A New Learning Control of Robot Manipulators

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Abstract

This paper illustrates a new learning control for robot manipulators using Lyapunov direct method. It has been shown that under the proposed learning control robot manipulators are always guaranteed to be asymptotically stable with respect to the number of trials. The proposed control is also robust in the sense that the exact knowledge of the nonlinear dynamics is not required except for bounding functions on the magnitude.

1 Introduction

There are a number of issues to address in designing advanced control system for robotic systems. The problem of obvious interest is the design of controllers to guarantee the system performance for the desired trajectory with acceptable accuracy. A new approach to solve this problem is *iterative learning control*. Iterative learning control is a technique for improving the performance of systems or processes that operate repetitively over a fixed time interval without exact knowledge of system dynamics. The applications of robotic systems contain a sequence of jobs and each task in the sequence has a finite time span. In this application standard robust (or adaptive) control does not have the ability to achieve good performance during a finite period of time. Instead, a learning control is therefore of great importance for such applications. In practice, robotic systems are nonlinear and their dynamics are not completely known *a priori*. In addition, if the unknown parameters of the system model is time-varying during the operation, the control is getting difficult. In contrast to the adaptive control schemes which fixed parameterization is required, the learning control schemes have no these requirements since they are searching the better control input for the desired transient response of the system for the same task on successive trials. At each trial the control input is updated by learning algorithms based on the previous performances of identical tasks. The intuition behind this approach is that we can take some advantages from the periodicity of the repeated task. From trial to trial, the periodic time function remains as a constant at any fixed instant of local time. So, if a learning control is designed properly in which it can learn the constant as the simplest form of unknowns, the control can compensate both uncertainties and nonlinearities of the system without the information of system dynamics by updating the control input over trials. To find such a learning control for a class of nonlinear systems, a Lyapunov argument based on periodicity may be derived.

There have been several investigated topics regarding the learning algorithm, the proof of system stability, and the controller implementation. A recent discussion on history and various approaches of learning control can be found in [10]. Since Arimoto's framework, the learning control has been based on two major practical and theoretical limitations; the requirements of acceleration feedback and Lipschitzian condition [3, 5, 6, 7, 8, 9, 11].

It shall be shown that a new class of learning control based on Lyapunov direct method always guarantees global asymptotic stability for robotic systems without those limitations. Although it only be presented a learning control of rigid-link robot manipulators, the principle can be applied to higher-order physical systems such as robotic systems in the presence of actuator dynamics and joint flexibilities because of its systematic technique using state transformation.

This paper is organized as follows. Section 2 formulates the tracking problem for rigid-link robot manipulators. Section 3 develops a learning control for the system. Section 4 presents the stability and the convergence analysis of the system under the proposed learning control. Section 5 illustrates the simulation results to show the effectiveness of the proposed learning control.

2 Problem Formulation

The dynamics of the rigid-link robot manipulator is described by the following nonlinear differential equation :

$$M(q)\ddot{q} + N(q,\dot{q}) = \tau \qquad (1)$$

where

$$N(q,\dot{q}) = V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} + F_s(\dot{q}) + T_L$$

where $q \in \Re^n$ is a vector of joint angle variables, $M(q)$ is an $[n \times n]$ inertia matrix, symmetric and positive definite; $V_m(q,\dot{q})$, $G(q)$, and $F_s(\dot{q})$ are $[n \times 1]$ vectors containing the centripetal and Coriolis terms, gravity term, and static friction term, respectively. F_d is an $[n \times n]$ diagonal matrix of dynamic friction coefficients and $\tau \in \Re^n$ is a vector of input torque.

Let q^d, $\dot{q}^d$, and $\ddot{q}^d$ denote the desired trajectory that the robot should track and we assume without loss of generality that they are uniformly bounded functions of time for all trials. Since the learning control objective is to obtain asymptotic link tracking, we define the tracking errors to be

$$e = q^d - q, \qquad \dot{e} = \dot{q}^d - \dot{q} \tag{2}$$

To design a control for asymptotic link tracking, write equation (1) in terms of the tracking error given by (2)

$$M(q)\ddot{q}^d - M(q)\ddot{e} + N(q, \dot{q}) = \tau \tag{3}$$

The important properties of the robot dynamics that will be used in this paper are as follows.

P1. Inertia matrix $M(q)$ is symmetric and positive definite for all $q \in \Re^n$.

P2. Coriolis/centripetal matrix $V_m(q, \dot{q})$ is linear in $\dot{q}$ and that its dependence on q is given as

$$V_m(q, \dot{q})w = V_m(q, w)\dot{q} \quad \forall\, q, w \in \Re^n$$

In addition, we have some assumptions from mechanical properties of robotic systems for closed-loop stability analysis as follows.

A1. Inertia matrix $M(q)$ satisfies

$$\underline{m}I \ \leq M(q) \ \leq \overline{m}I, \quad \forall\, q \in \Re^n$$

where $\underline{m}$ and $\overline{m}$ are positive, known constants, and I is the $[n \times n]$ identity matrix.

A2. From the property of coriolis/centripetal matrix $V_m(q, \dot{q})$, it follows that

$$\|V_m(q, \dot{q})\| \leq \rho_{Vm}(q)\|\dot{q}\|, \quad \forall\, q, \dot{q} \in \Re^n$$

where $\rho_{Vm}(q)$ is a known, positive definite function of q.

A3. For all possible disturbance T_L, it is assumed that

$$T_L = \beta_1 w + \beta_2 \quad \forall\, w \in \Re^n \qquad \|\beta 1\| \leq \rho_d,$$

where β_1 and β_2 are some unknown periodic functions, and ρ_d is a known constant. The Euclidean norm is denoted by $\|\cdot\|$.

3 Learning Control

Consider a first-order, i-th cascaded sub-system in an m-th order error system:

$$\begin{aligned} \dot{x}_{i,j} &= K_{i,j}^{-1}(x_{i-1,j}) f_{i,j}(x_{1,j}, \cdots, x_{i,j}, t)\zeta_i(t) \\ &\quad + g_{i,j}(x_{1,j}, \cdots, x_{i,j}, t) + K_{i,j}^{-1}(x_{i-1,j})u_{i,j}(t) \\ &\quad + K_{i,j}^{-1}(x_{i-1,j})[e_{i+1,j} - u_{i,j}(t)], \end{aligned} \tag{4}$$

where subscripts i and j are the indices of subsystems and the learning trials, respectively. Vector $x_i \in \Re^n$ is the transformed sub-state from the original sub-state $e_{i,j}$ of the i-th subsystem with $x_{1,j} = e_{1,j}$, added and subtracted $u_{i,j}(t) \in \Re^n$ is the fictitious control, $\zeta_i(t) \in \Re^l$ is a vector of unknown time functions that are invariant from trial to trial in order to be learned by learning control, and $f_i(x_{1,j}, \cdots, x_{i,j}, t) \in \Re^{n \times l}$ is known matrix function. Matrix $K_{i,j}(x_{i-1,j}) \in \Re^{n \times n}$ is not necessary known but symmetric and positive definite, and it is bounded as that

$$\begin{aligned} \underline{k}_i I_n \leq \ & K_{i,j} \ \leq \overline{k}_i I_n, \\ \|\frac{d}{dt}K_{i,j}(x_{i-1,j})\| \ & \leq \ \rho_{Ki}(x_{1,j}, \cdots, x_{i,j}, t), \end{aligned} \tag{5}$$

where $\underline{k}$ and $\overline{k}$ are known constants, and ρ_{ki} are known function. It is assumed that we can determine scaler function $\rho_{Ki}(\cdot)$ using dynamics of $(i-1)$-th subsystem.

The vector $g_{i,j}(x_{1,j}, \cdots, x_{i,j}, t) \in \Re^n$ is an unknown function which may include unknown time functions but bounded by known, well defined functions $\rho_{ik}(\cdot)$ as

$$\|g_{i,j}(x_{1,j}, \cdots, x_{i,j}, t)\| \leq \sum_{k=1}^{i} \rho_{ik}(x_{1,j}, \cdots, x_{i,j}, t) \cdot \|x_{k,j}\|, \tag{6}$$

which is locally uniformly bounded with respect to the state variables $x_{1,j}$ to $x_{i,j}$ and uniformly bounded with respect to t. It can be assumed without loss of any generality that both bonding functions $\rho_{Ki}(\cdot)$ and $\rho_{ik}(\cdot)$ are differentiable.

The proposed control is in the form of

$$\begin{aligned} u_{i,j} &= -\Big[\frac{\alpha}{2} f_{i,j} \cdot f_{i,j}^T x_{i,j} + (m - i + 1)x_{i,j} + \overline{k}_i\, x_{i,j}\, \rho_{ii} \\ &\quad + \frac{1}{2} x_{i,j} \cdot \rho_{Ki} + \frac{1}{4}\overline{k}^2 x_{i,j} \sum_{k=1}^{i-1} g_{ik}^2 + x_{i-1,j}\Big] - f_{i,j} \cdot \Delta_{i,j} \\ &\triangleq H_{i,j}(x_{1,j}, \cdots, x_{i,j}, t) + L_{i,j}(x_{1,j}, \cdots, x_{i,j}, t), \end{aligned} \tag{7}$$

where $\alpha > 0$ is a design parameter, m is the number of cascaded sub-systems in the overall system, $H_{i,j}(\cdot)$ is the feedforward, robust, control part, and $L_{i,j}(\cdot)$ is the learning part. The iterative learning contribution $\Delta_{i,j}$ that learns unknown time function is updated from trial to trial by the learning law:

$$\Delta_{i,j+1} = \Delta_{i,j} + \alpha f_{i,j}^T(x_{1,j}, \cdots, x_{i,j}, t)\, x_{i,j} \tag{8}$$

where $\Delta_{i,0} = 0$.

Rewrite equation (3) as the state-space form

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= M^{-1}(q)\,[M(q)\ddot{q}^d + N(q, \dot{q}) - \tau], \end{aligned} \tag{9}$$

where $\underline{e} = [e_1\, e_2]^T = [e\, \dot{e}]^T$.

In order to design the learning control, we rewrite the first subsystem in equation (9) as follows

$$\begin{aligned} \dot{x}_{1,j} &\triangleq \dot{e}_{1,j} \\ &= u_{1,j} + (\dot{e}_{1,j} - u_{1,j}) \\ &= u_{1,j} + x_{2,j} \end{aligned} \tag{10}$$

where $x_{2,j} = \dot{e}_{1,j} - u_{1,j}$.

To formulate the fictitious learning control, compare the first subsystem (10) to system (4), we have

$$\zeta_1 = 0, \quad f_1 = 0, \quad g_1 = 0, \quad K_1 = I_n,$$

where 0 denotes zero vector or matrix with proper dimension. Since there is no unknown time function in the first subsystem, learning part is not needed. It follows from (7) that the fictitious control is

$$u_j = -2x_{1,j} \tag{11}$$

The fictitious control $u_{1,j}$ provides stability property for system (10) and state transformation is defined to be $x_{2,j} = \dot{e}_{1,j} - u_{1,j}$. Thus, the objective of controlling of the second sub-system is to make $x_{2,j} \to 0$ as $j \to \infty$. To design the actual control law τ_j, we first establish dynamic equation of the second subsystem in equation (9) by taking derivative as follows :

$$\begin{aligned}\dot{x}_{2,j} &\triangleq \dot{e}_{2,j} - \dot{u}_{1,j} \\ &= \ddot{q}^d + M^{-1}(q_j)N(q_j,\dot{q}_j) - 4x_{1,j} + 2x_{2,j} \\ &\quad -M^{-1}(q_j)\tau_j \\ &\triangleq K_{2,j}^{-1} f_{2,j}\zeta_2(t) + g_{2,j} + K_{2,j}^{-1}(-\tau_j)\end{aligned} \tag{12}$$

where $K_{2,j} = M(q_j)$ and $\dot{u}_{1,j} = -2\dot{x}_{1,j} = 4x_{1,j} - 2x_{2,j}$.

Note that there are many possible choices of $f_{2,j}, \zeta_2(t)$, and $g_{2,j}$, and these choices yield different learning controls. Two typical choices are discussed in the following subsections.

3.1 Learning Control with the Simplest Learning Part

To get the simplest learning part, we include all functions which can be bounded by state variables into $g_{2,j}$ so that we can have the simplest choice of $f_{2,j}$ and $\zeta_2(t)$.

It follows from (12) that

$$\begin{aligned}&\ddot{q}^d + M^{-1}(q_j)N(q_j,\dot{q}_j) - 4x_{1,j} + 2x_{2,j} \\ &\quad \triangleq K_{2,j}^{-1} f_{2,j}\zeta_2(t) + g_{2,j},\end{aligned} \tag{13}$$

where $f_{2,j} = I_n$,

$$\begin{aligned}\zeta_2(t) &= M(q^d)\ddot{q}^d + N(q^d,\dot{q}^d) \\ &= M(q^d)\ddot{q}^d + V_m(q^d,\dot{q}^d)\dot{q}^d + G(q^d) \\ &\quad +F_d\dot{q}^d + F_s(\dot{q}^d) + T_L^d.\end{aligned}$$

and using properties and assumptions of robotic systems.

$$\begin{aligned}&M(q_j)g_{2,j} \\ =\;& [M(q_j)\ddot{q}^d - M(q^d)\ddot{q}^d + N(q_j,\dot{q}_j) - N(q^d,\dot{q}^d)] \\ & -M(q_j)(4x_{1,j} + 2x_{2,j}) \\ =\;& [M(q_j) - M(q^d)]\ddot{q}^d + [V_m(q_j,\dot{q}^d) - V_m(q^d,\dot{q}^d)]\dot{q}^d \\ & +2[\,V_m(q_j,\dot{q}^d) + V_m(q_j,\dot{q}_j) + F_d - 2M(q_j)\,]\,x_{1,j} \\ & -[\,V_m(q_j,\dot{q}^d) + V_m(q_j,\dot{q}_j) + F_d - 2M(q_j)\,]\,x_{2,j} \\ & +[G(q_j) - G(q^d)] + \beta_1(q_j,\dot{q}_j) + [F_s(\dot{q}_j) - F_s(\dot{q}^d)].\end{aligned}$$

It can be assumed without loss of any generality that disturbances can be separated such that $T_L^d = \beta_2$ includes all periodic time functions which can not be bounded by state variable and $(T_L - T_L^d)$ includes remaining disturbances.

To find the corresponding learning control, a bounding function on $g(\cdot)$ satisfying condition (6) is needed. It follows that

$$\begin{aligned}&\overline{m}\|g_{2,j}\| \\ &\le \|M(q_j) - M(q^d)\|\cdot\|\ddot{q}^d\| + 2[\,\|V_m(q_j,\dot{q}^d)\| + \|V_m(q_j,\dot{q}_j)\| \\ &\quad +\|F_d\| + 1.5\rho_d + 2\overline{m}\,]\cdot\|x_{1,j}\| \\ &\quad +[\,\|V_m(q_j,\dot{q}^d)\| + \|V_m(q_j,\dot{q}_j)\|\|\|F_d\| + \rho_d + 2\overline{m}\,]\cdot\|x_{2,j}\| \\ &\quad +\|V_m(q_j,\dot{q}^d) - V_m(q^d,\dot{q}^d)\|\cdot\|\dot{q}^d\| \\ &\quad +\|G(q_j) - G(q^d)\| + \|F_s(\dot{q}_j) - F_s(\dot{q}^d)\|.\end{aligned}$$

To design the feedforward control part, we define bounding functions as follows

$$\begin{aligned}&\|M(q_j) - M(q^d)\| \le \rho_M(q_j,q^d)\|x_{1,j}\| \\ &\|V_m(q_j,\dot{q}^d)\| \le \rho_{Vm1}(q_j)\|\dot{q}^d\| \\ &\|F_d\| \le \rho_{Fd} \\ &\|G(q_j) - G(q^d)\| \le \rho_G(q_j,q^d)\|x_{1,j}\| \\ &\|F_s(\dot{q}_j) - F_s(\dot{q}^d)\| \le \rho_{Fs}(q_j,\dot{q}^d)(2\|x_{1,j}\| + \|x_{2,j}\|),\end{aligned}$$

where $\rho_M(\cdot), \rho_{Vm1}(\cdot)$, and $\rho_{Fs}(\cdot)$ are known, positive definite functions. ρ_{Fd} is a known constant.

It follows that

$$\|\,g_{2,j}\,\| \le \rho_{21}(q_j,\dot{q}_j,q^d,\dot{q}^d)\,\|x_{1,j}\| + \rho_{22}(q_j,\dot{q}_j,q^d,\dot{q}^d)\,\|x_{2,j}\|,$$

where

$$\begin{aligned}\rho_{21}(\cdot) &= \underline{m}^{-1}[\rho_M(q_j,q^d)\|\ddot{q}^d\| + 2\rho_{Vm1}(q_j)\|\dot{q}^d\| + 2\rho_{Vm}(q_j)\|\dot{q}_j\| \\ &\quad +2\rho_{Fd} + 3\rho_d + \rho_G(q_j,q^d) + 2\rho_{Fs}(q_j,\dot{q}^d)] + 4\end{aligned}$$

and

$$\begin{aligned}\rho_{22}(\cdot) &= \underline{m}^{-1}\,[\,\rho_{Vm1}(q_j)\|\dot{q}^d\| + \rho_{Vm}(q_j)\|\dot{q}_j\| + \rho_{Fd} + \rho_d \\ &\quad +\rho_{Fs}(q_j,\dot{q}^d)\,] + 2\end{aligned}$$

where both $\rho_{21}(\cdot)$ and $\rho_{22}(\cdot)$ are some known positive definite functions.

It follows from (7) that actual control input τ_j is in form of

$$\begin{aligned}\tau_j &= \frac{\alpha}{2}x_{2,j} + x_{2,j} + \overline{m}_i\,x_{2,j}\,\rho_{22} + \rho_{Vm}(q_j)\|\dot{q}_j\|\,x_{2,j} \\ &\quad +\frac{1}{4}\overline{m}^2 g_{21}^2\,x_{2,j} + x_{1,j} + \Delta_{2,j} \\ \Delta_{2,j+1} &= \Delta_{2,j} + \alpha\,x_{2,j}\end{aligned} \tag{14}$$

where $\alpha > 0$ is an arbitrary design constant and $\Delta_{2,0} = 0$.

3.2 Learning Control with the Simplest Feedforward Part

Since non-parameterizable dynamics along the desired trajectory are can be compensated by learning control, it follows without loss generality that

$$\begin{aligned}&M(q_j)\ddot{q}^d + V_m(q_j,\dot{q}_j)\dot{q}_j + G(q_j) + F_d\dot{q}_j + F_s(\dot{q}^d) + \beta_2 \\ &\quad\triangleq f_{2,j}\zeta_2(t),\end{aligned} \tag{15}$$

Note that $f_2(\cdot)$ only includes q_j and $\dot{q}_j$ so that it can be some known matrix.

To design the feedforward control part, we derive the bounding functions for $g_{2,j}$.

$$g_{2,j} = M^{-1}(q_j)[\beta_1(q_j,\dot{q}_j) + F_s(\dot{q}_j) - F_s(\dot{q}^d)\,] - 4x_{1,j} + 2x_{2,j}.$$

It follows that

$$\begin{aligned}\|g_{2,j}\| &\le \|M^{-1}(q_j)\|\cdot[\,\|\beta_1(q_j,\dot{q}_j)\| + \|F_s(\dot{q}_j) - F_s(\dot{q}^d)\|\,] \\ &\quad +4\|x_{1,j}\| + 2\|x_{2,j}\| \\ &\le g_{21}(\cdot)\,\|x_{1,j}\| + g_{22}(\cdot)\,\|x_{2,j}\|,\end{aligned}$$

where

$$g_{21}(\cdot) = \underline{m}^{-1}[3\rho_d + 2\rho_{F_s}(q_j, \dot{q}^d)] + 4.$$

and

$$g_{22}(\cdot) = \underline{m}^{-1}[\rho_d + \rho_{F_s}(q_j, \dot{q}^d)] + 2.$$

where both $\rho_{21}(\cdot)$ and $\rho_{22}(\cdot)$ are some known positive definite functions.

It follows from (7) that actual control input τ_j is in form of

$$\begin{aligned} \tau_j &= \frac{\alpha}{2} f_2 \cdot f_2^T x_{2,j} + x_{2,j} + \overline{m}_i \rho_{22} x_{2,j} + \rho_{Vm}(q_j)\|\dot{q}_j\| x_{2,j} \\ &\quad + \frac{1}{4}\overline{m}^2 x_{2,j}\rho_{21}^2 + x_{1,j} + f_2 \cdot \Delta_{2,j} \\ \Delta_{2,j+1} &= \Delta_{2,j} + \alpha f_2^T x_{2,j} \end{aligned}$$

where $\Delta_{2,0} = 0$.

Note that the dynamics equations given by (10) and (12) can be viewed as two interconnected systems representing the overall closed-loop dynamics. In next section, we show the stability and the convergence of the system under the proposed learning control scheme.

4 Stability and Convergence Analysis

Consider the Lyapunov function candidate defined by

$$V_{i,j} = \int_0^{\delta T} \|\zeta_i(\tau) - \Delta_{i,j}\|^2 d\tau. \quad (16)$$

It is apparent that $[\zeta_i(t) - \Delta_{i,j}]$ is the learning error. Under learning control law (8), the difference of the learning errors between the $(j+1)$-th and j-th trial is given by

$$\begin{aligned} \delta[\zeta_i(t) - \Delta_{i,j}] &\triangleq [\zeta_i(t) - \Delta_{i,j+1}] - [\zeta_i(t) - \Delta_{i,j}] \\ &= -\alpha f_{i,j}^T x_{i,j}. \end{aligned} \quad (17)$$

It can be verified easily that the difference of Lyapunov function between the successive trial, $\delta V_{i,j} = V_{i,j+1} - V_{i,j}$, can be rewritten as

$$\begin{aligned} \delta V_{i,j} &= \int_0^{\delta T} \{ \delta[\zeta_i(\tau) - \Delta_{i,j}]^T \delta[\zeta_i(\tau) - \Delta_{i,j}] \\ &\quad + 2\delta[\zeta_i(\tau) - \Delta_{i,j}]^T [\zeta_i(\tau) - \Delta_{i,j}] \} d\tau. \end{aligned} \quad (18)$$

The effectiveness of the proposed learning control is shown by the following lemma and its proof.

Lemma 1 : *For system (9) under the control (14) or (16), the incremental change of Lyapunov function with respect to trial, δV_j, satisfies the inequality that*

$$\begin{aligned} \delta V_j &\equiv \delta V_{1,j} + \delta V_{2,j} \\ &\le -2\alpha \left[\frac{1}{2} x_{1,j}^T x_{1,j}\Big|_0^{\delta T} + \frac{1}{2} x_{2,j}^T K_{2,j} x_{2,j}\Big|_0^{\delta T} \right. \\ &\quad \left. + \int_0^{\delta T} \|x_{1,j}\|^2 d\tau + \int_0^{\delta T} \|x_{2,j}\|^2 d\tau \right] \end{aligned}$$

Proof: Under control (7), system (10) becomes

$$\dot{x}_{1,j} = -2x_{1,j} + x_{2,j},$$

and (12) becomes

$$\begin{aligned} \dot{x}_{2,j} &= K_{2,j}^{-1} f_{2,j}\zeta_2(t) + g_{2,j} - \frac{\alpha}{2} K_{2,j}^{-1} \cdot f_{2,j} \cdot f_{2,j}^T x_{2,j} \\ &\quad - K_{2,j}^{-1} x_{2,j} - \overline{k}_2 K_{2,j}^{-1} x_{2,j}\rho_{22} - \frac{1}{2} K_{2,j}^{-1} x_{2,j} \cdot \rho_{K2} \\ &\quad - \frac{1}{4}\overline{k}_2^2 K_{2,j}^{-1} x_{2,j}\rho_{21}^2 - K_{2,j}^{-1} x_{1,j} - K_{2,j}^{-1} \cdot f_{2,j} \cdot \Delta_{2,j} \\ &= g_{2,j} - K_{2,j}^{-1} \left[\frac{\alpha}{2} f_{2,j} \cdot f_{2,j}^T + 1 \right] x_{2,j} - \overline{k}_2 K_{2,j}^{-1} x_{2,j}\rho_{22} \\ &\quad - \frac{1}{2} K_{2,j}^{-1} x_{2,j} \cdot \rho_{K2} - \frac{1}{4}\overline{k}_2^2 K_{2,j}^{-1} x_{2,j}\rho_{21}^2 - K_{2,j}^{-1} x_{1,j} \\ &\quad + K_{2,j}^{-1} \cdot f_{2,j}[\zeta_2(t) - \Delta_{2,j}]. \end{aligned}$$

It follows that

$$f_{1,j} \cdot [\zeta_1(t) - \Delta_{1,j}] = \dot{x}_{1,j} + 2x_{1,j} - x_{2,j} \quad (19)$$

and

$$\begin{aligned} & f_{2,j} \cdot [\zeta_2(t) - \Delta_{2,j}] \\ &= K_{2,j}\dot{x}_{2,j} + \left[\frac{\alpha}{2} f_{2,j} \cdot f_{2,j}^T + 1 \right] x_{2,j} + \overline{k}_2 x_{2,j}\rho_{22} \\ &\quad + \frac{1}{2} x_{2,j} \cdot \rho_{K2} + \frac{1}{4}\overline{k}_2^2 x_{2,j}\rho_{21}^2 + x_{1,j} - K_{2,j} \cdot g_{2,j} \end{aligned} \quad (20)$$

where $K_{2,j} = M(q_j)$. Note that two typical choices are presented in the previous section for functions $f_{2,j}, \zeta_2(t), \rho_{22}, \rho_{21}$, and $g_{2,j}$ in equation (20).

Substituting equations (17), (19), and (20) into equation (18)

$$\begin{aligned} \delta V_j &= -\int_0^{\delta T} 2\alpha x_{1,j}^T [\dot{x}_{1,j} + 2x_{1,j} - x_{2,j}] d\tau \\ &\quad + \int_0^{\delta T} \Big\{ \alpha^2 x_{2,j}^T f_{2,j} \cdot f_{2,j}^T x_{2,j} - 2\alpha x_{2,j}^T \Big[K_{2,j}\dot{x}_{2,j} \\ &\quad + \frac{\alpha}{2} f_{2,j} \cdot f_{2,j}^T x_{2,j} + x_{2,j} + \overline{k}_2 x_{2,j}\rho_{22} + \frac{1}{2} x_{2,j}\rho_{K2} \\ &\quad + \frac{1}{4}\overline{k}_2^2 x_{2,j}\rho_{21}^2 + x_{1,j} - K_{2,j} \cdot g_{2,j} \Big] \Big\} d\tau \\ &= -2\alpha \int_0^{\delta T} 2x_{1,j}^T \dot{x}_{1,j} d\tau - 4\alpha \int_0^{\delta T} \|x_{1,j}\|^2 d\tau \\ &\quad + 2\alpha \int_0^{\delta T} x_{1,j}^T x_{2,j} d\tau - 2\alpha \int_0^{\delta T} x_{2,j}^T K_{2,j}\dot{x}_{2,j} d\tau \\ &\quad - 2\alpha \int_0^{\delta T} x_{2,j}^T x_{1,j} d\tau - 2\alpha \int_0^{\delta T} \|x_{2,j}\|^2 d\tau \\ &\quad - 2\alpha \int_0^{\delta T} x_{2,j}^T \Big[\overline{k}_2 x_{2,j}\rho_{22} + \frac{1}{2} x_{2,j}\rho_{K2} \\ &\quad + \frac{1}{4}\overline{k}_2^2 x_{2,j}\rho_{21}^2 - K_{2,j} \cdot g_{2,j} \Big] d\tau \\ &\le -\alpha x_{1,j}^T x_{1,j}\Big|_0^{\delta T} - \alpha x_{2,j}^T K_{2,j} x_{2,j}\Big|_0^{\delta T} \\ &\quad + \alpha \int_0^{\delta T} \|x_{2,j}\|^2 \|\dot{K}_2\| d\tau \\ &\quad - 4\alpha \int_0^{\delta T} \|x_{1,j}\|^2 d\tau - 2\alpha \int_0^{\delta T} \|x_{2,j}\|^2 d\tau \\ &\quad - 2\alpha \int_0^{\delta T} \Big[\overline{k}_2 \|x_{2,j}\|^2 \rho_{22} + \frac{1}{2}\|x_{2,j}\|^2 \rho_{K2} \\ &\quad + \frac{1}{4}\overline{k}_2^2 \|x_{2,j}\|^2 \rho_{21}^2 - \|x_{i,j}\| \cdot \|K_{2,j}\| \cdot \|g_{2,j}\| \Big] d\tau. \end{aligned}$$

Applying conditions (5) and (6), we have

$$\begin{aligned} & \delta V_j \\ & \le -\alpha x_{1,j}^T x_{1,j}\Big|_0^{\delta T} - \alpha x_{2,j}^T K_{2,j} x_{2,j}\Big|_0^{\delta T} \\ &\quad - 4\alpha \int_0^{\delta T} \|x_{1,j}\|^2 d\tau - 2\alpha \int_0^{\delta T} \|x_{2,j}\|^2 d\tau \\ &\quad - \frac{\alpha}{2} \int_0^{\delta T} \overline{k}_2^2 \|x_{2,j}\|^2 {\rho_{21}}^2 d\tau + 2\alpha \int_0^{\delta T} \overline{k}_2 \rho_{21} \|x_{1,j}\| \|x_{2,j}\| d\tau \\ & \le -2\alpha \left[\frac{1}{2} x_{1,j}^T x_{1,j}\Big|_0^{\delta T} + \frac{1}{2} x_{2,j}^T K_{2,j} x_{2,j}\Big|_0^{\delta T} \right. \end{aligned}$$

$$+\int_0^{\delta T} \|x_{1,j}\|^2 d\tau + \int_0^{\delta T} \|x_{2,j}\|^2 d\tau \Bigg],$$

in which inequality $a^2 + b^2 \geq 2ab$ is used. □

The lemma shows the property of Lyapunov function (16) for system (3). From the lemma, it can be easily concluded the stability and convergence of the system under the proposed learning control, as summarized by the following corollary.

Corollary 1. *Under the control (14) or (16), system (3) is globally asymptotically stable.*

Proof: From the result of lemma 1

$$\delta V_j = -\alpha \sum_{k=1}^{2} x_{k,j}^T K_{k,j} x_{k,j} \Bigg|_0^{\delta T} - 2\alpha \sum_{k=1}^{2} \int_0^{\delta T} \|x_{k,j}\|^2 d\tau,$$

where $K_{1,j} = I_n$. For learning control implementation, initial conditions at each trial are either manually set to zero or kept to the final conditions of the previous trail, no resetting. In the case of resetting of initial conditions, the term

$$-\alpha \sum_{k=1}^{2} x_{k,j}^T K_{k,j} x_{k,j} \Bigg|_0^{\delta T}$$

is non-positive. In other case, no resetting of initial condition, the sum of the above term from the first trial to the p-th trial is

$$-\alpha \sum_{j=1}^{p}\sum_{k=1}^{2} x_{k,j}^T K_{k,j} x_{k,j} \Bigg|_0^{\delta T} = -\alpha \sum_{k=1}^{2} x_{k,p}^T(\delta T)\, K_{k,p} x_{k,p}(\delta T)\, d\tau + \alpha \sum_{k=1}^{2} x_{k,0}^T(0)\, K_{k,0} x_{k,0}(0)\, d\tau,$$

in which there is only one possible positive term and it remains constant as p increases. Thus, we have that, in both cases of initial conditions,

$$\sum_{j=0}^{\infty} \delta V_j \leq C_{ini} - 2\alpha \sum_{j=1}^{\infty}\sum_{k=1}^{2} \int_0^{\delta T} \|x_{k,j}\|^2 d\tau$$

for some constant $C_{ini} \geq 0$, or equivalently,

$$2\alpha \sum_{j=1}^{\infty}\sum_{k=1}^{2} \int_0^{\delta T} \|x_{k,j}\|^2 d\tau \leq C_{ini} + \delta V_0$$

from which global asymptotic stability can be concluded. □

5 Simulations

We present simulation results of the theoretical development in previous sections using SIMNON©. A simple two-link manipulator is used to show the performance of the proposed learning control. The specification of the modeled manipulator is following:

$$m_1 = m_2 = 1.0\,\text{Kg}, \qquad l_1 = l_2 = 1.0\,\text{m}$$

The chosen initial conditions are : $q_1(0) = q_2(0) = \dot{q}_1(0) = \dot{q}_2(0) = 0$. And the given desired trajectory is $q_1^d(t) = q_2^d(t) = 1.0 - cos(t)$. The time span of each trial is chosen to be $\delta T = 2.5$ s. And the disturbance is given as

$$T_L = \begin{bmatrix} 5\cos(5t) \\ 5\cos(5t) \end{bmatrix},$$

The performance indices to evaluate the performance are given as

$$\|\mathcal{V}_j\|_\infty \triangleq \max_{0\leq t\leq \delta T} \mathcal{V}(t)_j, \qquad \|\mathcal{V}_j\|_2 \triangleq \int_0^{\delta T} \mathcal{V}(\tau)_j\, d\tau.$$

For a two-link robotic manipulator, we have

$$\mathcal{V}_j = \frac{1}{2}k_1(e_{1,j}^2 + e_{2,j}^2) + \frac{1}{2}[(e_{1,j} + k_2 e_{1,j})^2 + (e_{1,j} + k_2 e_{1,j})^2].$$

where $k_1 = 1$ and $k_2 = 2$ are chosen as design parameters.

Our simulated robot is modelled as links with point masses at distal ends of the links, so the dynamic equation of the manipulator can be written as

$$\begin{aligned}
\tau_1 &= m_2 l_2^2(\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 \cos(q_2)(2\ddot{q}_1 + \ddot{q}_2) \\
&\quad + (m_1 + m_2) l_1^2 \ddot{q}_1 - 2 m_2 l_1 l_2 \sin(q_2)\dot{q}_1\dot{q}_2 \\
&\quad - m_2 l_1 l_2 \sin(q_2)\dot{q}_2^2 + (m_1 + m_2) l_1 g \cos(q_1) \\
&\quad + m_2 l_2 g \cos(q_1 + q_2) + f_{r1}(\dot{q}_1) \\
\tau_2 &= m_2 l_1 l_2 \cos(q_2)\ddot{q}_1 + m_2 l_2^2(\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 \sin(q_2)\dot{q}_1^2 \\
&\quad + m_2 l_2 g \cos(q_1 + q_2) + f_{r2}(\dot{q}_2)
\end{aligned}$$

where g is the gravity, τ_i is the output torque of a motor reflected to the joint axes, and fri is friction function, $i = 1, 2$. Comparing the above equation to (15), we have $f_{2,j}$ and $\zeta_2(t)$ for the learning control with simplest feedforward part as follows

$$f_{2,j} = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix}, \qquad \zeta_2(t) = \begin{bmatrix} \zeta_1'(t) \\ \zeta_2'(t) \end{bmatrix}.$$

where

$$F_1 = \\ \begin{bmatrix} 1 & \cos(q_2) & \sin(q_2)\dot{q}_1\dot{q}_2 & \sin(q_2)\dot{q}_2^2 & \cos(q_1) & \cos(q_1 + q_2) \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 1 & \cos(q_2) & \sin(q_2)\dot{q}_1^2 & \cos(q_1 + q_2) & 0 & 0 \end{bmatrix}.$$

and

$$\zeta_1'(t) = \begin{bmatrix} m_2 l_2^2(\ddot{q}_1^d + \ddot{q}_2^d) + (m_1 + m_2) l_1^2 \ddot{q}_1^d + f_{S1}\sin(\dot{q}_1^d) \\ m_2 l_1 l_2 (2\ddot{q}_1^d + \ddot{q}_2^d) \\ -2 m_2 l_1 l_2 \\ -m_2 l_1 l_2 \\ (m_1 + m_2) l_1 g \\ m_2 l_2 g \end{bmatrix},$$

$$\zeta_2'(t) = \begin{bmatrix} m_2 l_2^2(\ddot{q}_1^d + \ddot{q}_2^d) + f_{S2}\, sign(\dot{q}_2^d) \\ m_2 l_1 l_2 \ddot{q}_1^d \\ m_2 l_1 l_2 \\ m_2 l_2 g \\ 0 \\ 0 \end{bmatrix}.$$

For the learning control with simplest learning part, $f_{2,j}$ is a $[2 \times 2]$ identity matrix.

The learning control (7) and (8) are implemented with the following choices:

$$\alpha = 5.0, \quad \overline{m} = 9, \quad \Delta_0 = 0.0 \quad \rho_{21} = \rho_{22} = 2.0 + \|\dot{q}_j\|.$$

The simulation results of the proposed learning control are given in Figures 1~2, they show the effectiveness of the control. Also the simulation results show that the case of simplest feedfoward control part is better than that of simplest learning control part for system convergence. This result implies that the first control scheme results in better performance as the learning control part compensates more uncertainties and nonlinearities of the system; that is, enhancing the learning capability is more effective than the other case to control the robotic system. So, it will be a future research topic to investigate better learning scheme. The convergence rate for both cases can be adjusted by α.

6 Conclusions

A new class of learning control law based on Lyapunov direct method is presented. The idea of recursive design is used to develop a Lyapunov argument. Using the new Lyapunov argument, continuous and differentiable fictitious learning controls can be designed to provide global asymptotic stability for the cascaded first-order sub-system separately. Then the fictitious controllers are actually embedded inside of an overall control strategy, actual control input in the last cascaded sub-system. Comparing with the existing learning control laws, this learning scheme has several advantages: (i) the recursive mapping provides systematic technique to design learning controllers; (ii) the differentiability for the controller because of the linear learning law; therefore the control scheme can be generalized and easily applied to multiple integrator systems such as robotic systems with actuator dynamics and/or flexible joints. The learning control requires only state feedback and repeatability of tasks, and removes all assumptions commonly required by existing results: acceleration measurement, Lipschitzian condition, resetting of initial tracking errors, etc. The simulation results show that the learning control effectively compensates for nonlinear uncertainties in the mechanical dynamics.

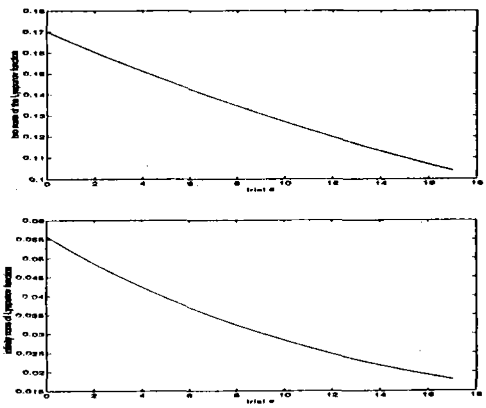

Figure 1: Simplest Learning Control Part

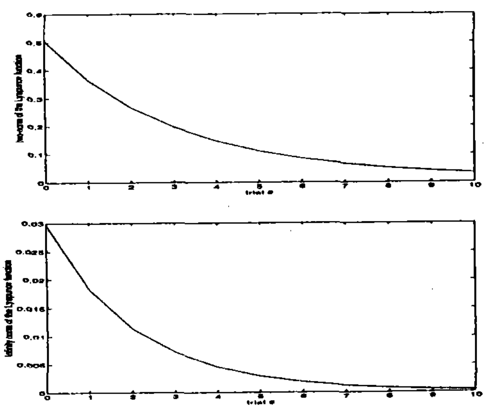

Figure 2: Simplest Feedforward Control Part

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