Efficient Channel Management for Cellular Mobile Communication Systems

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ABSTRACT

The minimum span problem (MSP) and minimum blocking problem (MBP) for cellular mobile systems with nonuniform traffic distributions are considered. MSP minimizes the span, i.e., the number of channels, necessary to satisfy a given grade of service (GOS) for the system subject to the co-channel, adjacent and co-site interference constraints. MBP minimizes the average blocking probability of the system subject to the interference constraints, given the number of available channels. In this paper, we suggest an efficient heuristic algorithm for MSP which uses a GOS value updating scheme, and compare the algorithm with existing other ones. Computational experiments show that this algorithm provides solutions with much smaller span than others. We also show that a simple modification of that algorithm provides encouraging computational results for MBP.

I. INTRODUCTION

With the enormous growth of mobile communication demand, the frequency spectrum allocated to the mobile system becomes a more scarce resource. Hence, it is a vital issue to use the spectrum more efficiently. An efficient way of increasing frequency spectrum utilization in a mobile communication system is the cellular structure approach which divides spatially the geographical region into a number of cells. In such a cellular mobile system, there are three types of hierarchically organized stations: a switch, base stations, and mobile stations. The switch and base stations are connected by a fixed-topology infrastructure. A base station is established in each cell, and every mobile station in the cell communicates through the base station via a channel of the available spectrum. The switch executes channel allocation algorithms in a cellular mobile system. It also hands off a call from one base station to another when the call can be handled better by the new base station.

To achieve a more efficient use of the limited spectrum, a frequency reuse mechanism is used in a cellular mobile system. The same channels cannot be used simultaneously in different cells of the system due to the co-channel interference. Furthermore, even adjacent frequencies are not allowed to be used simultaneously in the same cell and neighboring cells because of the co-site and adjacent interferences. The interferences depend on propagation of radio signals in the system, which is a complex function of different time-variable parameters. An efficient method of reusing frequency in different cells subject to the interference conditions is essential. This is true independent of the multiplexing technology being used. For a fixed spectrum assigned and a specific multiplexing technology used, the traffic carrying capacity of a cellular system depends on how the frequency channels are managed. In especial, in the design of an FDMA/TDMA (frequency division multiple access/time division multiple access) based system, channel management is one of the most important problems.

We consider two channel assignment problems MSP (minimum span problem) and MBP (minimum blocking problem) for cellular mobile systems with nonuniform traffic distributions. MSP minimizes the span, i.e., the number of channels, necessary to satisfy a given grade of service (GOS) for the system subject to the three interference constraints. MSP has already been investigated by many authors [1, 4, 6, 7, 11, 14], but their results are not quite satisfactory. Gamst [3] provides a lower bound for the minimum span necessary to satisfy a given GOS, but does not provide a feasible span. MBP minimizes the average blocking probability of the system subject to the interference constraints, given the number of available channels. Many researches [8, 9, 12, 13] have investigated MBP subject to only the co-channel interference constraint, but they have not suggested a practical algorithm for MBP considering all the interference constraints.

Recently, Chang [2] suggested a heuristic algorithm based on Lagrangean relaxation which works well for systems with special compatibility matrices.

In this paper, we suggest an efficient heuristic algorithm for MSP which uses a GOS value updating scheme. We compare the algorithm with existing other ones. Computational experiments show that our algorithm provides solutions with much smaller span than others. We also show that a simple modification of that algorithm provides encouraging computational results for MBP.

The paper is organized as follows. In section II, MSP and MBP are mathematically formulated. In section III, heuristic algorithms for MSP and MBP are suggested. Some computational results are given in section IV. Section V concludes the paper.

II. PROBLEM FORMULATION

II-1. Minimum Span Problem (MSP)

The minimum span problem (MSP) is to find the minimum bandwidth necessary to satisfy a given grade of service (GOS) for the system subject to the interference constraints. This problem can be formulated as follows:

We consider a cellular mobile system consisting of N cells. Let λ_i be the traffic demand in erlangs of cell i. Assume that frequency channels are represented by the positive integers $1, 2, \cdots$. The channel separation required between a call in cell i and a call in cell j is, then, represented by a nonnegative integer c_{ij}. The case $c_{ij} = 1$ $(i \neq j)$ implies that cells i and j cannot use the same channel (called a co-channel interference constraint). If $c_{ij} \geq 2$ $(i \neq j)$, the use of adjacent channels in cells i and j cannot be tolerated (called an adjacent-site interference constraint). And c_{ii} denotes the separation required between channels used in the same cell i (called a co-site interference constraint).

The blocking probability of a cell with traffic demand λ and m channels assigned to it is given by the Erlang B formula as

$$B(\lambda, m) = \left[\sum_{k=0}^{m} \frac{\lambda^k}{k!} \right]^{-1} \frac{\lambda^m}{m!}.$$

The maximum blocking probability among all cells is the grade of service (GOS) for the whole system. Hence the blocking probability of each cell is less than or equal to the GOS of the system.

Let $\bar{B}$ be the desired GOS for the whole system. Then the number of channels, m_i, in cell i with traffic demand λ_i should satisfy the following relation:

$$B(\lambda_i, m_i) \leq \bar{B}.$$

For a given λ_i, let $\bar{m}_i$ be the smallest integer satisfying the above relation.

To formulate the minimum span problem satisfying a given GOS, let us introduce the following variables

$$f_k^i : \text{the channel assigned to the } k\text{th call in cell } i,$$

where $1 \leq i \leq N,\ 1 \leq k \leq \bar{m}_i$.

(MSP)

$$\min \max_{i,k} f_k^i$$

$$\text{s.t. } |f_k^i - f_l^j| \geq c_{ij} \quad \text{for all } i, j, k, l \text{ except when } i = j \text{ and } k = l,$$

where $1 \leq i, j \leq N,\ 1 \leq k \leq \bar{m}_i$, and $1 \leq l \leq \bar{m}_j$.

MSP is known to be equivalent to the generalized graph coloring problem. If all the c_{ij}'s are 0's and 1's, this problem reduces to the classical graph coloring problem. Since the latter is known to be NP-complete, it follows that the generalized graph coloring problem is also NP-complete [5, 11].

II-2. Minimum Blocking Problem (MBP)

The minimum blocking problem (MBP) is to minimize the weighted average blocking probability of the whole system subject to the interference constraints, given the number of channels. This problem can be mathematically formulated as follows:

We consider a cellular mobile system consisting of N cells and M channels. Assume that cells and channels are represented by the positive integers 1, 2, $\cdots$, N and 1, 2, $\cdots$, M, respectively. The channel separation required between a call in cell i and a call in cell j is, then, represented by a nonnegative integer c_{ij}.

Let λ_i be the traffic demand in erlangs of cell i and let the number of channels available in the cell be m_i. Then the call blocking probability in the cell is given by the Erlang B formula as $B(\lambda_i, m_i)$ which is a strictly decreasing function of m_i. The weighted average blocking probability of the cellular system is given by

$$\sum_{i=1}^{N} w_i B(\lambda_i, m_i)$$

where $w_i = \lambda_i / \sum_{i=1}^{N} \lambda_i$ is the traffic weighting factor [8, 13].

Let the decision variable f_{ij} be a binary integer variable indicating channel allocation where $f_{ij} = 1$ represents that channel j is allocated to cell i and $f_{ij} = 0$ otherwise. Then the minimum blocking problem is given by

(MBP)

$$\min \sum_{i=1}^{N} w_i B(\lambda_i, m_i)$$

$$\text{s.t. } m_i = \sum_{j=1}^{M} f_{ij}, \text{ for } i = 1, \cdots, N, \tag{1}$$

$$f_{ij} + f_{kl} \leq 1, \text{ for } i, k = 1, \cdots, N,\ j, l = 1, \cdots, M$$
$$\text{such that } |j - l| < c_{ik} \text{ except when } i = k \text{ and } j = l, \tag{2}$$

$$f_{ij} = 0 \text{ or } 1, \text{ for } i = 1, \cdots, N,\ j = 1, \cdots, M. \tag{3}$$

This is a nonlinear combinatorial problem. The decision variable m_i, number of channels allocated to cell i, is determined by channel allocation f_{ij}'s as shown in equation (1). The equation (2) is a mathematical expression of the interference constraints. MBP is NP-hard because it can easily be reduced to the independent set problem which is known to be NP-complete [5, 9].

III. ALGORITHM

III-1. GOS Updating Algorithm for MSP

Our heuristic algorithm for MSP allocates each channel sequentially under the interference constraints. Suppose that we have already assigned channels $1, 2, ..., (k-1)$ to cells. Then, first, find cells satisfying a given GOS already, and cells to which the channel k can not be assigned due to the co-site and adjacent interference from previous assignments. Then, the channel k is assigned to some cells other than the cells found in the above process. In this assignment, the co-channel interference is considered.

The algorithm generates a sequence of grades of service $\{B^k\}$ that converges to the required GOS, $\bar{B}$, in a self-adjusting manner. The variable x_i is a binary integer variable indicating channel assignment where $x_i = 1$ represents that the current channel is assigned to cell i and $x_i = 0$ otherwise. Let m_i be the number of channels assigned to cell i. For a given $\bar{B}$, the algorithm goes as follows:

(Basic Algorithm)

Step 0 (*Initialization*) Choose $B^1 \geq \bar{B}$ and a smaller value $\Delta > 0$. Set $I = \{1, 2, ..., N\}$ and $m_i = 0$ for all $i \in I$. Consider the first channel $k = 1$.

Step 1 (*Updating the target GOS*) If $B(\lambda_i, m_i) \leq B^k$ for all i, then set $B^k = \max\{\bar{B}, B^k - \Delta\}$ repeatedly until $B(\lambda_i, m_i) > B^k$ for some i.

Step 2 (*Finding excluded cells*)
For each cell $i \in I$,
if $(B(\lambda_i, m_i) \leq \bar{B}$, i.e., $m_i \geq \bar{m}_i)$ or
$(k < f^j_{m_j} + c_{ij}$, for some cell j such that $m_j > 0)$,
then set $x_i = 0$ and $I = I - \{i\}$.

Step 3 (*Assignment*)

Step 3.1 For $i \in I$, compute the weighted marginal improvement of blocking probability in cell i, $d_i = w_i(B(\lambda_i, m_i) - B(\lambda_i, m_i + 1))$, and if $B(\lambda_i, m_i) > B^k$ then set $d_i = d_i + L$ where L is a relatively large positive number.

Step 3.2 Repeat *i*) & *ii*) s times, where s is a nonnegative integer:
i) Let p be the index such that $d_p = \max\{d_i, i \in I\}$.
ii) Set $x_p = 1$ and $I = I - \{p\}$. Then set $x_j = 0$ and $I = I - \{j\}$ for all $j \in I$ such that $c_{pj} > 0$ and $j \neq p$.

Step 3.3 Solve the following maximum weight node packing problem (MP):

$$\max \sum_{i \in I} d_i x_i$$
$$\text{s.t.} \quad x_i + x_j \leq 1, \quad \text{for } i, j \in I \text{ such that } c_{ij} > 0 \text{ and } i \neq j,$$
$$x_i = 0 \text{ or } 1, \quad \text{for } i \in I.$$

Step 3.4 For all i, if $x_i = 1$ then set $m_i = m_i + 1$ and $f^i_{m_i} = k$.

Step 4 (*Termination*) If $B(\lambda_i, m_i) \leq \bar{B}$, i.e., $m_i \geq \bar{m}_i$, for all i, then terminate with the minimum span k. Otherwise, set $B^{k+1} = B^k$ and $I = \{1, 2, ..., N\}$, and go to Step 1 with replacing $k + 1$ to k.

Step 1 obtains a new target GOS value B^k not being met in at least one cell. Step 2 finds cells satisfying a given GOS already, and cells to which the channel k can not be assigned due to the co-site and adjacent interference from previous assignments. Step 3 introduces a procedure which assigns the channel k to some cells other than the cells found at Step 2, subject to the co-channel interference constraint. This assignment procedure considers the weighted marginal improvement of blocking probability in each cell, and whether the target GOS value B^k is satisfied in each cell. This also has an effect on reducing the size of the maximum weight node packing problem (MP) at Step 3.3. The problem (MP) is easily solved by a branch-and-bound procedure since the set I has a few elements in all iterations except in the first iteration. In node packing, all of the variables that are integral in the solution of the linear integer programming relaxation (if any) keep these same integer values in some optimal solution to the integer program [10]. Hence, having solved the linear programming relaxation, we can fix the values of the integral variables and then eliminate them from the problem. This property can be applied at every node of the branch-and-bound tree.

Now we suggest a simple procedure to improve the channel assignment solution obtained by the Basic Algorithm. Let v and w be the index such that $f^v_{\bar{m}_v} = \max\{f^i_{\bar{m}_i}$, for all $i\}$ and $f^w_{\bar{m}_w} = \max\{f^i_{\bar{m}_i}$, for all $i \neq v\}$, respectively. Then, if the channel $f^v_{\bar{m}_v}$ assigned to the cell v is eliminated, the span reduces by the following amount:

$$f^v_{\bar{m}_v} - \max\{f^w_{\bar{m}_w}, f^v_{\bar{m}_v - 1}\}. \tag{4}$$

But we have to assign one channel to the cell v in order to meet GOS. Let $\bar{k}$ be the index at which the maximum value is attained from among $\{f^v_{k+1} - f^v_k, 1 \leq k \leq \bar{m}_v - 2\}$. Then, not violating the three interference constraints, we can assign one channel to the cell v with additional channels of the following amount:

$$\max\{2c_{vv} - (f^v_{\bar{k}+1} - f^v_{\bar{k}}), 2\bar{c}_v - 1\}, \tag{5}$$

where $\bar{c}_v = \max_{j \neq v} c_{vj}$. The first and second value of (5) is the number of channels required to satisfy the co-site and adjacent constraints, respectively. Thus if the value (4) is greater than the value (5), we can reduce the span by the value (4) − (5). This process is summarized at Step 5.

Step 5 (*Improvement*)

Step 5.1 Find v and w. Compute (4) and (5). If $(4) - (5) \leq 0$, terminate, otherwise, go to Step 5.2.

Step 5.2

Set $f_{k+1}^v = f_k^v + (5)$, for $k = (\bar{m}_v - 1), ..., (\bar{k} + 1)$, and set $f_{\bar{k}+1}^v = (f_{\bar{k}}^v + c_{vv})$.

For each cell $i \neq v$ and $k = 1, ..., \bar{m}_i$,

if $f_k^i \geq (f_{\bar{k}}^v + c_{vv} - \bar{c}_v)$, then set $f_k^i = f_k^i + (5)$.

Go to Step 5.1.

III-2. GOS Updating Algorithm for MBP

A heuristic solution for MBP can be obtained through a simple modification of the Basic Algorithm proposed in section III-1. The modified algorithm generates a sequence of target grades of service $\{B_k\}$ that decreases in a self-adjusting manner. The variable x_i is a binary integer variable indicating channel assignment where $x_i = 1$ represents that the current channel is assigned to cell i and $x_i = 0$ otherwise. Let m_i be the number of channels assigned to cell i and f_k^i be the kth channel assigned to cell i. For a given number of available channels M, the algorithm goes as follows:

Step 0 (*Initialization*) Choose $B^1, \Delta, \epsilon > 0$. Set $I = \{1, 2, \cdots, N\}$ and $m_i = 0$ for all $i \in I$. Consider the first channel $k = 1$.

Step 1 (*Updating target GOS*) If $B(\lambda_i, m_i) \leq B^k$ for all i, then set $B^k = B^k - \Delta$ repeatedly until $B(\lambda_i, m_i) > B^k$ for some i.

Step 2 (*Finding excluded cells*)

For each cell $i \in I$,

if $(k < f_{m_j}^j + c_{ij}$ for some cell j such that $m_j > 0)$ or

$(w_i B(\lambda_i, m_i) \leq \epsilon)$,

then set $x_i = 0$ and $I = I - \{i\}$.

Step 3 (*Assignment*) Equivalent to Step 3 of the Basic Algorithm given in section III-1.

Step 4 (*Termination*) If $(k = M)$ or $(w_i B(\lambda_i, m_i) \leq \epsilon$ for all $i)$, then terminate. Otherwise, set $B^{k+1} = B^k$ and $I = \{1, \cdots, N\}$, and go to Step 1 with replacing $k + 1$ to k.

Step 1 obtains a new target GOS value B^k not being met in at least one cell. Step 2 finds cells to which the channel k cannot be assigned due to the adjacent and co-site interferences derived from previous assignments. Step 3 introduces a procedure which assigns the channel k to some cells other than the cells found at Step 2, subject to the co-channel interference constraint.

IV. COMPUTATIONAL EXPERIMENTS AND REMARKS

Our heuristic algorithm for MSP and all the compared algorithms are coded in PASCAL, and all the computational experiments are made on IBM PC 486. We tested the algorithms through test examples. Test examples have a cellular system which consists of 49 (7 × 7) cells. Traffic demands for cells are generated randomly from Normal distribution with mean of 10 erlangs and variance of 10, 20, or 30 erlangs. Assume that the minimum co-channel reuse distance is $\sqrt{21}r$ where r is the cell radius.

The result of computational experiments is given in Table 1. In Table 1, CRF, CRR, CCF, CCR, DRF, DRR, DCF and DCR are heuristic algorithms presented in [11]. Our algorithm is tested for 4 different sets of parameters B^1, Δ. The parameters $\bar{B}$ and s (at Step 3.2) were set at 0.02 and 1, respectively. The entries in the table are the number of channels (span) required to satisfy GOS for the system. For each problem, the entries in bold faces denote the best result obtained for the problem. Our algorithm has found the best solution for all test problems. Each of the four cases of our algorithm provides much better solutions than any of the first eight algorithms. Especially, our algorithm with $(B^1, \Delta) = (0.9, 0.1)$, MS2, has found better solutions than the first eight algorithms for all test problems. A better parameter selection will improve our algorithm further.

Table 1. Computational results for MSP

m, v	csc, adc	CRF	CRR	CCF	CCR	DRF	DRR	DCF	DCR	MS1	MS2	MS3	MS4
10,10	5,2	245	222	218	255	272	234	224	240	201	208	202	210
		223	213	221	243	270	215	217	269	211	212	208	205
	7,2	264	276	236	285	281	227	233	275	211	212	213	214
		269	233	237	290	276	236	229	300	217	216	219	217
	7,3	376	284	310	340	400	291	312	325	278	281	273	283
		348	308	306	349	378	304	310	295	280	278	287	286
10,20	5,2	259	214	236	272	290	235	233	230	217	211	214	217
		272	228	231	277	262	242	241	260	215	216	213	216
	7,2	290	282	242	313	297	271	241	308	228	222	227	227
		275	226	246	310	292	282	249	311	223	225	228	228
	7,3	377	309	331	329	379	317	320	320	299	297	294	300
		353	329	324	360	392	329	323	351	298	298	295	294
10,30	5,2	264	237	246	260	284	235	249	250	222	220	221	224
		290	271	246	283	318	235	250	259	221	222	226	226
	7,2	309	261	268	299	311	282	255	353	238	233	236	239
		289	241	264	292	287	249	259	321	241	238	239	240
	7,3	376	317	339	326	382	325	347	342	306	306	309	307
		367	328	336	373	381	330	346	380	304	322	305	328

m, v : Mean and variance in erlangs of Normal distribution, respectively.
csc : Co-site constraint, i.e., channel separation in the same cell.
adc : Adjacent constraint, i.e., channel separation between adjacent cells.
MS1, MS2, MS3, MS4 : GOS updating algorithm for MSP with
$(B^1, \Delta) = (0.02, 0.01), (0.9, 0.1), (0.2, 0.01), (0.99, 0.01)$, respectively.

We tested our heuristic algorithm for MBP through test examples. Traffic demands for cells are generated randomly from Normal distribution with mean of 10 erlangs and variance of 10, 20, or 30 erlangs. Assume that the minimum co-channel reuse distance is $\sqrt{21}r$ where r is the cell radius. The parameter ϵ and s were set at 0.00001 and 1, respectively. We tested our algorithm with four different parameter pairs (B^1, Δ). The results of computational experiment are summarized in Table 2. In the table, the average blocking probability of the whole system and the maximum blocking probability among all cells are given. A better parameter selection will improve our algorithm. If the parameter B^1 is set to zero, the GOS updating scheme is not used. The table shows that the cases using the GOS updating scheme provide better solutions than that not using the scheme.

Table 2. Computational Results for MBP

m, v	csc, adc	M	MB1		MB2		MB3		MB4	
			BLO	GOS	BLO	GOS	BLO	GOS	BLO	GOS
10,10	5,2	200	1.16	3.49	0.79	2.43	0.97	2.84	1.11	2.95
		200	1.09	3.97	1.08	3.48	1.19	3.97	1.18	3.80
	7,2	200	1.57	4.35	1.50	4.41	1.37	3.97	1.74	4.41
		200	1.65	4.76	1.71	4.50	1.58	4.19	1.60	3.97
	7,3	250	2.43	8.19	2.23	6.55	2.22	6.17	2.26	8.19
		250	2.22	6.10	2.13	6.10	2.50	7.36	2.18	8.28
10,20	5,2	200	1.60	5.11	1.30	3.70	1.40	3.70	1.59	4.94
		200	1.39	5.05	1.49	4.10	1.44	4.71	1.71	4.81
	7,2	200	2.44	8.39	2.26	6.72	1.93	5.12	2.43	8.19
		200	1.87	11.38	2.22	7.53	2.25	7.53	2.40	9.53
	7,3	250	2.94	8.88	2.94	8.88	2.91	10.51	3.17	10.51
		250	3.21	11.81	3.26	9.87	3.17	9.87	3.05	11.86
10,30	5,2	200	2.06	5.44	1.63	4.91	1.37	3.57	2.05	6.50
		200	2.16	10.79	2.17	5.92	2.10	5.92	2.57	8.14
	7,2	200	3.57	11.10	3.08	9.02	3.11	9.02	3.14	11.10
		200	3.30	10.07	2.93	12.25	3.23	10.07	3.41	12.25
	7,3	250	4.11	14.84	3.95	12.16	3.65	12.59	4.01	14.84
		250	3.76	14.64	4.05	17.25	4.09	20.06	4.34	14.64

M : Number of available channels
csc : Co-site constraint, i.e., channel separation in the same cell
adc : Adjacent constraint, i.e., channel separation between adjacent cells
BLO : average blocking probability of the whole system (%)
GOS : maximum of blocking probability among all cells (GOS) (%)
MB1, MB2, MB3, MB4 : GOS updating algorithm for MBP with
$(B^1, \Delta) = (\cdot, \cdot), (0.1, 0.01), (0.2, 0.01), (0.99, 0.01)$, respectively

In MBP, we considered only the average blocking probability of system. Hence the solution of MBP may lack fairness among cells. In other words, some cells may have high blocking probabilities compared with other cells, which may not be desired. This fact can be considered by introducing a maximum allowable blocking probability level (GOS). The constraints $B(\lambda_i, m_i) \leq \text{GOS}, i = 1, ..., N$, is appended to the problem MBP. This problem can be easily solved by successively applying algorithm for MSP and a simple modification of algorithm for MBP.

V. CONCLUSIONS

We have suggested an efficient heuristic algorithm which finds the minimum span necessary to satisfy a given GOS for the system. The algorithm sequentially assigns available channels to cells, using a GOS value updating scheme. We compared the algorithm with existing other algorithms. Computational experiments showed that our algorithm provides solutions with much smaller span than other algorithms. We also showed that a simple modification of that algorithm provides encouraging computational results for MBP.

We have investigated our solving procedures in a cellular mobile system consisting of identical hexagonal cells. Real systems may consist of non-hexagonal cells with different cell sizes. However, our procedures are applicable to any cellular system, not only those consisting of identical hexagonal cells.

REFERENCES

[1] F. Box, "A heuristic technique for assigning frequencies to mobile radio nets," *IEEE Trans. Veh. Technol.*, vol. VT-27, pp. 57-64, 1978.

[2] K.-N. Chang, "Channel Management Models in Cellular Mobile Systems," Ph. D. Dissertation, Dept. of Management Science, KAIST, 1994.

[3] A. Gamst, "Some Lower Bounds for a Class of Frequency Assignment Problems," *IEEE Trans. Veh. Technol.*, vol. VT-35, pp. 8-14, 1986.

[4] A. Gamst and W. Rave, "On frequency assignment in mobile automatic telephone systems," *Proc. IEEE GLOBECOM*, pp. 309-315, 1982.

[5] M. R. Garey and D. S. Johnson, *Computers and Intractability : A Guide to the Theory of NP-Completeness*, W. H. Freeman and Co., New York, 1979.

[6] W. K. Hale, "New spectrum management tools," *Proc. IEEE Int. Conf. Eletromagn. Comp.*, pp. 47-53, 1981.

[7] W. K. Hale, "Frequency assignment: Theory and applications," *Proc. of IEEE*, vol. 68, pp. 1497-1514, 1980.

[8] D. Hong and S. S. Rappaport, "Heuristic channel assignments for cellular land mobile radio systems" *Proc. IEEE GLOBECOM*, pp. 997-1001, 1985.

[9] R. Mathar and J. Mattfeldt, "Channel assignment in cellular radio networks," *IEEE Trans. Veh. Technol.*, vol. VT-42, pp. 647-656, 1993.

[10] G. L. Nemhauser and L. A. Wolsey, *Integer and combinatorial optimization*, John Wiley & Sons, 1988.

[11] K. N. Sivarajan, R. J. McElece and J. K. Ketchum, "Channel assignment in cellular radio," *Proc. IEEE VTC*, pp. 846-850, 1989.

[12] K. L. Yeung and T. S. Yum, "The optimization of nominal channel allocation in cellular mobile systems," *Proc. IEEE ICC*, pp. 915-919, 1993.

[13] M. Zhang and T. S. Yum, "The nonuniform compact pattern allocation algorithm for cellular mobile systems," *IEEE Trans. Veh. Technol.*, vol. VT-40, pp. 387-391, 1991.

[14] J. A. Zoellner and C. L. Beall, "A breakthrough in spectrum conserving frequency assignment technology," *IEEE Trans. Electromagn. Comp.*, vol. EMC-19, pp 313-319, 1977.