GEOMETRY OF BED FORMS IN THE LOWER FLOW REGIME

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1. Introduction

In recent years it has been made clear that until a fundamental understanding of the reasons of formation and the characteristics of bed forms is achieved, precise approach to problems of bed configurations remains largely on a rather roughly empirical formula. In general, it might be suggested that the prediction of the sand wave geometrical shape requires a full understanding of two main fundamentals : one is the initiation process of sand waves, the second is the growth process of the sand waves. If the two processes were precisely understood, it would be possible to predict the shape of sand waves. In particular, it might be essential to correlate the wave length or height with the flow resistance, bed load transport rate and bed roughness.

Most of previous studies, for the geometrical shape of sand waves in open channels and alluvial rivers, concentrated on height of dunes and antidunes. On the contrary, length of bed forms such as dunes and ripples has a lack of interest. The biggest part of these studies introduced the wave number and the Froude number relation as a boundary for bed stability.

The purpose of this study is to explain the geometrical shape of sand waves from a more theoritical point of view according to the study by Hirano[1]. Characteristic dimensionless wave number equation is derived from the real part of the dimensionless complex propagation velocity, which represents the logarithmic growth rate of an initial disturbance. The functional relation is numerically solved, and then the effect of various parameters on the wave number is clarified. As a result, a simple form of wave number equation is obtained. Finally a comparison between theoritical results and data collected from rivers and experimental flumes are performed.

2. Length of Sand Waves

2.1 Linear stability analysis

In the theory, according to the study by Hirano[1], small scale sand waves are treated as two dimensional bed configurations in the sence that their formation is independent of the channel width.

First, continuity and momentum equations for one dimensional flow are used to discribe the flow properties as follows : Continuity equation for flow :

$$\frac{\partial h}{\partial t}+\frac{\partial (u_m h)}{\partial x}=0 \qquad (1)$$

where t is the time, x is the coordinate in downstream direction on the undisturbed bed, h is the local water depth, u_m is the local mean velocity over the cross section in x-direction.

Momentum equation for flow :

$$\frac{\partial u_m}{\partial t}+u_m\frac{\partial u_m}{\partial x}=g\sin\theta-g\frac{\partial (\lambda h+z)}{\partial x}\cos\theta-\frac{\tau_b}{\rho h} \qquad (2)$$

where g is the acceleration gravity, θ is the slope angle of undisturbed bed, λ is the Jaeger coefficient, z is the elevation of bed taken from the average bed surface, ρ is the mass density of water and τ_b is the bed shear stress.

Second, a continuity equation for sediments and an equation for bed load discharge in non-equilibrium state are used to discribe the sedimentation phenomena, respectively.

Continuity equation for sediments :

$$\frac{\partial z}{\partial t}+\frac{(q_{Be}-q_B)}{(1-\varepsilon)l}=0 \qquad (3)$$

where ε is the sediment porosity, l is the average step length of a sediment particle, qbe and q_B are transport rates of bed load per unit time and unit width of bed in equilibrium and non-equilibrium state, respectively. Equation for bed load discharge in non-equilibrium state :

$$\frac{\partial (C_B a*)}{\partial t}+\frac{\partial q_B}{\partial x}=\frac{(q_{Be}-q_B)}{l}+C_B w_0\{\Phi(\sigma)-F(\sigma)\}\left[1-\frac{C_B\{\Phi(\sigma)-F(\sigma)\}}{C_B\{\Phi(\sigma)-F(\sigma)\}}\right] \qquad (4)$$

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where C_B is the bed load concentration in the bed layer, a_* is the thickness of bed layer, w_0 is the fall velocity of sediment particle and $\phi(\sigma)$, $F(\sigma)$ are functions of $\sigma=(w_0 / 0.93 u_*)$, in which u_* is the shear velocity.

The formation of sand waves is a result of local erosion and deposition produced by the irregularity of sediment transport in the flow direction. The flow over wavy beds accelerates and decelerates, espesially in shallow water depths, and a separation zone is formed in the downstream face of bed forms. Considering these factors, the local bed shear stress is given by

$$\frac{\tau b}{\rho}=\frac{1}{\phi_0^2}\left(\frac{1-\Delta_0/3}{1-\Delta_0}\right)^2 u_m^2\left(\frac{1-\Delta}{1-\Delta/3}\right)^2 \tag{5}$$

where

$$\Delta=\Delta_0+\alpha\frac{\partial h}{\partial x},\quad \Delta_0=6/(\phi_0+2) \tag{6}$$

where D is the velocity defect at the bed, Δ_o , ϕ_o are values of Δ and $\phi_0 = u_m/u_*$ for uniform flow, respectively, and α is the empirical parameter denoting asymmetric distribution of bed shear stress.

By using the following dimensionless quantities :

$$\zeta=z/h_0,\quad X=x/h_0,\quad T=t.q_{B0}/\{(1-\varepsilon)h_0^2\} \tag{7}$$

a sinusoidal bed is introduced in the form

$$\zeta=\xi e(\gamma t+i\beta x) \tag{8}$$

where ξ is the normalized amplitude of disturbance, g is the dimensionless complex propagation velocity, and b is the dimensionless wave number defined by$(2\pi h_0/L)$, where L is the wave length . Subscripts 'o' indicates the undisturbed flow quantity. By using an infinitesimal wave theory and assuming a quasi-uniform flow, g can be derived from above equations as

$$\gamma=\frac{\beta^2 M(1-\lambda_1\beta^2F^2)\{\beta^2(E_*q\alpha+2/\beta^2)+i\beta(q\alpha-2E_*)\}}{\{\beta^2E^2+(1+EW)^2\}\{-3S_0+i\beta\{1-S_0q\alpha-(1+\lambda_2\beta^2)F^2\}]} \tag{9}$$

in which

$$F=\frac{U}{\sqrt{gh_0}},\quad E=\frac{\lambda_d d(1+A_*\phi_{bo})}{h_0},\quad E_*=E+\frac{(1+EW)W}{\beta^2},\quad W=\frac{k_0 w_0\{\phi(\sigma)-F(\sigma)\}}{\zeta_* u_{*0}}$$

$$M=\frac{m}{1-\psi_c/\psi_0},\quad q=\frac{0.75}{(1-\Delta_0)(1-\Delta_0/3)},\quad k_0=\frac{1}{k(1-u_{*c}/u_{*0})} \tag{10}$$

where F is the Froude number, U is the mean flow velocity, $\phi_{Bo}=q_{Bo}/\sqrt{sgd^3}$, is the specific weight of sediment in water, $\psi_0=u_{*0}^2/sgd$ is the dimensionless tractive force, $\psi_c=u_{*c}^2/sgd$ is the dimensionless critical tractive force, in which u_{*c} is the critical shear velocity, S_0 is the longitudinal slope for undisturbed bed, a = 5 , l_d =100 , A_* =10 , k =8.5 , x_* = 0.05 and m = 3/2. The parameter (E) represents the non-equilibrium state of bed load transport process. The parameter (W), which increases rapidly with the decrease of w_0 / u_* , denotes the effect of suspended sediment. The parameters λ_1 and λ_2 represent the centrifugal effects of curvilinearity of bed surface and water surface, respectively. These were introduced by Iwasa and Kennedy[2] as sub-coefficients in the equation of pressure correction coefficient λ, which known as the Jaeger coefficient. The value of λ_1 was given in the range of (0.4 - 0.55), and the value of λ_2 was given in the range of (0.2 - 0.4).

2.2 Length of dunes and ripples

2.2.1 Dominant wave length concept

Flow-generated bed configurations in the lower flow regime, such as dunes and ripples, have a characteristic wave length which depends on the properties of flow, fluid and bed material. From stability analysis by using perturbation technique, the bed wave length could be obtained successfully from the linear terms of the expanded equations. The sand wave length can be measured as soon as the sand wave begins to form and does not change noticeably with the time. Accordingly, the linear stability analysis may be used to obtain the length for fully developed sand waves even the theory is developed for the initiation of sand waves. The real part of Eq. 9 is represnted by g_1 which is derived in the form

$$\gamma_1=\frac{\beta^2M(1-\lambda_1\beta^2F^2)\left[\left\{1-(1+\frac{q\alpha}{\phi_0^2}+\lambda_2\beta^2)F^2\right\}\{q\alpha-2E_*\}-\frac{3F^2}{\phi_0^2}(E_*q\alpha+\frac{2}{\beta^2})\right]}{\{\beta^2E^2+(1+EW)^2\}\left[\left\{1-(1+\frac{q\alpha}{\phi_0^2}+\lambda_2\beta^2)F^2\right\}^2+\frac{9F^4}{\beta^2\phi_0^4}\right]} \tag{12}$$

The parameter g_1 represents the logarithmic growth rate of sand waves and its sign identificates the stable and unstable condition of bed. The dominant wave number b_d corresponds to the maximum growth rate of bed disturbance. Therefore, b_d is obtained by differentiating g_1 with respect to b , and then the first derivative is equated with zero.

$$\frac{\partial \gamma_1}{\partial \beta} = 0 \qquad (13)$$

The general solution for b_d from Eq.13 is given in the form

$$\sum_{n=0}^{6} A_n \beta_d^{2n} = 0 \qquad (14)$$

in which, the coefficients from A_o to A_6 contain the parameters F, f_o , E ,W and q . Also they contain the constants a , λ_1 , λ_2 . According to Tsubaki and Saito[3], the parameter a is taken as 5. According to Iwasa and Kennedy[2],the values of parameters l1 and l2 may be taken as constants and equal to 0.5 and 0.4, respectively.

The solution of this equation gives more than one value for wave number b_d . To clarify which value is the dominant, the relation between wave number b and the growth rate of amplitude of sand waves is shown in Fig.1 by using Eq.12 for Froude number F = 0.3, velocity coefficient f_o = 16, dimensionless tractive force y_o= 0.1 and suspended sediment coefficient w_0 / u_* = 0.1. In the figure, when the sign of gl is negative, the bed is stable and sand waves do not occur. When it is positive, the bed is unstable and the sand waves are formed. Also it is quite clear that the wave number b = 5.5 is the dominant wave number b_d which gives the maximum growth rate of amplitude for sand waves g_m. From the previous discussion, the value of b_d which gives the maximum growth rate of amplitude for bed forms g_m should achieve the following two conditions :

$$\frac{\partial^2 \gamma_1}{\partial \beta^2} < 0 \qquad (15)$$

and g_m is the maximum of the extreme values.

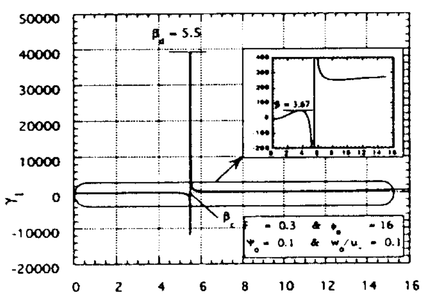

Fig. 1 Relation between growth rateγ_1 andβ

2.2.2 Characterictics of dominant wave number

To examine the main factors which have effect on wave number, Eq.14 could be expressed as the following function

b_d = f_1 (F , f_o , E , W , q) (16)

By using the relations $S_0 = F^2/\phi_0^2$ and $\psi_0 = h_0 S_0/s_d$, E is rewritten as a function of y_0,

$$E = \frac{\lambda_d(1 + A*\phi_{B0})F^2}{S\phi_0^2\psi_0} \qquad (17)$$

Based on the model of Hirano[4] for suspended sediments, the parameter W is a function of w_0 / u_* , as already mentioned. The parameter q is a function of f_o from Eqs. 6 and 10. By using the above relations, Eq. 16 reduces to more simple and fundamental one:

b_d = f_2 (F , y_o , f_o , w_o /u_*) (18)

Numerical solution for Eq.14 is derived in order to investigate the four previous variables effects on wave number values. Fig. 2 shows the relation between the Froude number F and the wave number b_d for y_0 value equal to 0.1, f_o equal to a reasonable moderate value 14 and wo/u* equal to 0.1, 0.3, 0.8, respetively. The figure shows a descending in b_d with F, and the curve changes from linear to curvilinear form. It is also noticed that b_d increases with the decrease of w_o/u_* for F > 0.6, and the curves for different values of w_o/u_* tend to merge into the same single line with the decrease of the Froude number F. In Fig. 3, the relation between b_d and y_0

with a parameter w_o/u_* is shown for F = 0.3 and f_o = 14 by using Eq.14. The figure shows that neither y_0 nor w_o/u_* has any effect on the wave number value. The effect of the velocity coefficient f_o on the wave number b_d is investigated. Figure 4 shows a relation between b_d and the velocity coefficient f_o with a parameter w_o/u_* for F = 0.3 and y_0 = 0.1 by using Eq.14. It is noticed that w_o/u_* does not affect b_d value, while f_o has a little effect on that value, especially for f_o <10.

As been concluded from Figs. 2,3 and 4, the suspended sediment parameter wo/u_* may be eliminated in Eq.18 for low values of Froude number. Also from Fig.3, the effect of parameter y^0 may be eliminated in Eq.18. Since the value of b_d is approximately constant for the change of f_o value as shown in Fig. 4, the parameter f_o may be eleminated in Eq. 18 without fear of accuracy. Consequently, the only dominant variable which has effect on the wave length formation is the Froude number F. From above figures, Eq. 18 may be simplified by neglecting the less effective parameters as

$$b_d = f_3 (F) \tag{19}$$

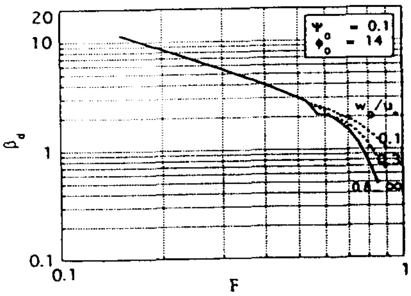

Fig. 2 Relation between F and β_d with parameter w_o/u_*.

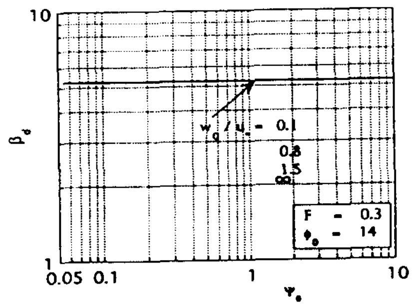

Fig. 3 Relation between Ψ_o and β_d with parameter w_o/u_*.

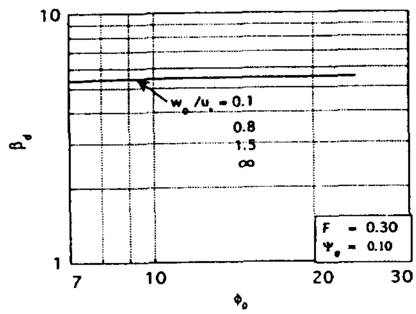

Fig. 4 Relation between Φ_o and β_d with parameter w_o/u_*

2.2.3 Practical formula for wave length

For the sake of treating with sand wave steepness, there is more particular interest of getting a practical formula of sand wave length in the lower flow regime is demanded. From Fig.1, it is noticed that the dominant value of b is quit near the marginal value bc , which gives the wave length before sand waves are washed out, or which separates stable and unstable beds (gl = 0). On the other hand, from the study by Watanabe and Hirano[1],the three boundary conditions for bed stability are expressed as follows

$$1-\lambda_1\beta^2F^2=0 \tag{20}$$

$$1-(1+\lambda_2\beta^2)F^2=0 \tag{21}$$

$$q\alpha-2E*=0 \tag{22}$$

Fig. 5 shows the stability diagram represented by b and the Froude number F. The solid lines represent the three boundary curves which separate stable and unstable conditions, in other words, sand waves and flat bed form. The dotted line represents Eq.14 with all parameters. The boundary curve from Eq.21 shows good agreement with Eq.14 curve, especially for F < 0.6. From this conclusion one can put bd = bc in the lower flow regime. By using Eq. 21, the practical equation for dominant wave number equation may be expressed in the form

$$\beta_d=1.58(\frac{1}{F^2}-1)^{0.5} \tag{23}$$

The practical equation of sand waves length in the lower flow regime is expressed in the form of Eq.24.

$$\frac{L}{h}=\frac{4F}{(1-f^2)^{0.5}} \tag{24}$$

Most of previous studies of wave number and wave length have been made from distinctly different approaches. Based on the theory of potential flow, using the complex stream function, Anderson introduced the relation between the Froude number and wave number as follows

$$F^2 = \frac{\sinh 2\beta}{\beta(\tanh\beta \sinh 2\beta - 2)} \tag{25}$$

Ten years later, Kennedy modified Anderson's theory and proposed equation which relates the wave number of the dominant wave length to the parameter F ,

$$F^2 = \frac{2+\beta\tanh\beta}{\beta^2+3\tanh\beta} \tag{26}$$

Tsuchiya and Ishizaki assumed that the surface wave has a small amplitude, which is half the wave length of the dune, then the following expression was introduced :

$$F = \sqrt{\frac{2\tanh 2\beta}{\beta}} - \sqrt{\frac{\tanh\beta}{\beta}} \tag{27}$$

Hayashi modified the study of Kennedy3) and introduced the region of occurrence of sand waves in F -b plane by using the following equation :

$$F_{1,2}^2 = \frac{\{c+2\mp\sqrt{(c+2)^2-8c\tanh^2\beta}\}}{4\beta\tanh\beta} \tag{28}$$

in which c is a dimensionless parameter and taken equal to 2. From Eq. 28, the Froude number has two values F1 and F2 for one value of b and there relation could be represented by two curves Hayashi 1 and Hayashi 2 , as shown in Figs. 6 and 7. Nakagawa and Tsujimoto introduced the limiting curve for $F < 1$ by using the following equation

$$F^2 = \frac{\tanh\beta}{\beta} \tag{29}$$

From all previous expressions, it is needless to say that the only dominant variable which has effect on wave number value is F. In Fig.6, the data for dunes show scattering in a wide range, and neither the presented curve nor reference curves, except one by Eq. 27, give a close agreement with scattered data. A modification is conducted in Eq.23 to obtain the fitting curve for b_d with the plotted data as

$$\beta_d = 0.63\left(\frac{1}{F^2}-1\right)^{0.5} \tag{30}$$

Eq. 30 is completely compatible with Eq. 27 by Tsuchiya and Ishizaki curve for $F > 0.20$ as shown in Fig. 6. In Fig.7, despite considerable scattering of the plotted data for ripples, the curve given by Eq. 23 shows the trend of data and lies the average of scattering more than the other comparable curves. A modification is conducted in Eq.23 to obtain the fitting curve for b_d with the plotted data as

$$\beta_d = 1.4\left(\frac{1}{F^2}-1\right)^{0.5} \tag{31}$$

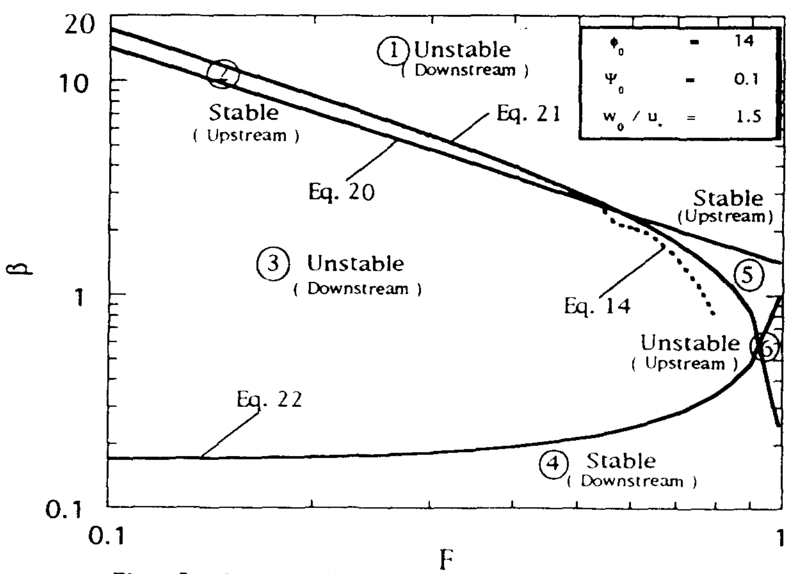

Fig. 5 Stability diagram for lower regime

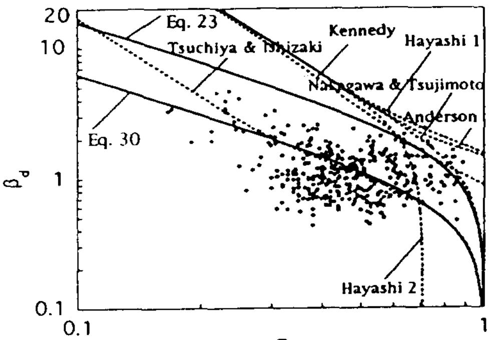

Fig. 6 Comparison with various theoretical results and the experimental data for dunes .

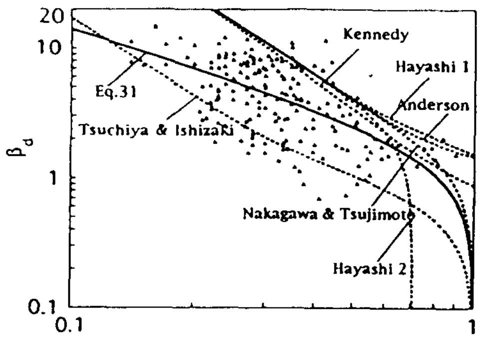

Fig. 7 Comparison with various theoretical results and the experimental data for ripples .

3. Conclusions

Using linear stability analysis, bed forms in the lower flow regime was examined analytically for determining the dominant wave length. The result was compared with the extensive bed forms data collected from rivers and experimental flumes. From this study the following conclusions are obtained.

1) The dominant wave number value lies in the unstable zone for bed and it is very close to the marginal stability value. It indicates that the linear stability analysis is applicable approach to the problem.
2) The suspended sediment factor w_0 / u_* has a weak effect on the dominant wave length. For the low Froude number F, one could neglect this factor in the wave number equation. On the contrary, for Froude number F > 0.6, suspended load should be considered.
3) For different values of Froude number, the dimensionless tractive force y_0 has no effect on wave number value.
4) The velocity coefficient fo does not strongly affect the wave number value. The wave number bd slightly increases with the increase of velocity coefficient fo effect and for more simplification f_0 may be neglected.
5) The apparant fact in this study is that the Froude number F is the major factor which affects the wave length.
6) Using the previous results, a practical equations for wave number bd for both dunes and ripples are obtained.
7) To demonstrate the applicability of the theoritical approach, the practical equations had been compared with the previous studies and data from rivers and flumes in (F - b) plane for dunes and ripples, respectively. The data show a good agreement with the proposed equations.

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