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A TRAJECTORY ESTIMATION STUDY OF A HYPERSONIC VEHICLE

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Abstract A method of trajectory error estimation of a hypersonic vehicle, by a covariance analysis technique is presented and discussed. The method itself is a wellkown technique, however, the thema has been rarely treated. As the importance is increasing, it is explained here and some of our newly deviced techniques are also presented.

1. INTRODUCTION

The trajectory estimation error propagation study by the covariance analysis has been already conducted in early 1960's, and the program has beeen developed for example the MATS[1] (The Mission Analysis and Trajectory Simulation Program). which has been applied from Apollo project through the trajectory estimation of the Minnitman's multiple warheads. The program has been developed by TRW Corporation and introduced to Japanese NASDA (National Space Development Agency) through Mitsubishi-TRW Corporation (current Mitubishi Space Software Corporation), which has been employed for the guidance and orbit control of Japanese BS (Broadcasting Sattellite) and CS (Communication Sattellite). Therefore the technique itself is wellknown to our space engineer, however, the experience of the recent Gulf War has kept our attention again to the importance of the trajectory estimating technique of unguided hypersonic vehicles. We have been studying the technique for intending to apply for future anti-ballistic missile system, or re-entry phase of a space vehicle. As the importance of the technique is increasing, it will be of some use to the readers to explain the method with some of our newly deviced technique, and some experiences.

2. THE EQUATIONS OF THE COVARIANCE PROPAGATION

In this paper, the trajectory error estimation process of a relatively short range surface launched vehicle is explained.

Let the vehicle system equations are expressed as follows.

$$\dot{x} = f(x) + w(t) \tag{1}$$

where x is a state vector, and w is a pure gaussian random process. Now we separate the x vector with the nominal state vector x^* and the deviation vector from x^*; δx, then

$$\dot{x} = \dot{x}^* + \delta\dot{x} = f(x^*) + \partial f/\partial x \cdot \delta x + w(t) \tag{2}$$

Let x^* should satisfy the next equation

$$\dot{x}^* = f(x^*) + E[w(t)] \tag{3}$$

where E expresses the expected (avarage) value, then

$$\delta\dot{x} = \partial f/\partial x \cdot \delta x + u(t) \tag{4}$$
$$u(t) = w(t) - E[w(t)] \tag{5}$$

Let us replace this small deviation vector as x, and $\partial f/\partial x$ which is a time function as $F(t)$, then

$$\dot{x} = F(t)x + u(t) \tag{6}$$

where $x(t)$ is a stochastic process state vector, and $u(t)$ becomes a white gaussian vector.

Let $Q(t)$ be the intensity matrix of $u(t)$ (the integral of the auto-correlation function of $u(t)$), then the propagation of the covariance matrix of x; X is expressed by the following equation.[2]

$$\dot{X} = F(t)X + XF^T(t) + Q(t) \tag{7}$$

where X is expressed as

$$X = E\{[x - E(x)] \cdot [x - E(x)]^T\} \quad (8)$$

This derivation of the equations is a little defferent from that of Ref.(2). The reason is as follows. In Eq.(6), if $u(t)$ is a collored noise, the deviation value of x often becomes quite large and our quasi-linearized equations become inadequate. Therefore we have included the bias components of $w(t)$ into Eq.(3) and elliminate the effect to x in Eq.(6). Another reason is that the vehicle flight time is relatively short, therefore the effect of unknown bias noise is assumed to be small. As the expected value of x follows to Eq.(3), and $u(t)$ is a white gaussian noise, the expected value of the perturbation about x* is zero, therefore

$$X = E(x \cdot x^T) \quad (9)$$

3. CALCULATION METHOD

The employed vehicle is a vertically launched first stage surface to surface missile with the range about 950km and maximum altitude about 300km. For a covariance error analysis within this range, the difference between employing the flat earth model or the circular earth model is very small, therefore for simplicity, we explain it by employing the former. (However, for a precise trajectory simulation, we need the oblate spheroidal earth model, and the geopotential function expressed in terms of zonal, tesseral, and sectorial coefficients, as well as the launching point data in relation to the geodetic altitude, earth rotational rate and wind model) By employing the point mass and the flat earth model, the missile equations are given as follows.

$$\dot{v} = (T\cos\alpha - D)/m - g\sin\gamma \quad (10)$$
$$\dot{\gamma} = (L + T\sin\alpha)\cos\phi/(mv) - (g/v)\cos\gamma \quad (11)$$
$$\dot{\psi} = (L + T\sin\alpha)\sin\phi/(mv \cdot \cos\gamma) \quad (12)$$
$$\dot{x} = v\cos\gamma\cos\psi \quad (13)$$
$$\dot{y} = v\cos\gamma\sin\psi \quad (14)$$
$$\dot{h} = v\sin\gamma \quad (15)$$
$$\dot{m} = -T/(g \cdot I_{SP}) \quad (16)$$

where

$$L = 1/2\rho v^2 s C_L, \quad C_L = C_{L\alpha}(\alpha - \alpha_0) \quad (17)$$
$$D = 1/2\rho v^2 s C_D, \quad C_D = C_{D0} + kC_L^2 \quad (18)$$

The aerodynamic derivative coefficients $C_{L\alpha}$, C_{D0} and k are given as functions of Mach number M, which is a function of v and h, and the air density ρ is a function of h;

$$\rho = \rho(h), \qquad M = M(v,h) \quad (19)$$
$$C_{L\alpha} = C_{L\alpha}(M),\ C_{D0} = C_{D0}(M),\ k = k(M) \quad (20)$$

By employing Eqs.(10) through (20), the missile nominal trajectory from launching to landing with noise-free condition is obtained first. If there is any predicted and modeled bias error, these equations are modified by adding the E[w(t)] of Eq.(3). The error estimation of the drop point is implemented through three phases.

In the first phase where the missile is boosting and the infrared ray emanated from the exhaust is observable from the sattellite or the airborne optical aparatus, the missile states v, γ, ψ, x, y, h, and m are estimated by an extended Kalman filter. It is assumed that the missile flights unguided after the boost, and it can be observed only in the boosting stage. Let t_0 be the time when rocket engine is stopped, then the covariance matrix $X(t_0)$ of the state vector x at t_0

$$x = (\delta v, \delta\gamma, \delta\psi, \delta x, \delta y, \delta h)^T \quad (21)$$

gives the dominant effect for the error propagation. In Eqs.(10) through (16), m is treated as a state variable to be estimated, however, after the engine is stopped, $m = m(t_0)$ is constant. Therfore the effect of the estimation error of $m(t_0)$ is studied separately, and is not treated as a state variable in (21).

In the second phase of the error analysis, Eq.(7) is integrated in accompany with Eqs.(10)~(20) by employing the $x(t_0)$ and $X(t_0)$ which are obtained in the first phase as the initial values, and obtain the error propagation until the missile approaches to the atmosphere, e.g. at 80km. In the calculation, $F(t) = \partial f/\partial x$ is evaluated along with the nominal trajectory. Let the final time be noted as t_1.

Until this time we have employed the point mass model, because if we employ the rigid body model, states estimation by a Kalman filter and calculation of the covariance propagation becomes quite complicated. However, from the re-entry phase through collision to the earth, the error propagation is very sensitive to the employed vehicle model, therefore at least six degree-of-freedom rigid body model has to be employed. In the third phase of the error analysis, the following axis symmetric missile equations of motion with the flat earth model is employed.

$$\dot{p} = M_u / I \quad (22)$$
$$\dot{q} = [M_v - (I - I_T) rp] / I_T \quad (23)$$
$$\dot{r} = [M_w - (I_T - I) pq] / I_T \quad (24)$$
$$\dot{\phi} = p + (q \cdot s\phi + r \cdot c\phi) \tan\theta \quad (25)$$
$$\dot{\theta} = q \cdot c\phi - r \cdot s\phi \quad (26)$$
$$\dot{\psi} = (q \cdot s\phi + r \cdot c\phi) / c\theta \quad (27)$$
$$\dot{u} = F_u / m - qw + rv \quad (28)$$
$$\dot{v} = F_v / m - ru + pw \quad (29)$$
$$\dot{w} = F_w / m - pv + qu \quad (30)$$

where u, v, w and p, q, r are the velocity vector components and angular rate components, F_u, F_v, F_w and M_u, M_v, M_w are the force vector components and moment vector components, all of them are measured in the missile body fixed axes, respectively. In these equations,"c"and "s" are the abreviations of "cos" and "sin", respectively. The inertial coordinates of the missile are obtained by integrating

$$\dot{x} = u \cdot c\theta c\psi + v(-c\phi s\psi + s\phi s\theta c\psi) + w(s\phi s\psi + c\phi s\theta c\psi) \quad (31)$$
$$\dot{y} = u \cdot c\theta s\psi + v(c\phi c\psi + s\phi s\theta s\psi) + w(-s\phi c\psi + c\phi s\theta s\psi) \quad (32)$$
$$\dot{z} = -u \cdot s\theta + v \cdot s\phi c\theta + w \cdot c\phi c\theta \quad (33)$$

In Eqs.(22)～(24), the following is assumed.

$$I_{xx} = I,\ I_{yy} = I_{zz} = I_T,\ I_{xy} = I_{xz} = I_{yz} = 0 \quad (34)$$

The Euler angles ψ, θ and ϕ are selected so that the body coordinates are taranslated to the inertial coordinates through the rotation ψ , about z axis, and θ about y axis and ϕ about x axis in this order. In the simulation, other aerodynamic coefficients, which are not employed in the point mass model, such as $C_{m\alpha}, C_{mq}, C_{n\beta}$ etc. are required. The initial condition of the states of the third phase, $x(t_1)$ is distributed about the $x^*(t_1)$ with the covariance $X(t_1)$. We have conducted Monte Carlo simulations in this phase. By combining the results of phase 1 through 3 and with some statistical treatment, we can obtain the hit point probability distribution of the missile. As for the estimation error about the $m(t_0)$ or other constant parameters, we can estimate the effect just by conducting a small number of simulations without a difficulty.

4.OTHER DISCUSSIONS

Figures 1 and 2 show an example of flight simulations. Figure 1 shows the missile down-range versus altitude trajectory, and Fig.2 shows the time histories of the missile velocity and the flight path angle. The missile is vertically launched and at its boost stage, the attitude is controlled to enter into a ballistic trajectory. The attitude is trimmed even in a very low air density, and the angle of attack is constantly increasing, but relatively small in the ascent phase. In the desent phase, α is still increasing to reach the maximum value of about 40 deg. In the re-entry phase, and at about 80km's altitude, α quickly decreases and induces a Phugoid mode. This Phugoid mode makes it difficult to implement the covariance analysis, therefore the Monte Carlo simulations are conducted in this phase. The employed airdensity models in the MATS of those days are US standard 1962 and ARDC 1959 etc. Recently we can employ more renovated models, but as far as the relatively short range ballistic vehicle in the paper is concerned, the effect of employing the different air density model is very small. However, in the case of a space vehicle which stays in an orbit longer time, we recommend to employ the newest version of CIRA, Jaccia or other sophisticated air density model. In the case, the point mass model is described by employing orbital element parameters, and the precise gravity model must be employed. In practice, $u(t)$ in Eq.(6) or $Q(t)$ in Eq.(7) does not give large effect in our case, because the effect of $X(t_0)$ is dominant, however, it is useful to calculate the effect of the applied noise in an arbitrary time (for example, the effect of the collision of a small meteorite). In the paper, the missile body is assumed to be symmetric. In practice, the existance of a small asymmetry induces a roll, and unless a roll stabilizing control in the missile, a small α and β couples with the induced roll and cause a roll resonance, which makes more difficult to estimate the missile trajectory.

CONCLUSION

A method of trajectory error estimation and the calculation of hit point error probability distribution of a surface to surface missile is proposed and explained in detail. The algorithm is developed to combines the point mass model simulation, the raigid body model simulation and covariance propagation simulation. The method gives a quick overview of the estimation error of the missile drop point distribution, without conducting massive numbers of Monte Carlo simulations.

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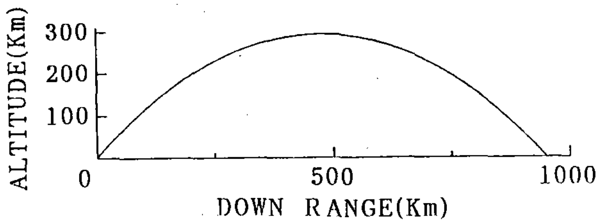

Fig.1 Missile nominal trajectory

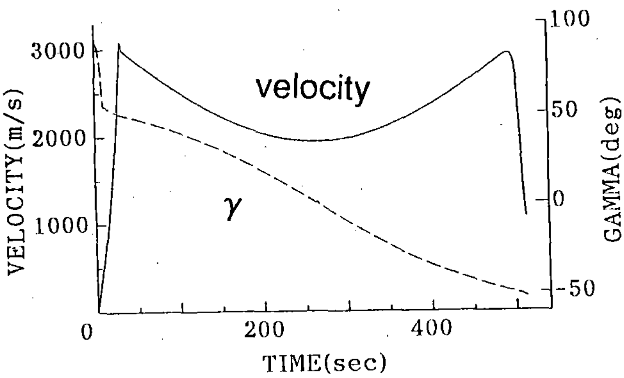

Fig.2 Missile velocity and flight path angle