'94 KACC (1994. 10. 17 ~ 20)

Guidance Law for a Missile After Thrust Cutoff

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Key Word : Missile, Proportional Navigation, Guidance and Control

Abstract

In the previous paper, we presented a new guidance law for a missile during boost phase. Thus, this paper deals with the guidance law for a missile after the thrust cutoff against an accelerating and turning target. It is essentially based on the concept of proportional navigation. Some simulation studies were performed using a three dimensional mathmatical model of an air-to-air missile and the effectiveness of the guidance law presented was shown.

Nomenclature

A_t = lateral acceleration vector of target
A_m = lateral acceleration vector of missile
a_y = missile y-axis lateral acceleration
a_z = missile z-axis lateral acceleration
a_{yc} = missile y-axis lateral acceleration command signal
a_{zc} = missile z-axis lateral acceleration command signal
V_t = target velocity vector
V_m = missile velocity vector
M = present missile position
T = present target position
I = predicted hit position
R = relative position vector from M to T
P = relative position vector from T to I
Q = relative position vector from M to I
σ = line-of-sight (LOS) angle
$\dot{\sigma}$ = line-of-sight (LOS) rate vector
θ = missile flight-path angle
$\dot{\theta}$ = missile flight-path rate vector
ϕ_t = target flight-path angle to LOS
ϕ_m = missile flight-path angle to LOS
μ = $\phi_t - \phi_m$
N = navigation constant
N_e = effective navigation constant
m = missile total mass after thrust cutoff
g = gravitational acceleration
t_{go} = time-to-go
ρ = air density
S = missile reference area
k = induced drag coefficient
C_{D0} = missile zero-lift-drag coefficient
D_m = missile drag
γ_m = missile pitch angle
ψ_m = missile yaw angle
τ_m = missile time constant

1 Introduction

It is well known that the conventional proportional navigation (PN) is one of the most effective guidance law when missile and target velocities are constant [1]. But a missile has an acceleration due to thrust during boost phase. Thus, two of the authors presented the new guidance law for short range missiles with thrust [2]. However, a missile may track a target after the thrust cutoff. In this case a missile has the axial deceleration due to air drag and the velocity decreases rapidly. This velocity change may seriously degrade the performance of the missile guided by PN. Chadwick derived the approximate analytical solution for the miss distance of PN missile with axial deceleration after thrust cutoff, but he mentioned nothing about a guidance law [3].

Therefore, first taking into account the varying velocity due to air drag, we derive a guidance law for a missile against a target with constant acceleration. Though the guidance law derived gives the theoretical acceleration to guide a missile on a collision course, it is very difficult to inplement on most existing tactical missiles. Thus, the on-board approximation of the guidance law is derived and also the technique for implementation of the guidance law is shown. In this paper, we call the approximated guidance law "NEW GUIDANCE LAW (NGL)" for sake of convenience. Finally, the performance of NGL is compared with that of PN, using the simulation studies of a three dimensional mathmatical model of an air-to-air missile.

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2 Derivation of a Guidance Law

Figure 1 shows the intercept geometry of a missile against a target. M and T represent the actual positions of a missile and a target at time t, respectively, and I indicates the predicted hit point, that is, the triangle ITM is a collision triangle at time t. Let us assume that a target is flying with constant acceleration A_t. From Fig.1, the line-of-sight (LOS) rate is given by the following vector equation :

$$\dot{\sigma}=\frac{1}{R^2}\{R\times(V_t-V_m)\} \tag{1}$$

Since I indicates the predicted hit point, the missile should be guided along MI, which is the shortest or optimal course for the missile to intercept a target. Assuming that the correct missile velocity vector along with MI is $\widetilde{V}_m$ and the deviation of the real missile velocity vector from $\widetilde{V}_m$ is ΔV_m, we have

$$V_m=\widetilde{V}_m+\Delta V_m \tag{2}$$

Substituting Eq.(2) into Eq.(1), we obtain

$$\dot{\sigma}=\frac{R\times(V_t-\widetilde{V}_m)}{R^2}+\frac{R\times(-\Delta V_m)}{R^2} \tag{3}$$

The first term of the right side of Eq.(3) represents the correct LOS rate when the missile flies along the collision course. The second term is the deviation of the LOS rate from the correct one. If a missile is guided with a flight-path rate in proportion to the deviation of the LOS rate, assuming no missile dynamic lag, the missile flight-path rate becomes

$$\dot{\theta}=N\frac{R\times(-\Delta V_m)}{R^2} \tag{4}$$

where N is the navigation constant. Then, the required acceleration for a missile to be guided to the correct collision course is given by

$$\begin{aligned}F&=\dot{\theta}\times V_m\\&=-\frac{N}{R^2}[\{(\widetilde{V}_m-V_m)\times R\}\times V_m]\end{aligned} \tag{5}$$

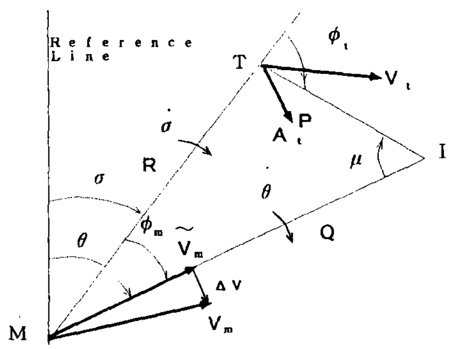

Figure 1 : Intercept Geometry

From Fig.1, $\widetilde{V}_m$ can be written as

$$\widetilde{V}_m=V_m\left(\frac{\sin\phi_m P}{\sin\phi_t P}+\frac{\sin\mu R}{\sin\phi_t R}\right) \tag{6}$$

Substituting Eq.(6) into Eq.(5), we obtain

$$F=\frac{N}{R^2}\left[\left\{\left(V_m-V_m\frac{\sin\phi_m P}{\sin\phi_t P}\right)\times R\right\}\times V_m\right] \tag{7}$$

and from Fig.1, we have

$$\frac{\sin\phi_m}{\sin\phi_t}=\frac{\overrightarrow{TI}}{\overrightarrow{MI}}=\frac{P}{Q} \tag{8}$$

Substituting Eq.(8) into Eq.(7), we obtain

$$F=\frac{N}{R^2}\left[\left\{\left(V_m-\frac{V_m}{Q}P\right)\times R\right\}\times V_m\right] \tag{9}$$

This is the basic equation of the guidance law for a missile to hit a target at the point I. In order to realize this guidance law, P and Q must be predicted. Since the target is moving with the constant acceleration A_t, defining t_{go} as time-to-go, P can be approximated by

$$P=V_t\, t_{go}+\frac{1}{2}A_t\, t_{go}^2 \tag{10}$$

Next, let us derive the equation for Q. The forces acting on a missile after thrust cutoff are air drag and gravity but the gravity is neglected because its effect is very small [4]. Thus taking only zero-lift air drag into account, we have the following equation of motion for a missile.

$$\dot{V}_m=\frac{1}{2}\rho V_m^2 SC_{D0}/m \tag{11}$$

Let us define λ as follows :

$$\lambda=\frac{\rho SC_{D0}}{2m} \tag{12}$$

If we assume that λ is constant, Eq.(11) can be solved analytically and we have

$$V_m(t)=\frac{V_{m0}}{1+\lambda V_{m0}t} \tag{13}$$

where V_{m0} is an initial value. Q is computed from the following equation.

$$Q=\int_0^{t_{go}}V_m(t)dt \tag{14}$$

Integrating Eq.(14), we obtain

$$Q=\frac{1}{\lambda}(1+\lambda V_{m0}t_{go}) \tag{15}$$

As it is clear from Eq.(10) and (15), we need t_{go} in order to determine P and Q. From Fig.1, the equation for t_{go} is obtained as follows:

$$Q^2-P^2-R^2-P\cdot R=0 \tag{16}$$

Assuming that V_m, V_t, λ and R are known, t_{go} can be computed from Eq.(16). Q can be obtained from Eq.(15) and then the required guidance acceleration command is computed from Eq.(9).

3 Implementation of New Guidance Law

Substituting Eq.(10) into Eq.(9), we obtain

$$F=\frac{N}{R^2}\left[\left\{\left(V_m-\frac{V_m t_{go}}{Q}V_t-\frac{t_{go}^2 V_m}{2Q}A_t\right)\times R\right\}\times V_m\right] \quad (17)$$

Let us define k_1, k_2 as follows :

$$k_1=\frac{V_m t_{go}}{Q} \quad (18)$$

$$k_2=\frac{t_{go}^2 V_m}{2Q} \quad (19)$$

Substituting Eqs. (18) and (19) into Eq. (17) and rearranging it, we have

$$F=\frac{Nk_1}{R^2}[\{(V_m-V_t)\times R\}\times V_m] + \frac{N(1-k_1)}{R^2}\{(V_m\times R)\times V_m\} + \frac{Nk_2}{R^2}\{(R\times A_t)\times V_m\} \quad (20)$$

The first term represents PN with the navigation constant Nk_1, the second one represents pure pursuit navigation (PPN) with navigation constant $N(1-k_1)$ and the last one is the correcting force for target's maneuver. We can set up the guidance system given by Eq.(20) combining PN, PPN and correcting term. The block diagram representation of Eq.(20) is shown in Figure 2, where the effective navigation constant N_e, defined by the following equation, is used instead of N.

$$N_e=\frac{NV_m\cos\phi_m}{V_c} \quad (21)$$

where V_c is the closing velocity. Also, we used Λ_t which is defined by the following equation

$$\Lambda_t=\left|\frac{R\times A_t}{R}\right| \quad (22)$$

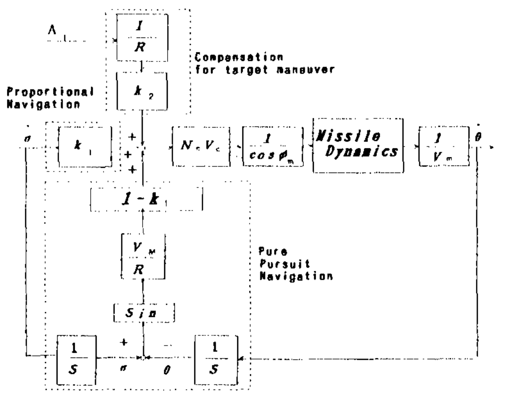

Figure 2 : Block Diagram of New Guidance Law

In order to realize the guidance law shown in Fig.2, we assume that $\dot{\sigma}$, Λ_t, θ, ϕ_m, R and V_c are measured. Besides, the value of V_m, V_t, λ and t_{go} are needed to compute k_1 and k_2. λ can be determined in advance ; V_t is assumed to be constant if it is difficult to measure or estimate it ; and V_m and t_{go} are estimated using the following equations ;

$$V_m=\frac{V_{m0}}{1+\lambda V_{m0}t} \quad (23)$$

$$t_{go}=\frac{R}{V_c} \quad (24)$$

We call the guidance law given by Eq.(20) NEW GUIDANCE LAW (NGL).

4 Simulation

4.1 Simulation model

Let us apply the new guidance law presented to the three dimensional mathmatical model of an air-to-air missile after thrust cutoff and compare the result with that achieved with conventional proportional navigation (PN). Figure 3 shows the intercept geometry in three dimensions. Here, the origin of the inertial axis XYZ is on the earth surface, Z-axis is directed vertically down and X-axis coincides with the initial target velocity vector.

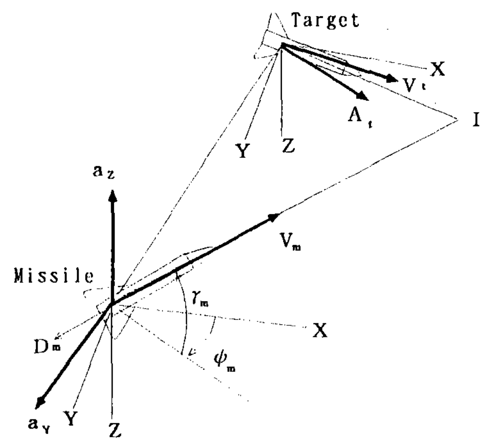

Figure 3 : Intercept Geometry in 3 Dimensions

In this figure, both the missile and the target are particles. The target is flying straight with constant speed or turning with the constant lateral acceleration of 5g. On the other hand, the total dynamics of the guidance system, including the missile dynamics, a noise filter, etc., is given by a first-order lag with time constant 0.4 sec. The other conditions are as follows : The target initial velocity is Mach 2.0 ; the effective navigation constant is set equal to 6.0 regardless of the guidance law; the missile initial speed is Mach 4.0; the missile reference cross section is 0.04 m^2 ; C_{D0} and k are the functions of Mach number, that is, 0.70 (0.9), 0.98 (1.0), 0.83 (2.0), 0.67 (3.0), 0.6 (4.0) for C_{D0} and 0.03 (0.9) , 0.02 (1.2) , 0.03 (2.0) , 0.035 (3.0) , 0.039 (4.0) for k ; the missile total mass after thrust cutoff is 170 *kg* ; and the blind distance is set equal to 80m. The employed equations of motion are as follows.

$$\dot{V}_m = \frac{1}{m}(-D_m - mg\sin\gamma_m) \tag{25}$$

$$\dot{\psi}_m = \frac{a_{yc}}{V_m\cos\gamma_m} \tag{26}$$

$$\dot{\gamma}_m = -\frac{1}{V_m}(a_{zc} + g\cos\gamma_m) \tag{27}$$

$$\dot{a}_{yc} = (a_y - a_{yc})/\tau_m \tag{28}$$

$$\dot{a}_{zc} = (a_z - a_{zc})/\tau_m \tag{29}$$

$$\dot{x}_m = V_m \cos\gamma_m \cos\psi_m \tag{30}$$

$$\dot{y}_m = V_m \cos\gamma_m \sin\psi_m \tag{31}$$

$$\dot{z}_m = -V_m \sin\gamma_m \tag{32}$$

where D_m is given by

$$D_m = \left(\frac{1}{2}\rho S C_{D0}\right)V_m^2 + \left(\frac{2m^2k}{\rho S}\right)\frac{(a_{yc}^2 + a_{zc}^2)}{V_m^2} \tag{33}$$

The first term represents the zero-lift-drag , second term represents the induced drag and k is an induced drag coefficient .

4 . 2 Simulation Results

One of the simulation results is displayed in Fig.4, where the target is flying straight and the missile initial heading is toward the intercept point. The target initial position is (0 ,0 ,-5000) and the missile one is (-1000 ,-1000, -4000) . The solid line shows the flight-path achieved with NGL and the broken line shows that with PN. For reference , the flight-paths projected onto two dimensions are shown in Figs. 5 and 6 .

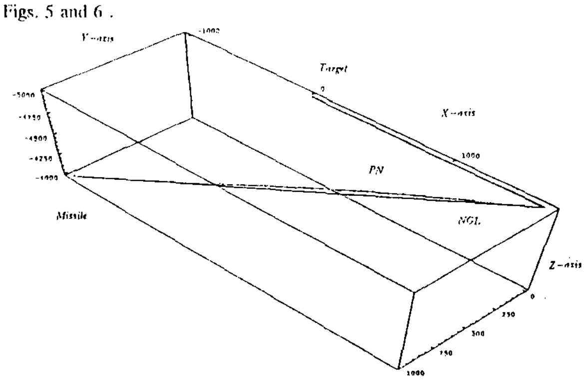

Figure 4 Flight-path in three dimensions

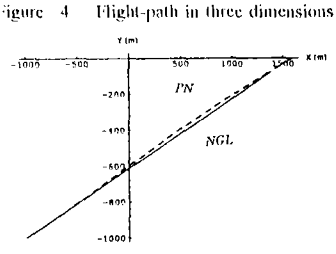

Figure 5 Flight-paths in X-Y plane

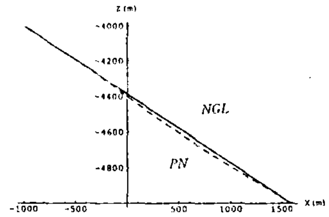

Figure 6 Flight-paths in X-Z plane

Figures 7 and 8 depict the time histories of the missile longitudinal and lateral accelerations with the same flight as shown in Fig. 4. From these figures , we see that the missile guided by NGL flies almost straight with little acceleration commands but the trajectory achieved with PN is curved significantly and requires large acceleration command.

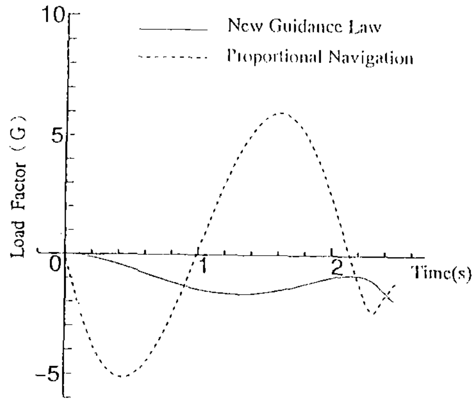

Figure 7 Time histories of the missile longitudinal acceleration

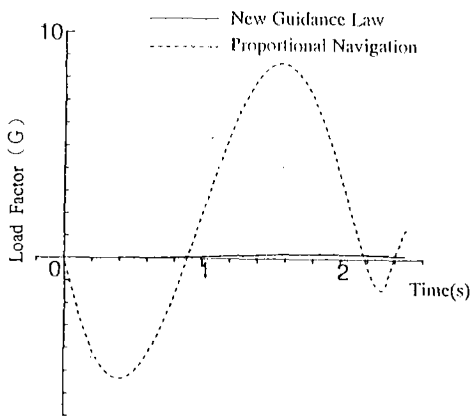

Figure 8 Time histories of the missile lateral acceleration

Figures 9 and 10 display the miss distance as the function of the missile initial velocity and heading error. The flight conditions except the missile initial velocity and heading are the same as in Fig.4. In Figs.9 and 10, MD represents miss distance, VM represents missile initial velocity and IE represents missile initial heading error. From Fig.9 , we see that the miss distance of NGL becomes less than 3m in case the initial heading error is within the range of $\pm 10^{\circ}$. On the other hand , Fig.10 shows that the miss of PN becomes less than 3m only with the very limited values of the initial velocity and heading error. From these simulation results , it can be said that NGL has far better off-boresight ability than PN.

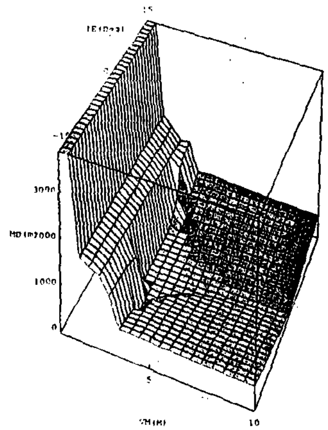

Figure 9 Miss as a function of the missile initial velocity and heading error (NGL)

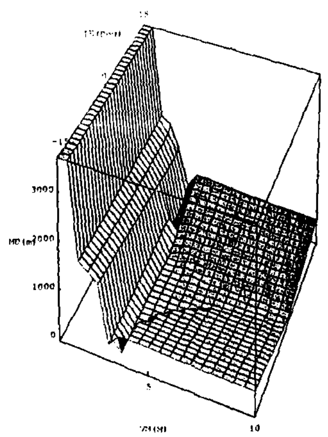

Figure 10 Miss as a function of the missile initial velocity and heading error (PN)

Figure 11 explain that what number is the best effective navigation constant (N_e) of the guidance laws. In each point, 2500 missiles are fired. The simulation conditions are as follows ; the initial altitudes of a missile and a target are equal to 5000m ; the initial target position is always the origin ; on the other hand , the initial missile position is changed at intervals of 200m within X=-4800m to 4800m and Y=-4800m to 4800m ; the effective miss for hit is assumed to be less than 3m ; and other conditions are the same as before. From Fig.11 , the reasonable value of N_e is 2-10 for the new guidance law and 5-8 for PN. Considering the noises , N_e was set equal to 6.0 for each guidance law .

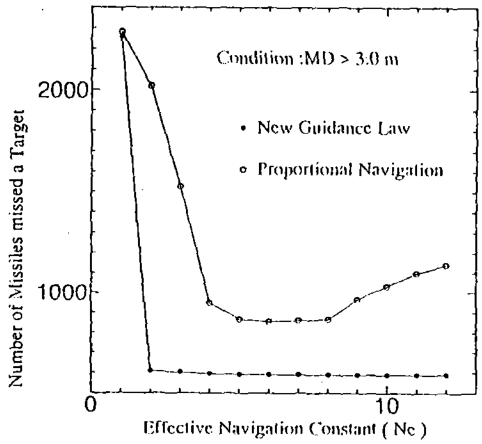

Figure 11 Number of missiles missed a target vs effective navigation constant

4 . 3 Effective area for a missile to intercept a target

Figures 12-15 contain the results from the simulation runs performed to define the effective area for a missile to intercept a target. The target is located at the center of the plot and its flight trajectory is indicated by an arrow, in each figure . The bounded regions represent the set of points from which a missile cannot intercept a target. The initial conditions for simulation are as follows ; missile and target speeds are 4.0 and 2.0 Mach, respectively ; a missile has an appropriate lead angle because it has been guided against a target since launching ; and other conditions are the same as before , except the altitude and the limited load factor .

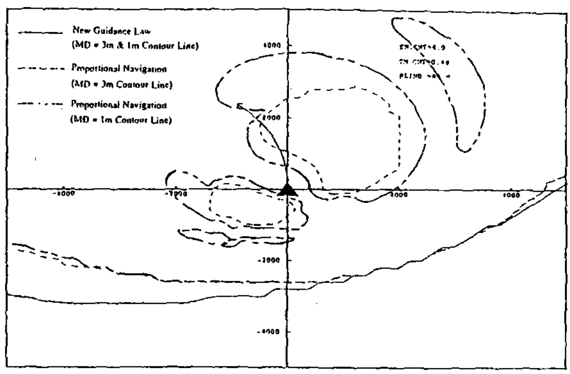

Figure 12 Effective area (altitude=5000)

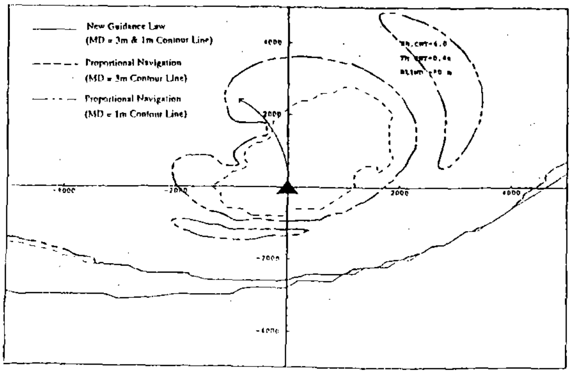

Figure 13 Effective area (altitude=4000)

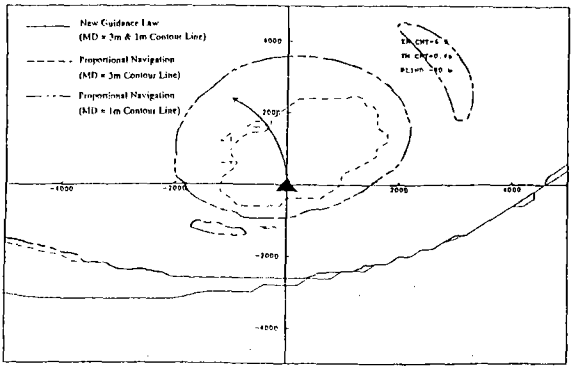

Figure 14 Effective area (altitude=6000)

In Figs.12 and 15 , a missile and a target are at the same altitude of 5000m ; in Fig.13 , missile and target altitudes are 4000m and 5000m, respectively ; and in Fig.14 , missile and target altitudes are 6000m and 5000m , respectively . On the other hand , the limited load factor of a missile is 40g in Fig.12,13,14 and 20g in Fig.15 .

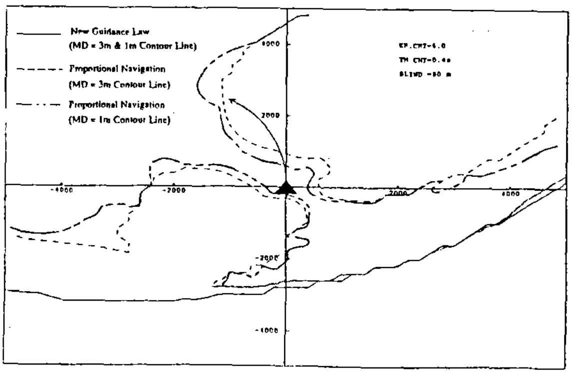

Figure 15 Effective area (altitude=5000m)

As shown in each figure , there are the regions near the target and in the far rear of it from which NGL missile can intercept a target but PN missile cannot . The regions near the target are generated because PN missile cannot track the target ; and the regions in the far rear of the target are generated because the induced drag of PN missile becomes greater than that of NGL missile and then PN missile cannot overtake the target .

On the other hand , NGL missile can intercept a target as long as the missile hold a position at the point from which it can overtake a target .

Figs.12–15 show that the effective area of a NGL missile is larger than that of PN.

5 Conclusion

A new guidance law for a missile after thrust cutoff has been presented . First, the guidance law is derived theoretically and then the real implementation of the guidance law is discussed. The new guidance law presented can be implemented by combining proportional navigation , pure pursuit navigation and compensation for the target maneuver . From the simulation studies, the following results are obtained. The missile trajectory achieved with the proportional navigation is quite curved and large acceleration command is required to intercept a target but the missile guided by the new guidance law flies nearly straight and intercepts a target with a little acceleration command. Also , the effective regions from which a missile can intercept a target are generated from many simulation runs . These regions show that the new guidance law presented provide an overall performance improvement over proportional navigation . If this guidance law was combined with the guidance law for a missile with thrust in the previous paper, it would show the outstanding performances.

Reference

[1] Zarchan, P., "*Tactical and Strategic Missile Guidance*", AIAA, Washington, D.C.,1990

[2] Baba,Y.,Takehira,T.and Takano,H., "*Guidance Law for a Missile with Thrust*,"Proceedings of Pacific International Conference on Aerospace Science and Technology (PICAST1),Vol. II 1993, pp.776-781

[3] Chadwick, W.R., "*Miss Distance of Proportional Navigation Missile with Varying Velocity*", Journal of Guidance, Control, and Dynamics ,Vol.8, No.5, 1985 , pp.662-666

[4] Baba,Y.,Takehira,T.and Takano,H.,"*Guidance Law for a Free-Flying Projectile*"Proceedings of First Asia Control Conference (ASCC) ,Vol. II 1994, pp437-440