

Design of Discrete-Time Integral Controllers for Non-Minimum Phase Plants via LTR Techniques

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Abstract

In this paper, we discuss an application of LTR techniques to integral controller design for discrete-time non-minimum phase plant models. It is shown that the feedback property obtained by enforcing the conventional LTR procedure can be achieved by the partial LTR technique. In addition, we point out that the partial LTR technique provides more design freedom in shaping a target feedback property.

1. Introduction

Although the perfect recovery of a target feedback property is impossible for non-minimum phase plant models, the LTR (Loop Transfer Recovery) techniques are still useful to approximately recover the target feedback property [1][2]. As a systematic design procedure for non-minimum phase plant models, Moore and Xia [3] have proposed a partial LTR technique for continuous-time LQG control systems. For discrete-time LQG control systems, Ishihara [4] have recently clarified the relation between the enforcement of the conventional LTR procedure and the partial LTR technique.

In this paper, we discuss an application of the LTR techniques to integral controller design for discrete-time non-minimum phase plant models. As a basic integral controller design, we adopt the method proposed by Guo *et al.* [5]-[7]. We focus our attention to feedback property at the plant input side. By reformulating the design method of Guo *et al.* [5] for a minimum phase/all-pass decomposition of the plant transfer function matrix, we construct an observer-based integral controller required for the partial LTR. We show that the feedback property

obtained by enforcing the conventional LTR procedure can be achieved by the partial LTR technique. This result provides clear system-theoretic interpretation on the enforcement of the conventional LTR procedure. In addition, we point out that the partial LTR technique provides more design freedom in shaping a target feedback property via the selection of a performance index.

2. Preliminaries

Consider a discrete-time plant described by

$$x(t+1) = Ax(t) + Bu(t), \quad y(t) = Cx(t), \quad (2.1)$$

where $x(t) \in R^n$ is a state vector, $u(t) \in R^m$ is a control vector and $y(t) \in R^r$ is an output vector. We assume that the pairs (A, B) and (C, A) are controllable and observable, respectively, and that the matrix CB is nonsingular. To guarantee the existence of an integral controller, we assume that the plant model has no zero at $z=1$.

If the model (2.1) is obtained by discretization with a zeroth order holder, the transfer function matrix $G(z) = C(zI - A)^{-1}B$ is frequently non-minimum phase. Then, we can decompose $G(z)$ as

$$G(z) = C(zI - A)^{-1}B_m G_n(z), \quad (2.2)$$

where $C(zI - A)^{-1}B_m$ is a minimum-phase part and $G_n(z)$ is an all-pass part satisfying $G_n'(z^{-1})G_n(z) = I$. All unstable zeros of $G(z)$ are contained in $G_n(z)$.

Let $x_m(t)$ and $x_n(t)$ denote the state of the minimum phase part and that of the all-pass part, respectively. Define the argument state vector as

$$\chi(t) = [x_m'(t) \quad x_n'(t)]'. \quad (2.3)$$

Let $\{A_n, B_n, C_n, D_n\}$ denote a state space representation of the all-pass part. Then, we can

construct a stochastic model of the plant as

$$\begin{aligned}\chi(t+1) &= \Phi\chi(t) + \Gamma u(t) + \Theta w(t), \\ y(t) &= H\chi(t) + v(t),\end{aligned}\quad (2.4)$$

where

$$\begin{aligned}\Phi &= \begin{bmatrix} A & B_m C_a \\ 0 & A_a \end{bmatrix}, \quad \Gamma = \begin{bmatrix} B_m D_a \\ B_a \end{bmatrix}, \\ \Theta &= \begin{bmatrix} B_m & B_m D_a \\ 0 & B_a \end{bmatrix}, \quad H = [C \quad 0].\end{aligned}\quad (2.5)$$

$w(t) \in R^p$ is disturbance vector

$$w(t) = [w'_f(t) \quad w'_s(t)]', \quad (2.6)$$

and $v(t) \in R^r$ is an observation noise. Note that $w_s(t)$ is a disturbance entering in the plant input and $w_f(t)$ is a fictitious disturbance inserted at the input of the minimum-phase part to achieve the partial LTR.

We assume that $w(t)$ is a zero-mean white noise sequence with the covariance matrix

$$Q = \text{diag}[Q_f I \quad Q_s I], \quad (2.7)$$

where $Q_f \geq 0$, $Q_s \geq 0$. The observation noise $v(t)$, which is assumed to be independent of $w(t)$, is a zero-mean white noise sequence with the covariance matrix $R \geq 0$.

It can easily be checked that if $\{A, B, C\}$ is minimal, (Φ, Γ) and (H, Φ) are stabilizable and detectable, respectively. In addition, we can show that the matrix

$$\Omega = \begin{bmatrix} \Phi - I & \Gamma \\ H & 0 \end{bmatrix} \quad (2.8)$$

is nonsingular provided the model (2.1) has no zero at $z=1$.

3. Minimum Phase State Feedback Controller

Assuming that all the state of (2.3) is perfectly measurable, we construct an integral controller accounting unit computation delay based on the efficient design method proposed by Guo *et al.* [5]. The algorithm of this controller is given by

$$u(t+1) = s(t+1) - \Lambda[\Phi\chi(t) + \Gamma u(t)], \quad (3.1)$$

$$s(t+1) = s(t) + M[r(t) - y(t)], \quad (3.2)$$

where $r(t)$ is a step reference input, $s(t)$ is the state of

the integrator. The matrices Λ and M in (3.1) are determined by the linear matrix equation

$$[\Lambda \quad M]\Omega = [\Psi\Phi \quad I + \Psi\Gamma], \quad (3.3)$$

where Ψ is the state feedback gain matrix of a regulator problem for the plant (3.1) and Ω is the matrix defined by (2.8). Let us rewrite the controller algorithm (3.1)-(3.2) such that only the minimum-phase state is used for the feedback. First, partition the feedback gain matrix defined by (3.3) as

$$\Lambda = [L_m \quad L_a], \quad (3.4)$$

where L_m and L_a are the feedback gain matrix for the minimum-phase part and that of the all-pass part, respectively. Then, from the equation (3.1) and (3.4), we have

$$\begin{aligned}u(t+1) &= s(t+1) - L_m[Ax_m(t) + B_m C_a x_a(t) \\ &\quad + B_m D_a u(t)] - L_a[A_a x_a(t) + B_a u(t)].\end{aligned}\quad (3.5)$$

Denoting the z -transforms of $x_a(t)$ and $u(t)$ by $x_a(z)$ and $u(z)$, respectively, we have

$$\begin{aligned}x_a(z) &= (zI - A_a)^{-1} B_a u(z), \\ C_a x_a(z) + D_a u(z) &= G_a(z) u(z).\end{aligned}\quad (3.6)$$

Taking the z -transforms of the both sides of (3.5) and substituting (3.6) for $x_a(z)$, we obtain

$$\begin{aligned}zu(z) &= zs(z) - L_m[Ax_m(z) + B_m G_a(z)u(z)] \\ &\quad - L_a[A_a(zI - A_a)^{-1} + I]B_a u(z) \\ &= z(z-1)^{-1}M[r(z) - y(z)] - L_m[Ax_m(z) \\ &\quad + B_m G_a(z)u(z)] - zL_a(zI - A_a)^{-1}B_a u(z),\end{aligned}\quad (3.7)$$

where $x_m(z)$, $s(z)$, $y(z)$ and $r(z)$ are the z -transforms of $x_m(t)$, $s(t)$, $y(t)$ and $r(t)$, respectively. Define the transfer function matrices

$$T(z) = \Delta_a(z)M, \quad L(z) = \Delta_a(z)L_m, \quad (3.8)$$

where

$$\Delta_a(z) = [I + L_a(zI - A)^{-1}B_a]^{-1}. \quad (3.9)$$

Then, we can rewrite (3.7) as

$$\begin{aligned}u(z) &= (z-1)^{-1}T(z)[r(z) - y(z)] \\ &\quad - z^{-1}L(z)[Ax_m(z) + B_m G_a(z)u(z)].\end{aligned}\quad (3.10)$$

Note that $[A_m x_m(z) + B_m G_a(z)]$ in the right side of the above equation is just equal to the z -transform of the one-step ahead prediction of the minimum-phase state. Formally, the control law (3.10) is obtained by replacing the constant controller matrices in the original integral controller [5] by the dynamic

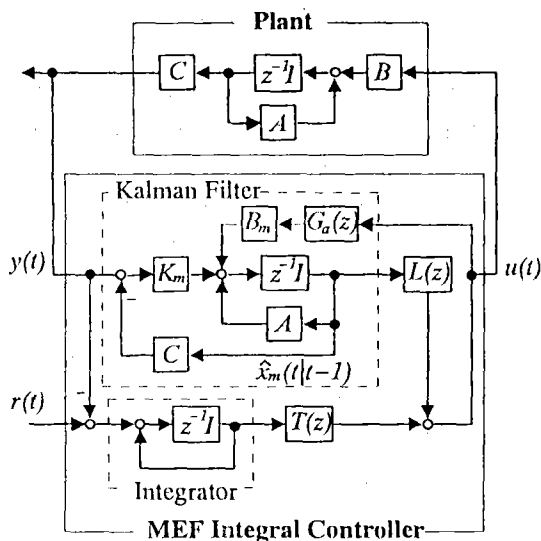


Fig.1 Structure of MEF Integral Controller

matrices. We call the controller of (3.10) as the MSF (Minimum-Phase State Feedback) integral controller.

For the sensitivity matrix at the plant input side of the above control system, we can obtain the following result.

Proposition 3.1: The input sensitivity matrix $\Sigma_{\text{MSF}}(z)$ for the MSF integral controller is given by

$$\Sigma_{\text{MSF}}(z) = z^{-1}(z-1)\Sigma(z)[I + L_a(zI - A_a)^{-1} B_a + z^{-1}L_m B_m G_a(z)], \quad (3.11)$$

where

$$\Sigma(z) = [I + \Psi(zI - \Phi)^{-1}\Gamma]^{-1} \quad (3.12)$$

is the input sensitivity matrix of the regulator for the system (2.4) with feedback gain matrix Ψ .

The expression (3.11) does not only manifest the effect of introducing the integrators but also gives the explicit relation to the regulator used in the design.

Although the above proposition holds for any state feedback gain matrix Ψ that makes $\Phi - \Gamma\Psi$ stable, we now consider the case where Ψ is determined by using the quadratic performance index

$$V = \sum_{t=0}^{\infty} [y'(t)y(t) + \rho u'(t)u(t)], \quad (3.13)$$

where $\rho \geq 0$. Then we can obtain the following result.

Lemma 3.1: Assume that the transfer function matrix $G(z) = C(zI - A)^{-1}B$ has a single unstable zero q

($|q| > 1$). Let η and ξ denote a unit zero-direction vector and a zero-state vector, respectively. Namely,

$$\begin{bmatrix} qI - A & -B \\ -C & 0 \end{bmatrix} \begin{bmatrix} \xi \\ \eta \end{bmatrix} = 0, \quad \eta'\eta = 1 \quad (3.14)$$

holds. Choose the matrix B_m in (2.2) and $\{A_a, B_a, C_a, D_a\}$ as

$$B_m = B_q J_q, \quad (3.15)$$

$$A_a = \frac{1}{q}, \quad B_a = \frac{1}{q}(q - \frac{1}{q})\eta', \quad C_a = \eta, \quad (3.16)$$

$$D_a = I - (1 + \frac{1}{q})\eta\eta',$$

where

$$B_q = B - (q - \frac{1}{q})\xi\eta', \quad (3.17)$$

$$J_q = I - (q+1)\eta\eta'. \quad (3.18)$$

Let F and $\Psi = [F_m \ F_a]$ denote the optimal feedback gain matrix of the regulator problem for (2.1) and that for (2.4), respectively, under the performance index (3.13). Then the following relations hold:

$$F_m = F, \quad F_a = qI\xi \quad (3.19)$$

□

Using the above lemma, we can obtain the following result for the feedback gain matrices of the integral controller.

Lemma 3.2: Assume that $G(z) = C(zI - A)^{-1}B$ has a single unstable zero q . Consider the realization of the all-pass part $G_a(z)$ given in Lemma 3.1. Define the matrices L and T by the linear matrix equation

$$[L \ T]E = [FA \ I + FB], \quad (3.20)$$

where F is the optimal feedback gain matrix of the regulator problem for (2.1) under the performance index (3.13) and

$$E = \begin{bmatrix} A - I & B \\ C & 0 \end{bmatrix}. \quad (3.21)$$

Then, for the regulator feedback gain matrix given by (3.19), the solution of the matrix equation (3.3) can be expressed as

$$L_m = L, \quad L_a = qL, \quad M = T. \quad (3.22)$$

Proof: Using (2.3) and (2.4), we can rewrite the matrix equation for L_m , L_a and M as

$$L_m(A - I) + MC = F_m A, \quad (3.23)$$

$$L_m B_m C_a + L_a (A_a - I) = F_m B_m C_a + F_a A_a, \quad (3.24)$$

$$L_m B_m D_a + L_a B_a = I + F_m B_m D_a + F_a B_a. \quad (3.25)$$

From (3.14), we have $(qI - A)\xi = B\eta$. Then, from (3.15) and (3.16), we have

$$\begin{aligned} B_m C_a &= B_q J_q C_a = -q[B\eta - (q - \frac{1}{q})\xi] \\ &= (qA - I)\xi. \end{aligned} \quad (3.26)$$

Noting that $J_a D_a = I$ holds from (3.16) and (3.18), we have

$$B_m D_a = B_q J_q D_a = B_q. \quad (3.27)$$

Using (3.19), (3.26) and (3.27), we can rewrite (3.23)–(3.25) as

$$L_m (A - I) + MC = FA, \quad (3.28)$$

$$L_m (qA - I)\xi + L_a (\frac{1}{q} - 1) = qFA\xi, \quad (3.29)$$

$$L_m [B - (q - \frac{1}{q})\xi\eta'] + L_a \frac{1}{q} (q - \frac{1}{q})\eta' = I + FB. \quad (3.30)$$

Noting that L and T are defined by (3.20), we can easily show that L_m , L_a and M defined in (3.22) satisfy (3.28) and (3.30). Substituting (3.20) into the left side of (3.29) and noting that L satisfies (3.20), we have

$$\begin{aligned} L(qA - I)\xi + (1 - q)L\xi \\ = qL(A - I)\xi = q(FA - TC)\xi. \end{aligned} \quad (3.31)$$

Since $C\xi = 0$ holds, it follows from (3.32) that (3.29) is satisfied. Hence, the matrices L_m , L_a and M defined in (3.22) satisfy the matrix linear equation (3.3). Since the uniqueness of the solution of (3.3) is guaranteed by the non-singularity of Ω , the solution of (3.3) is given by (3.22). ■

Remark: The matrices L and T defined in (3.20) are the feedback gain matrices of an integral controller for the original plant model (2.1).

Using Proposition 3.1 and the above lemma, we have the following expression.

Proposition 3.2: Assume that $G(z) = C(zI - A)^{-1}B$ has a single unstable zero. Consider the realization of the all-pass part $G_a(z)$ given in Lemma 3.1. Consider the control law (3.10) using the matrices (3.22). Then, the sensitivity matrix at the plant input side can be expressed as

$$\begin{aligned} \Sigma_{MSF}(z) &= z^{-1}(z - 1)S(z)\{I + L(zI - A)^{-1} \\ &\quad [B - B_m G_a(z)] + z^{-1}LB_m G_a(z)\}, \end{aligned} \quad (3.32)$$

where

$$S(z) = [I + F(zI - A)^{-1}B]^{-1}. \quad (3.33)$$

Proof: From (3.16) and (3.19), we have

$$F_a(zI - A)^{-1}B_a = \frac{q^2 - 1}{qz - 1}F\xi\eta'. \quad (3.34)$$

Also using (3.14)–(3.18), we obtain

$$B_m G_a(z) = B - \frac{q^2 - 1}{qz - 1}(zI - A)\xi\eta'. \quad (3.35)$$

Multiplying the both sides of (3.35) by $F(zI - A)^{-1}$, we have

$$F(zI - A)^{-1}B_m G_a(z) = F(zI - A)^{-1}B - \frac{q^2 - 1}{qz - 1}F\xi\eta'. \quad (3.36)$$

From (3.34) and (3.36), we have the following relation for the feedback gain matrix defined in (3.19):

$$\begin{aligned} F_a(zI - A)^{-1}B_a + F_m(zI - A)^{-1}B_m G_a(z) \\ = F(zI - A)^{-1}B \end{aligned} \quad (3.37)$$

Substituting (3.37) into (3.12), we have (2.21). Also, for the feedback gain matrix defined in (3.21), the following relation holds:

$$\begin{aligned} L_a(zI - A)^{-1}B_a + L_m(zI - A)^{-1}B_m G_a(z) \\ = L(zI - A)^{-1}B \end{aligned} \quad (3.38)$$

Noting that $L_m = L$ holds and substituting (3.38) into (3.11), we have (3.32). ■

4. Partial Recovery

For the output feedback case, we can construct an integral controller by replacing the state $\chi(t)$ in (2.3) with an estimate $\hat{\chi}(t)$ generated by a prediction type Kalman filter. The estimate defined as

$$\hat{\chi}(t) = [\hat{x}'_m(t) \quad \hat{x}'_a(t)]' \quad (4.1)$$

is generated by the following algorithm:

$$\begin{aligned} \hat{x}'_m(t + 1) &= A\hat{x}'_m(t) + B_m G_a(z)u(t) \\ &\quad + K_m[y(t) - C\hat{x}'_m(t)] \end{aligned} \quad (4.2)$$

$$\hat{x}'_a(t + 1) = A_a \hat{x}'_a(t) + B_a u(t) \quad (4.3)$$

Let $\hat{x}'_m(z)$ denote the z -transform of the estimate $\hat{x}'_m(t)$. Replacing $z^{-1}[Ax'_m(z) + B_m G_a(z)]$ in the right side of (3.10) by $\hat{x}'_m(z)$, we can obtain the output feedback integral controller as

$$u(t) = (z-1)^{-1}T(z)[r(z) - y(z)] - L(z)\hat{x}_m(z). \quad (4.4)$$

We call this controller as the MEF (Minimum-phase Estimate Feedback) integral controller.

For the controller transfer function matrix of the MEF integral controller, we have the following result. *Lemma 4.1:* The transfer function from $y(z)$ to $u(z)$ of the MEF integral controller given by (4.4) can be expressed as

$$C_{\text{MEF}}(z) = D_c^{-1}(z)N_c(z), \quad (4.5)$$

where

$$D_c(z) = I + L(z)(zI - A + K_m C)^{-1} B_m G_a(z), \quad (4.6)$$

$$N_c(z) = (z-1)^{-1}T(z) + L(z)(zI - A + K_m C)^{-1} K_m. \quad (4.7)$$

Using the above lemma, the input sensitivity matrix for the output feedback case can be obtained as follows.

Lemma 4.2: The input sensitivity matrix for the MEF integral controller (4.4) can be expressed as

$$\begin{aligned} \Sigma_{\text{MEF}}(z) = & z^{-1}(z-1)\Sigma(z)\{I + L_a(zI - A_a)^{-1}B_a \\ & + z^{-1}L_m[I + (zI - A + K_m C)^{-1} \\ & (A - K_m C)]B_m G_a(z)\}, \end{aligned} \quad (4.8)$$

where $\Sigma(z)$ is the matrix defined in (3.12).

Proof: Using the relation of (4.5), we can write the input sensitivity matrix as

$$\begin{aligned} \Sigma_{\text{MEF}}(z) = & [I + C_{\text{MEF}}(z)G(z)]^{-1} \\ = & [D_c(z) + N_c(z)G(z)]^{-1} D_c(z). \end{aligned} \quad (4.9)$$

Noting that the matrix identity

$$\begin{aligned} (zI - A + K_m C)^{-1} \\ = z^{-1}I + z^{-1}(zI - A + K_m C)^{-1}(A - K_m C) \end{aligned} \quad (4.10)$$

holds, we can express $D_c(z)$ as

$$\begin{aligned} D_c(z) = & I + z^{-1}\Delta_a(z)L_m[I + (zI - A + K_m C)^{-1} \\ & (A - K_m C)]B_m G_a(z). \end{aligned} \quad (4.11)$$

Then the denominator of (4.9) can be rewritten as

$$\begin{aligned} & D_c(z) + N_c(z)G(z) \\ & = I + L(z)(zI - A + K_m C)^{-1}\{B_m + K_m G_m(z)\} \\ & \quad G_a(z) + (z-1)^{-1}T(z)G(z) \\ & = I + L(z)(zI - A)^{-1}B_m G_a(z) + \\ & \quad (z-1)^{-1}T(z)G(z) \\ & = \Delta_a(z)(z-1)^{-1}\{(z-1)I + \Gamma\Lambda + [\Lambda(\Phi - I) \\ & \quad + M\mathbf{H}](zI - \Phi)^{-1}\Gamma\} \end{aligned}$$

$$= z(z-1)^{-1}\Delta_a(z)\Sigma(z), \quad (4.12)$$

where we have used (3.3) to obtain the last expression. Substituting (4.11) and (4.12) into (4.9), we can obtain (4.8). ■

Using Proposition 3.1 and Lemma 4.2, we have the following result for the partial LTR.

Proposition 4.1: Consider the MEF integral controller with the prediction type Kalman filter for the disturbance covariance matrix Q and observation noise covariance matrix R given by

$$Q = \text{diag}[\sigma_f I \quad \sigma_s I], \quad R = I, \quad (4.13)$$

where $\sigma_f \geq 0$, $\sigma_s \geq 0$. Then, as $\sigma_f \rightarrow \infty$, the input sensitivity matrix approaches the matrix $\Sigma(z)$ defined by (3.11).

Proof: Using the result obtained by Shaked [8] for the choice of covariance matrices (4.13), we can easily show that the Kalman filter gain matrix K_m approaches

$$K_m^* = AB_m(CB_m)^{-1} \quad (4.14)$$

as $\sigma_f \rightarrow \infty$. Noting that

$$(A - K_m^*)B_m = 0, \quad (4.15)$$

we can show that (4.8) approaches (3.11). ■

For the LQG problem, it has been clarified that, under the common choice of the performance index, the feedback property obtained by enforcing the conventional LTR procedure can be achieved by the partial LTR technique [4]. In the following, we show that the same result holds for the integral controller.

Proposition 4.2: Assume that the transfer function matrix $G(z) = C(zI - A)^{-1}B$ has a single unstable zero q ($|q| > 1$). Consider the integral controller using the prediction type Kalman filter and the control parameter determined by (3.19) where F is the optimal feedback gain matrix for the regulator problem under the performance index (3.13). Choose the disturbance covariance matrix as $\sigma BB'$ and the observation noise covariance matrix as the identity matrix. Then, the asymptotic input sensitivity matrix obtained by letting $\sigma \rightarrow \infty$ for this integral controller coincides with the input sensitivity matrix for the MSF integral controller given by (3.32).

Proof: The input sensitivity matrix for this integral controller has been given in [5] as

$$S_{\text{INT}}(z) = z^{-1}(z-1)S(z)\{I + L(zI - A + KC)^{-1}B\}, \quad (4.16)$$

where K is a prediction type Kalman filter gain matrix for the plant (2.1). For the given choice of the disturbance covariance matrix and the observation noise covariance matrix, the asymptotic value of K as $\sigma \rightarrow \infty$ is given by (4.14). Using the matrix inversion lemma, we have the following identity:

$$L(zI - A + KC)^{-1}B = z^{-1}LB_m G_a(z) + L(zI - A)^{-1}[B - B_m G_a(z)] \quad (4.17)$$

Substituting (4.17) into (4.16), we can easily check that the asymptotic value of (4.16) coincides with (3.32). ■

The above result justifies the enforcement of the conventional LTR procedure for non-minimum phase plants since the partial LTR technique has clear system-theoretic meaning: it attempts to recover only the minimum phase part of a target feedback property.

The above result also suggests that, in order to achieve the target feedback property achieved by the partial LTR technique, it is not necessary to use the MEF integral controller which has higher order than the conventional integral controller. Note that this is true only if the both controllers have been designed by the common choice of the performance index (3.13). The partial LTR technique admits a performance index more general than (3.13). Exploiting this design freedom, we can achieve a feedback property that can not be obtained by the conventional LTR technique.

6. Conclusions

We have discussed an application of the partial LTR techniques to integral controller design for discrete-time non-minimum phase plant models. We have shown that, for the common choice of the performance index, the feedback property obtained by enforcing the conventional LTR technique can be achieved by the partial LTR technique. In addition, we have pointed out that the partial LTR technique provides more design freedom in shaping a target feedback property via the selection of a performance index.

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