NEURAL CHANDRASEKHAR FILTERING METHOD FOR STATIONARY SIGNAL PROCESSES

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Abstract-In this paper we show the performances of neural Chandrasekhar filtering which is a special case for the new method of neural filtering using the artificial neural network systems developed recently for the filtering probems of linear and nonlinear, stationary and nonstationary stochastic signals. The neurofilter developed has either the finite impulse response(FIR) structure or the infinite impulse response(IIR) structure. The neurofilter differs from the conventional linear digital FIR and IIR filters because the artificial neural network system used in the neurofilter has nonlinear structure due to the sigmoid function.

Numerical studies for the estimation of a second order Butterworth process are performed by changing the structures of the neurofilter in order to evaluate the performance indices under the changes of the output noises or disturbances In the numerical studies both Chandrasekhar filtering esitmates and true signals are used as the training signals for the neurofilter. The results obtained from the studies verified the capabilities which are essentially necessary for on-line filtering of various stochastic signals.

Keywords-Neural filtering, Stochastic signals, FIR and IIR structures.

INTRODUCTION

The methods of neural filtering using the artificial neural network systems(denoted as ANNSs) for general stochastic signal processes provide us effective tools for estimating the signals whose characteristics change unexpectedly or suddenly by the disturbances and defaults, etc.. Recently a few studies on developing the neural filtering methods have been done in the fields of estimation problems[1]. However, only a conceptual idea how to construct the structures for the neural filtering based on the digital Chandrasehkar filtering formulas has been presented and the numerical studies have not been shown in the reference [1].

Recently a new method of neural filtering using the ANNSs has been proposed for the filtering probems of general stochastic signals[2-4]. The neurofilter developed has either the FIR structure or the IIR structure, differs from the conventional linear digital FIR and IIR filters because the ANNS which constructs the neurofilter has nonlinear structure due to the sigmoid function, and uses the back propergation method in order to minimize the errors between the outputs of the neurofilter(or the ANNS) and the training signals.

In this paper, we show the performances of the neural Chandrasekhar filtering which uses the Chandrasekhar filtering estimates as the training signals for the ANNSs. Both the estimates of the signals obtained from the Chandrasekhar filter[5-9] and the true signals, which are assumed to be obtained in off-line fashion, are used as the training signals in the neurofilter. The true signals can be obtained or can be known for some cases in practical applications.

Numerical results for the estimation of the second order Butterworth process are presented by changing the structures of the neurofilter in order to evaluate the performance indices under the changes of the output noises or disturbances. The results obtained verified the capabilities which are essentially necessary for on-line filtering of various stochastic signals.

2. NEUROFILTERS

For the reason of space, we will show only the IIR structure of the neurofilter in the followings because the FIR structure is considered as a special case of the IIR structure. The IIR structure is illustrated in Fig.1. In this figure, the inputs to the neurons in the input layer are both the sequence of the $(m+1)$ observed signals given by the set $\{y_t, ..., y_{t-m}\}$ and the sequence of the n estimated value of the signals given by the set $\{\hat{x}_{t-1}, ..., \hat{x}_{t-n}\}$. The output from the output layer is the filtering estimate $\hat{x}_t$. Here t denotes the present discrete time, and m and n denote the orders of the observed data and the filtering estimate, respectively.

In order to facilitate the understanding of the nerofilter with the IIR structure, we note that the fundamental equation of the linear digital IIR filter is specified by

$$\hat{x}_t = \sum_{i=0}^{m} a_i y_{t-i} + \sum_{j=1}^{n} b_j \hat{x}_{t-j}, \qquad (1)$$

where a_i and b_j are the coefficients which affect the filtering effects. Namely, the filtering estimate produced by the above equation is a linear combination of both the past measurement data and the past filtering estimates calculated. Therefore, the linear digital filters do not have the adaptive characteristics for sudden disturbances and unexpected faults in the system of signal generations.

In the meanwhile, the fundamental equation of the neurofilter with the IIR structure is represented by

$$\hat{x}_t = f_t(y_t, ..., y_{t-m}, \hat{x}_{t-1}, ..., \hat{x}_{t-n}), \quad (2)$$

where f_t is a special nonlinear transformation with adaptive characteristics due to the nonlinearlities of the sigmoid function in the ANNS. In the next section, we will show the methods how to train the neurofilter with the IIR structure.

3. TRAINING OF NEUROFILTERS

In order to train the neurofilter presented in Section 2, the back propergation method is used for minimizing the errors between the outputs from the neurofilter and the training signals. Two signals are employed as the training signals. One is the filtering estimates obtained by calculating the Chandrasekhar filter [5-9] and the other is the true signals which are obtained by assuming the true signals to be measured.

The blockdiagrams of the training schemes are shown in Fig.2a-b where Fiq.2a shows usual training approach based on the filtering estimates and Fig.2b shows unusual training approach based on the true signals, but often used. In the figures, $u(t)$ is the zero-mean white Gaussian input noise with covarince matrix $Q(\geq 0)$, $x(t)$ is the stochastic signal to be estimated, $v(t)$ is the zero-mean white Gaussian output noise with covariance matrix $R(> 0)$, and $y(t)$ is observation specified by

$$\dot{x}(t) = Fx(t) + Gu(t), x(0) = x_0, \quad (3)$$

$$y(t) = z(t) + v(t), z(t) = Hx(t), \quad (4)$$

where x_0 is the zero-mean white Gaussian initial state noise with covariance matrix K_0, and the noises u, v, and x_0 are uncorrelated each other. Refer to [5-9] for the Chandrasekhar filtering formuras.

4. NUMERICAL RESULTS

As a example for the numerical simulation studies in order to investigate the performances of the filtering effects of the neurofilter proposed, we consider the second order Butterworth signal process realized by the analog RC electric circuit illustrated in Fig. 3.

Let the voltages across the capacitors C_1 and C_2 be the states x_1 and x_2, respectively, in Fig.3. Hence, we see that x=col(x_1,x_2). We used the following values for the paramters in the circuit in oredr for the system to be stable.

$$R_1 = 1, R_2 = C_1 = C_2 = 2.$$

Using the above values for the parameters and the Lyapunov equation, we can evaluate F, G, H, and K_0 as follows[9].

$$F = \begin{bmatrix} -\frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{bmatrix}, G = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}, H = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$K_0 = \frac{Q}{10}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

Using the parameters specified above, the numerical studies of the neurofilter are performed on a NEC 9801 microcomuter by changing both the structure of the neurofilter, viz., m and n in Eq.(2), and the measurement noise variance expressed by R. The results shown in this section used the structure and the variance of the input noise sepcified by $m = n = 1$ and Q=1, respectively, and the ANNS with 3 neurons in the input layer, 15 neurons in the hidden layer, and 1 neuron in the output layer.

In order to evaluate the performance indexes of the neurofilter, we use the mean square errors specified by

$$e = \sum_{i=1}^{K} \| \hat{x}(t_i) - x(t_i) \|^2 / K,$$

where $\| \|$ denotes the norm and, x and $\hat{x}$ are equal to z ans $\hat{z}$, respectively, for our example. Fig.4 shows the mean square errors of both the neurofilter(denoted as the white circule or Neuro) and the Chandrasekar filter(denoted as the black circule or Filter) when the Chandrasekhar filtering estimates are used as the training signals and R is changed. Similarly, Fig.5 shows the same results as that in Fig.4 when the true signals are used as the training signals. We see from Figs. 4 and 5 that if we use the true signals as the traing signals for the case of large output noise variance the performance of the neurofilter reduces compared with that of the Chandrasekhar filter. This result is quite reasonable.

We performed other numerical calculations by changing the structure of the neurofilter and obtained intersting results. However, they are not given in this paper for the reason of space and will appear in a comming paper. The numerical results obtained verified the capability which are essentially necessary for on-line filtering of the general stochastic signals.

5. CONCLUSIONS

In this paper we showed the performances of the neural Chandrasekhar filtering which is a special case for the new method of neural filtering using the ANNSs developed recently[2-4] for the filtering probems of linear and nonlinear, stationary and nonstationary stochastic signals. The neurofilter developed has either the finite impulse response(FIR) structure or the infinite impulse response(IIR) structure. Our attentions are mainly focused on the performance index of the neurofilter with the IIR structure.

Numerical studies for the filtering of the second order Butterworth process are performed in order to evaluate the performance indices of the neurofilter under the changes of the output noises or disturbances. The results obtained from the studies verified the capabilities which are essentially necessary for on-line filtering of various stochastic signals. However, in this study, the numerical experiments for examples of the nonlinear filtering problems are not performed and will be presented in future studies.

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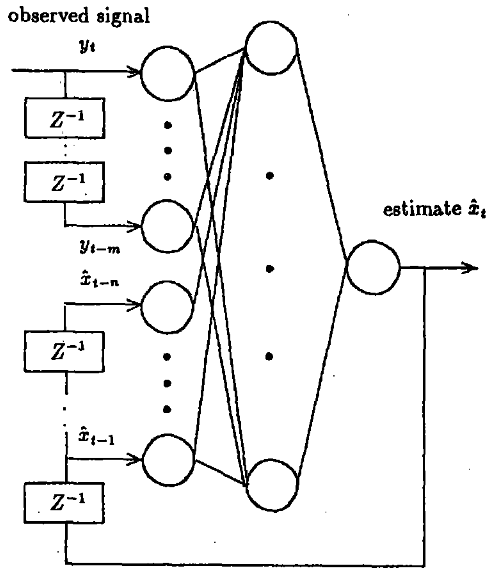

Fig.1 Neurofilter with IIR structure

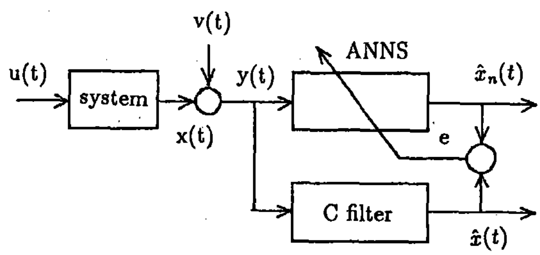

Fig.2a Training by Chandrasekhar filter

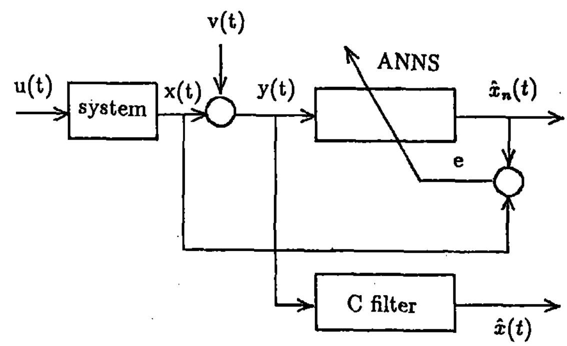

Fig.2b Training by true signals

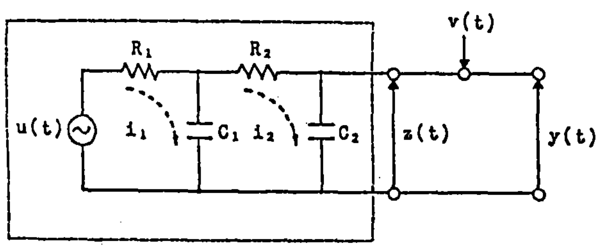

Fig.3 Analog RC circuit

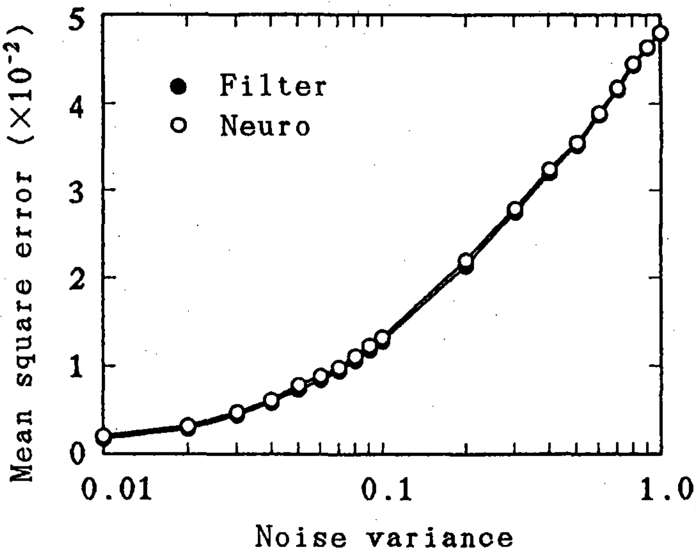

Fig.4 Mean square errors corresponding Fig.4a when R is changed

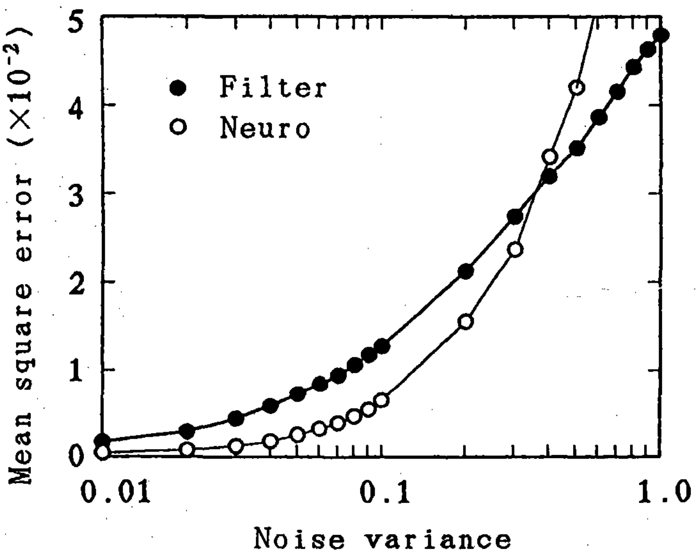

Fig.5 Mean square errors correponding Fig.5a when R is changed