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LAYERWISE FORMULATION OF PIEZOELECTRIC LAMINATED COMPOSITES

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Abstract

A layerwise theory for the dynamic response of a laminated composite plate with integrated piezoelectric actuators and sensors subjected to both mechanical and electrical loadings is proposed. The formulation is derived form the variational principle with consideration for both total potential energy of the structures and the electrical potential energy of the piezoceramics. The governing equations of the present theory account for direct and converse effects of piezoelectrics, and layerwise variation of displacement field through the thickness of a laminate.

I. Introduction

Piezoelectric materials have received considerable attention due to their potential use in actively controling the elastic deformations of structures. Piezoelectric materials respond to mechanical load and generates an electric charge, which is called the *direct piezoelectric effect*[1]. Conversely, application of an electric field to the material can produce mechanical stress or strain which is referred to the *converse piezoelectric effect*. The sensing and actuating piezoelectric elements can be either surface bonded or embedded within the laminated structures.

One of the earliest studies concerned with piezoelectric plates is the work of Tiersten[2]. In his monograph, the basic equations governing the behavior of linear, piezoelectric media are developed and applied to various wave and vibration problems. Recently, a number of investigators have considered incorporating layers of piezoelectric materials into structural systems, providing a means for altering the structure's response through sensing, actuation, and control. Bailey and Hubbard[3] used a PVDF(Polyvinylidene fluoride) polymer as a distributed actuator for a Bernoulli-Euler beam model. Some other analyses and numerical

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models have also been developed to study cantilever beam with segmented piezoelectric actuators[4,5].

Lee[7] presented a formulation of distributed piezoelectric sensors and actuators based on the classical lamination theory (CLT) for bending and torsional modal control. Wang and Rogers[8] developed spatially distributed induced strain actuation model based on CLT. Tauchert[9] investigated piezothermoelastic behavior of a laminated composite under staionary thermal and electric fields. Buckling and postbuckling analyses with piezoelectric actuator studied by Chandrashekhara and Bhatia[6] by finite element method based on the first order shear deformable theory (FSDT). A fully nonlinear theory for the dynamics of anisotropic plates with piezoelectric layer was presented by Pai et al.[10] based on a higher-order shear deformable theory (HSDT). Tzou and his companion[11] presented a multilayered thin shell model with a piezoelectric layer based on Love's theory.

All the literature above used equivalent single-layer two dimensional theories (CLT, FSDT, HSDT), which model a multi-layered composite as an equivalent single-layer homogeneous plate. These theories are adequate for global response of laminates. In these equivalent single-layer theories, however, the coupling effects coming from material anisotropy may be overlooked. Furthermore, when the piezoelectric layers are taken into account in the analysis, these coupling effects become even larger. In this regard, a full three-dimensional piezoelectric finite element model was developed by several researchers. Among them, Ha et al.[12] implemented eight-node composite brick element based on the total potential energy of the structures and the electrical potential energy of the piezoceramics. Sunar and Rao[13] considered distributed thermopiezoelectric sensors and actuators by using three-dimensional finite lement method. Hagood et al.[14] used extended Hamilton's principle to derive the governing equations of coupled piezoelectric laminates and applied Ritz method to solve the problem. Ray et al.[15] considered a piezoelectric plate under cylindrical bending subjected to sinusoidal mechanical loading and electric potential on the top surface. Robbins and Reddy[16] performed static and dynamic analyses of an isotropic beam using layerwise finite element model to investigate the piezoelectric behavior of beam structures. However, the direct piezoelectric effect was not considered in that analysis.

In the present study, the layerwise plate theory[17] is extended for laminated composites with distributed piezoelectric sensors and actuators. The governing equations are obtained by the application of the variational principle. The present layerwise theory accounts for direct and converse effects of piezoelectrics, and a linear variation of displacement field through the thickness of a laminate for each layer.

II. Theoretical Formulation

A. Kinematics

An N-layer fiber-reinforced composite plate containing distributed piezoelectric sensors and actuators is considered. The resulting displacements U_α and U_3 at a generic point x_1, x_2, x_3 in the laminate are assumed to be of the form:

$$U_\alpha(x_\beta, x_3, t) = u_\alpha(x_\beta, t) + \phi^j(x_3) u^j_\alpha(x_\beta, t)\,, \tag{1}$$

$$U_3(x_\beta, x_3, t) = u_3(x_\beta, t)\,. \tag{2}$$

The usual Cartesian indicial notation is employed where Greek subscripts are assumed to have values 1 to 2. Superscript j ranges from 1 to N, where N is the number of layers. The terms u_α and u_3 are the displacements of a point (x_β, t) on the reference surface of the laminate, u^j_α are nodal values of the displacements in the x_α direction of each single-layer, and $\phi^j(z)$ is the linear Lagrangian interpolation function through the thickness of the laminate which accounts for linear variation of displacement field within each layer.

The strain tensor associated with small-displacement theory of elasticity are given by

$$\epsilon_{\alpha\beta} = \epsilon^{\circ}_{\alpha\beta} + \phi^j \epsilon^j_{\alpha\beta} , \tag{3}$$

$$\gamma_{\alpha 3} = \gamma^{\circ}_{\alpha 3} + \phi^j_3 \gamma^j_{\alpha 3} , \tag{4}$$

where

$$\epsilon^{\circ}_{\alpha\beta} = \frac{1}{2}(u_{\alpha,\beta} + u_{\beta,\alpha}) , \tag{5}$$

$$\epsilon^j_{\alpha\beta} = \frac{1}{2}(u^j_{\alpha,\beta} + u^j_{\beta,\alpha}) , \tag{6}$$

$$\gamma^{\circ}_{\alpha 3} = u_{3,\alpha} , \qquad \gamma^j_{\alpha 3} = u^j_\alpha . \tag{7}$$

B. Constitutive Equations.

The constitutive equations of a piezoelectric media are of the following form[18] :

$$\epsilon_{ij} = S_{ijkl}\sigma_{kl} + d_{nij}E_n , \tag{8}$$

$$D_m = d_{mkl}\sigma_{kl} + \varepsilon_{mn}E_n , \tag{9}$$

or, alternatively

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} - e_{kij}E_k , \tag{10}$$

$$D_m = e_{mkl}\epsilon_{kl} + \varepsilon_{mk}E_k , \tag{11}$$

where, σ_{ij} and ϵ_{ij} denote the stress and strain components; D_m are the electric displacements; E_m are the electric field vectors. Also, S_{ijkl} and C_{ijkl} denote elastic compliances and moduli, respectively; d_{mij} and e_{nij} are the piezoelectric coefficients; ε_{mk} are the dielectric constants. As shown in Eqs. (10) and (11), the piezoelectrics couple the mechanical and electrical equations. The coupling terms are the piezoelectric constants, e_{nij} which correlate stress to applied electric filed, electric displacement to applied strain. The first index in subscripts refers to electrical axis while the second and third indices are mechanical.

By using the assumption $\sigma_{33} = 0$ in the constitutive relation of Eqs. (10) and (11), the following expressions can be obtained for monoclinic materials,

$$\sigma_{\alpha\beta} = Q_{\alpha\beta\gamma\omega}\epsilon_{\gamma\omega} - e^*_{3\alpha\beta}E_3 , \tag{12a}$$

$$\sigma_{\alpha 3} = Q_{\alpha 3\beta 3}\gamma_{\beta 3} - e^*_{\beta\alpha 3}E_\beta , \tag{12b}$$

$$D_\alpha = e^*_{\alpha\beta 3}\gamma_{\beta 3} + \varepsilon_{\alpha\beta}E_\beta , \tag{12c}$$

$$D_3 = e^*_{3\alpha\beta}\epsilon_{\alpha\beta} + \varepsilon^*_{33}E_3 , \tag{12d}$$

where $Q_{\alpha\beta\gamma\omega}$, $e^*_{k\alpha\beta}$ and ε^*_{33} are reduced quantities defined by

$$Q_{\alpha\beta\gamma\omega} = C_{\alpha\beta\gamma\omega} - \frac{C_{\alpha\beta 33}C_{33\gamma\omega}}{C_{3333}}\,, \tag{13}$$

$$e^*_{k\alpha\beta} = e_{k\alpha\beta} - \frac{C_{\alpha\beta 33}e_{333}}{C_{3333}}\,, \tag{14}$$

$$\varepsilon^*_{33} = \varepsilon_{33} + \frac{e^2_{333}}{C_{3333}}\,. \tag{15}$$

C. Variational Formulation

The general system is composed of an elastic body with inclusion of piezoelectric material which are poled and electroded arbitrarily. The extented Hamilton's principle is used to derive the equations of motion of the present theory[18],

$$0 = \int_{t_1}^{t_2} [\delta(\mathcal{T} - \mathcal{U} + \mathcal{W}_e) - \delta\mathcal{W}]dt\,, \tag{16}$$

where $\mathcal{T}, \mathcal{U}, \mathcal{W}_e$ and $\delta\mathcal{W}$ are kinetic energy, strain energy, electric energy and the first variation of work done by external forces, respectively, given by

$$\mathcal{T} = \frac{1}{2}\int_v \rho\dot{U}_i\dot{U}_i dv\,, \tag{17}$$

$$\mathcal{U} = \frac{1}{2}\int_v \sigma_{ij}\epsilon_{ij} dv\,, \tag{18}$$

$$\mathcal{W}_e = \frac{1}{2}\int_v D_i E_i dv\,, \tag{19}$$

$$\delta\mathcal{W} = \int_v f_i\delta U_i dv + \oint_s \hat{t}_i\delta U_i ds + \oint_s \hat{q}\delta V ds\,. \tag{20}$$

In the above Equations, ρ is the density, $\hat{t}_i$ are the specified surface traction forces, $\hat{q}$ is electric charge, f_i are the body forces ($i = 1,2,3$), and V denotes electric potential. v is the volume of the plate and s is the boundary of v on which the tractions are specified.

The electric field is related to the electric potential V by

$$E_i = -V_{,i}\,, \tag{21}$$

and the electric potential is assumed to be the following form as for the displacement field:

$$V(x_\beta, x_3, t) = v^\circ(x_\beta, t) + \phi^j(x_3)v^j(x_\beta, t)\,. \tag{22}$$

Then the electric field vector becomes

$$E_\alpha = E^\circ_\alpha + \phi^j E^j_\alpha\,, \tag{23}$$

$$E_3 = \phi^j_{,3}E^j_3\,, \tag{24}$$

where

$$E^{\circ}_{\alpha} = -v^{\circ}_{,\alpha}\,; \qquad E^{j}_{\alpha} = -v^{j}_{,\alpha}\,; \qquad E^{j}_{3} = -v^{j}\,. \tag{25}$$

Substituting Eqs. (17), (18), (19) and (20) into Eq. (16) and defining specified traction and electric charge resultants as

$$(\hat{N}_{\alpha}, \hat{N}^{j}_{\alpha}) = \int_{z_k}^{z_{k+1}} \hat{t}_{\alpha}(1, \phi^{j}) dx_3\,, \tag{26}$$

$$\hat{Q}_3 = \int_{z_k}^{z_{k+1}} \hat{t}_3 dx_3\,, \tag{27}$$

$$(\hat{G}, \hat{G}^{j}) = \int_{z_k}^{z_{k+1}} \hat{q}(1, \phi^{j}) dx_3\,, \tag{28}$$

Eq. (16) takes the following form in the absence of body forces after integrating through the thickness

$$\begin{aligned} 0 = \int_{t_1}^{t_2} \Bigg\{ & \int_{\Omega} [I^{\circ}(\dot{u}_{\alpha}\delta\dot{u}_{\alpha} + \dot{u}_3\delta\dot{u}_3) + I^{j}(\dot{u}_{\alpha}\delta\dot{u}^{j}_{\alpha} + \dot{u}^{j}_{\alpha}\delta\dot{u}_{\alpha}) + I^{jk}\dot{u}^{j}_{\alpha}\delta\dot{u}^{k}_{\alpha}] d\Omega \\ & - \int_{\Omega} [N_{\alpha\beta}\delta u_{\alpha,\beta} + N^{j}_{\alpha\beta}\delta u^{j}_{\alpha,\beta} + Q_{\alpha 3}\delta u_{3,\alpha} + Q^{j}_{\alpha 3}\delta u^{j}_{\alpha} - p\delta u_3] d\Omega \\ & - \int_{\Omega} [G_{\alpha}\delta v^{\circ}_{,\alpha} + G^{j}_{\alpha}\delta v^{j}_{,\alpha} + G^{j}_{3} v^{j}] d\Omega \\ & - \int_{\Gamma_1} \hat{N}_{\alpha}\delta u_{\alpha} ds - \int_{\Gamma_2} \hat{Q}_3 \delta u_3 ds - \int_{\Gamma_3} \hat{N}^{j}_{\alpha}\delta u^{j}_{\alpha} ds - \int_{\Gamma_4} \hat{G}\delta v^{\circ} ds - \int_{\Gamma_5} \hat{G}^{j}\delta v^{j} ds \Bigg\} dt\,, \end{aligned} \tag{29}$$

where p is a specified distributed transverse load and $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$ are the portions of the boundary of the reference surface Ω on which $\hat{N}_{\alpha}, \hat{Q}_3, \hat{N}^{j}_{\alpha}, \hat{G}, \hat{G}^{j}$, respectively, are specified. In Eq. (29), the stress and electric resultants are defined as

$$[N_{\alpha\beta}, N^{j}_{\alpha\beta}] = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \sigma_{\alpha\beta}[1, \phi^{j}] dx_3\,, \tag{30}$$

$$[Q_{\alpha 3}, Q^{j}_{\alpha 3}] = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \sigma_{\alpha 3}[1, \phi^{j}_{,3}] dx_3\,, \tag{31}$$

$$[G_{\alpha}, G^{j}_{\alpha}] = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} D_{\alpha}[1, \phi^{j}] dx_3\,, \tag{32}$$

$$G^{j}_{3} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} D_3 \phi^{j}_{,3} dx_3\,. \tag{33}$$

The inertia terms are defined as

$$[I^{\circ}, I^{j}, I^{jk}] = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \rho[1, \phi^{j}, \phi^{jk}] dx_3\,. \tag{34}$$

D. Governing Equations

The actuator equations of motion and the sensor equations of the present theory can be derived by integrating the derivatives of the varied quantities by parts and collecting the coefficients of $\delta u_{\alpha}, \delta u_3, \delta u^{j}_{\alpha}, \delta v^{\circ}, \delta v^{j}$:

$$N_{\alpha\beta,\beta} = I^{\circ}\ddot{u}_{\alpha} + I^{j}\ddot{u}^{j}_{\alpha} \tag{35}$$

$$Q_{\beta 3,\beta} + p = I^{\circ}\ddot{u}_{3} \tag{36}$$

$$N^{j}_{\alpha\beta,\beta} - Q^{j}_{\alpha 3} = I^{j}\ddot{u}_{\alpha} + I^{jk}\ddot{u}^{k}_{\alpha}, \tag{37}$$

$$G_{\alpha,\alpha} = 0 \tag{38}$$

$$G^{j}_{\alpha,\alpha} - G^{j}_{3} = 0 \qquad \text{in } \Omega\,. \tag{39}$$

Eqs. (35), (36) and (37) are the equations of motion (actuator equations) induced by the applied load and electric field, while Eqs. (38) and (39) are the sensor equations which relate the displacements of the plate to the closed-circuit output charge signal. The complete set of Eqs. (35) through (39) ((4 + 3N) differential equations) describes the mechanical and electrical state of the piezoelectric laminated composite plate in (4 + 3N) variables $(u_{\alpha}, u_{3}, u^{j}_{\alpha}, v^{\circ}, v^{j})$.

The natural boundary conditions associated with Eqs. (35) to (39) are of the form:

$$N_{\alpha\beta}n_{\beta} - \hat{N}_{\alpha} = 0 \quad \text{on } \Gamma_{1} \tag{40}$$

$$Q_{\beta 3}n_{\beta} - \hat{Q}_{3} = 0 \quad \text{on } \Gamma_{2} \tag{41}$$

$$N^{j}_{\alpha\beta}n_{\beta} - \hat{N}^{j}_{\alpha} = 0 \quad \text{on } \Gamma_{3} \tag{42}$$

$$G_{\beta}n_{\beta} - \hat{G} = 0 \quad \text{on } \Gamma_{4} \tag{43}$$

$$G^{j}_{\beta}n_{\beta} - \hat{G}^{j} = 0 \quad \text{on } \Gamma_{5}\,, \tag{44}$$

where n_{β} is a unit normal in β direction. Recall that Γ_1 through Γ_5 complement the total boundary Γ such a way that essential boundary conditions are specified:

$$u_{\alpha} = \hat{u}_{\alpha} \quad \text{on } \Gamma - \Gamma_{1} \tag{45}$$

$$u_{3} = \hat{u}_{3} \quad \text{on } \Gamma - \Gamma_{1} \tag{46}$$

$$u^{j}_{\alpha} = \hat{u}^{j}_{\alpha} \quad \text{on } \Gamma - \Gamma_{1} \tag{47}$$

$$v^{\circ} = \hat{v}^{\circ} \quad \text{on } \Gamma - \Gamma_{1} \tag{48}$$

$$v^{j} = \hat{v}^{j} \quad \text{on } \Gamma - \Gamma_{1}\,. \tag{49}$$

E. Laminate Constitutive Equations

Substitution of Eq. (12) into Eqs. (30), (31), (32) and (33) gives the constitutive equations of the laminate:

$$\begin{Bmatrix} N_{\alpha\beta} \\ N^{j}_{\alpha\beta} \\ G^{j}_{3} \end{Bmatrix} = \begin{bmatrix} A_{\alpha\beta\gamma\omega} & B^{k}_{\alpha\beta\gamma\omega} & L^{k}_{3\alpha\beta} \\ B^{j}_{\alpha\beta\gamma\omega} & D^{jk}_{\alpha\beta\gamma\omega} & F^{kj}_{3\alpha\beta} \\ L^{j}_{3\gamma\omega} & F^{jk}_{3\gamma\omega} & H^{jk}_{33} \end{bmatrix} \begin{Bmatrix} \epsilon^{\circ}_{\gamma\omega} \\ \epsilon^{k}_{\gamma\omega} \\ -E^{k}_{3} \end{Bmatrix},$$

$$\begin{Bmatrix} Q_{\alpha 3} \\ Q^{j}_{\alpha 3} \\ G_{\alpha} \\ G^{j}_{\alpha} \end{Bmatrix} = \begin{bmatrix} A_{\alpha 3\beta 3} & B^{k}_{\alpha 3\beta 3} & L_{\beta\alpha 3} & F^{k}_{\beta\alpha 3} \\ B^{j}_{\alpha 3\beta 3} & D^{jk}_{\alpha 3\beta 3} & L^{j}_{\beta\alpha 3} & F^{jk}_{\beta\alpha 3} \\ L_{\alpha\beta 3} & L^{k}_{\alpha\beta 3} & H_{\alpha\beta} & H^{k}_{\alpha\beta} \\ F^{j}_{\alpha\beta 3} & F^{kj}_{\alpha\beta 3} & H^{j}_{\alpha\beta} & H^{jk}_{\alpha\beta} \end{bmatrix} \begin{Bmatrix} \gamma^{\circ}_{\beta 3} \\ \gamma^{k}_{\beta 3} \\ -E^{\circ}_{\beta} \\ -E^{k}_{\beta} \end{Bmatrix}, \tag{50}$$

where $A_{\alpha\beta\gamma\omega}$, $B^j_{\alpha\beta\gamma\omega}$, $D^{jk}_{\alpha\beta\gamma\omega}$ are the stiffnesses of the laminate and $L_{\alpha\beta 3}$, $F^j_{\alpha\beta 3}$, etc. are the piezoelectric quantities given by

$$(A_{\alpha\beta\gamma\omega}, B^j_{\alpha\beta\gamma\omega}, D^{jk}_{\alpha\beta\gamma\omega}) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \bar{Q}^{(k)}_{\alpha\beta\gamma\omega}(1,\, \phi^j,\, \phi^j\phi^k)dx_3\,, \tag{51a}$$

$$(A_{\alpha 3\beta 3}, B^j_{\alpha 3\beta 3}, D^{jk}_{\alpha 3\beta 3}) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \bar{Q}^{(k)}_{\alpha 3\beta 3}(1,\, \phi^j_{,3},\, \phi^j_{,3}\phi^k_{,3})dx_3\,, \tag{51b}$$

$$(L_{\alpha\beta 3}, L^j_{\alpha\beta 3}, F^j_{\alpha\beta 3}, F^{jk}_{\alpha\beta 3}) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \bar{e}^{*(k)}_{\alpha\beta 3}(1,\, \phi^j_{,3},\, \phi^j,\, \phi^j_{,3}\phi^k)dx_3\,, \tag{51c}$$

$$(L^j_{3\alpha\beta}, F^{jk}_{3\alpha\beta}) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \bar{e}^{*(k)}_{3\alpha\beta}(\phi^j_{,3},\, \phi^j_{,3}\phi^k)dx_3\,, \tag{51d}$$

$$(H_{\alpha\beta}, H^j_{\alpha\beta}, H^{jk}_{\alpha\beta}) = -\sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \bar{\varepsilon}^{(k)}_{\alpha\beta}(1,\, \phi^j,\, \phi^j\phi^k)dx_3\,, \tag{51e}$$

$$H^{jk}_{33} = -\sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \bar{\varepsilon}^{*(k)}_{33}\phi^j_{,3}\phi^k dx_3\,, \tag{51f}$$

where "$-$" denotes transformed quantities for each layer.

III. Closure

A two-dimensional, layerwise theory of laminated composite plates with distributed piezoelectric sensors and actuators is presented. The theory accounts for direct as well as converse piezoelectric effects, and a linear variation of displacement field through the thickness of a laminate for each layer. Thus, coupling effects coming from strong material anisotropy are systematically included in the formulation. Extension to finite element model awaits attention.

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