddots & & & & \\ A_p'P_0 & & & -P_{1p} & & & \\ F'B_1'P_0 & \vdots & & & -P_{21} & & \vdots \\ \vdots & & & & & \ddots & \\ F'B_q'P_0 & 0 & & & \dots & & -P_{2q} \end{bmatrix}$$

$$N = (A_0 + B_0F)'P_0 + P_0(A_0 + B_0F) + \sum_{i=1}^{p} P_{1i} + \sum_{j=1}^{q} P_{2j}.$$

Hence, the Lyapunov derivative (5) is negative definite if the inequality (4) is satisfied [6].

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LEMMA 2 *The Riccati inequality (4) is equivalent to the LMI*

$$\begin{bmatrix} M & A_1Q_0 & \dots & A_pQ_0 & B_1Y & \dots & B_qY \\ Q_0A_1' & -Q_{11} & & & \dots & & 0 \\ \vdots & & \ddots & & & & \\ Q_0A_p' & & & -Q_{1p} & & & \\ Y'B_1' & \vdots & & & -Q_{21} & & \vdots \\ \vdots & & & & & \ddots & \\ Y'B_q' & 0 & & & \dots & & -Q_{2q} \end{bmatrix} < 0 \quad (6)$$

where

$$Q_0 = P_0^{-1}, Q_{1i} = P_0^{-1}P_{1i}P_0^{-1}, Q_{2j} = P_0^{-1}P_{2j}P_0^{-1}, Y = FP_0^{-1},$$

and

$$M = A_0Q_0 + Q_0A_0' + B_0Y + Y'B_0' + \sum_{i=1}^{p} Q_{1i} + \sum_{j=1}^{q} Q_{2j}.$$

The following theorem provides a delay independent stabilizer for the state and input delayed system (1).

THEOREM 1 *If there exist* Y, $Q_0 > 0$, $Q_{1i} > 0$, *and* $Q_{2j} > 0$ *which satisfy the LMI (6), then the state feedback control*

$$u(t) = YQ_0^{-1}x(t) \quad (7)$$

stabilizes the input delayed system (1) for all $h_{1i}, h_{2j} \geq 0$.

Proof: If there exist Y, Q_0, Q_{1i}, and Q_{2j} which satisfy the LMI (6), then the state feedback control gain F is equal to YQ_0^{-1}. Hence we obtain the theorem.

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We have obtained the stability conditions and the stabilizing controller for the state and input delayed system (1). The following theorem provides a stabilizing memoryless controller which guarantees the H_∞ norm bound γ of the closed loop transfer function of the system (1).

THEOREM 2 *If there exist* Y, $Q_0 > 0$, $Q_{1i} > 0$, *and* $Q_{2j} > 0$ *which satisfy the LMI*

$$\begin{bmatrix} M & A_1Q_0 & \cdots & A_pQ_0 & B_1Y & \cdots & B_qY & D & Q_0E' \\ Q_0A_1' & -Q_{11} & & & \cdots & & & & 0 \\ \vdots & & \ddots & & & & & & \\ Q_0A_p' & & & -Q_{1p} & & & & & \\ Y'B_1' & \vdots & & & -Q_{21} & & & & \vdots \\ \vdots & & & & & \ddots & & & \\ Y'B_q' & & & & & & -Q_{2q} & & \\ D' & & & & & & & -\gamma I & \\ EQ_0 & 0 & & & \cdots & & & & -\gamma I \end{bmatrix} < 0 \quad (8)$$

then the state feedback control

$$u(t) = YQ_0^{-1}x(t) \quad (9)$$

stabilizes the system (1) and guarantees the H_∞ norm bound of the closed loop transfer function T_{zw} in (3), i.e. $\| T_{zw} \|_\infty < \gamma$.

Proof: If Y, $Q_0 > 0$, and $Q_1 > 0$ are the solutions of the LMI (8), then the solutions also satisfy the LMI (6). Hence from the theorem 1, the state feedback control (9) stabilizes the state and input delayed system (1).

The LMI (8) is equal to the following Riccati inequality

$$A_0Q_0 + Q_0A_0' + B_0Y + Y'B_0' + \sum_{i=1}^{p} Q_{1i} + \sum_{j=1}^{q} Q_{2j} + \sum_{i=1}^{p} A_iQ_0Q_{1i}^{-1}Q_0A_i' + \sum_{j=1}^{q} B_jYQ_{2j}^{-1}Y'B_j' + \frac{1}{\gamma}DD' + \frac{1}{\gamma}Q_0E'EQ_0 < 0. \quad (10)$$

Using the relations, $P_0 = Q_0^{-1}$, $P_{1i} = Q_0^{-1}Q_{1i}Q_0^{-1}$, $P_{2j} = Q_0^{-1}Q_{2j}Q_0^{-1}$, and $F = YQ_0^{-1}$, the inequality (10) can be rewritten as

$$P_0A + A'P_0 + P_0B_0F + F'B_0'P_0 + \sum_{i=1}^{p} P_{1i} + \sum_{j=1}^{q} P_{2j} + \sum_{i=1}^{p} P_0A_iP_{1i}^{-1}A_i'P_0 + \sum_{j=1}^{q} P_0B_jFP_{2j}^{-1}F'B_j'P_0 + \frac{1}{\gamma}P_0DD'P_0 + \frac{1}{\gamma}E'E < 0. \quad (11)$$

Following the similiar procedure as in the proof of the theorem 1 in [3], we can show that γ is the H_∞ norm bound of the closed loop transfer function T_{zw} (3).

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The theorem 2 provides an LMI problem to obtain a controller which guarantees some H_∞ performance. In order to find the H_∞ controller which provides the smallest possible H_∞ norm bound, we must solve the minimization problem for γ subject to conditions $Q_0 > 0$, $Q_{1i} > 0$, and $Q_{2j} > 0$, and the LMI (8). It is also noted that the LMI (8) is reduced to the LMI (6) when the H_∞ norm bound γ approaches to the infinity.

3 Conclusion

In this paper, a memoryless state feedback H_∞ controller for state and input delayed systems is presented. The proposed controller not only stabilizes the state and input delayed system, but also guarantees the H_∞ norm bound. The controller is obtained by solving linear matrix inequalities with some existing convex optimization algorithms.

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