'95 KACC (1995. 10. 23 ~ 25)

Slewing Maneuver Control of Flexible Space Structure Using Adaptive CGT

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Abstract This paper concerns an adaptive control scheme which is an extension of the simplified adaptive control. Originally, the SAC approach was developed based on the command generator tracker (CGT) theory for perfect model tracking. An attractive point of the SAC is that a control input can be synthesized without any prior knowledge about plant structure. However, a feedforward dynamic compensator of the CGT is removed from the basic structure of the SAC. This deletion of the compensator makes perfect model tracking difficult against even a step input. In this paper, an adaptive control system is redesigned to achieve perfect model tracking for as long as possible by reviving the dynamic compensator of the CGT. The proposed method is applied to slewing control of a flexible space structure and compared to the SAC responses.

Keywords Command Generator Tracker, DMRAC, Simplified Adaptive Control, Exact Model Matching, Large Space Structure.

1. INTRODUCTION

In recent years, direct model reference adaptive control (DMRAC) or simplified adaptive control (SAC) has attracted the attention of researchers.

The SAC, developed by Sobel, Kaufman and Mabious and extended by Bar-Kana, is a very simple adaptive control approach [1-3]. This simplicity is quite advantageous to the SAC particularly for large flexible space structure (LFSS) control. This is because, unlike the conventional MRAC based on the exact model matching (EMM) method, the controller can be designed without accurate knowledge about plant structure.

The SAC approach is based on the command generator tracker (CGT) theory generalized by Broussard [4]. This CGT theory is a feedforward-type perfect model tracking or EMM approach for MIMO systems. However, for the sake of simplicity a feedforward dynamic compensator of the CGT is removed from the basic structure of SAC. Therefore, it becomes impossible for the SAC to achieve EMM. This treatment of the compensator seems to be due to the inappropriate assumption that a command signal is constant [5].

In this study, an adaptive control system is redesigned to recover perfect model tracking for as long as possible, by reviving the dynamic compensator of the CGT.

As a numerical example, the proposed adaptive CGT will be applied to slewing maneuver control of a flexible space structure.

2. CGT THEORY

We consider a controllable and observable plant represented by

$$\dot{x}_p = A_p x_p + B_p u_p \tag{1a}$$
$$y_p = C_p x_p + D_p u_p \tag{1b}$$

and its reference model to be followed, represented by

$$\dot{x}_m = A_m x_m + B_m u_m \tag{2a}$$
$$y_m = C_m x_m + D_m u_m \tag{2b}$$

where $x_p \in R^n$, y_p, $u_p \in R^m$, $x_m \in R^{nm}$, y_m, $u_m \in R^m$.

According to the CGT theory, when the plant's outputs track the reference model's outputs perfectly, the plant can be represented by the following expressions:

$$x_p^* = A_p x_p^* + B_p u_p^* \tag{3a}$$
$$y_p^* = C_p x_p^* + D_p u_p^* = y_m \tag{3b}$$

Here x_p^*, u_p^*, y_p^* denote the ideal state, control input and output, respectively. In this situation, the result of the CGT shows that the ideal trajectories x_p^* and u_p^* are given by the next equations.

$$\begin{bmatrix} x_p^* \\ u_p^* \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} x_m \\ u_m \end{Bmatrix} + \begin{bmatrix} \Omega_{11} \\ \Omega_{21} \end{bmatrix} v \tag{4}$$

$$\Omega_{11} \dot{v} = v - S_{12} \dot{u}_m, \quad v \in R^n \tag{5}$$

Eq. (5) is a feedforward dynamic compensator whose poles are equal to the inverses of the plant's zeros (hereafter, we refer to the feedforward dynamic compensator as the inverse dynamics). The coefficient matrices in Eqs. (4) and (5) are given by Eqs. (6) and (7).

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = \begin{bmatrix} A_p & B_p \\ C_p & D_p \end{bmatrix}^{-1} \tag{6}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \begin{bmatrix} S_{11} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_m & B_m \\ C_m & D_m \end{bmatrix} \tag{7}$$

In the previous work [5], the author introduced a new variable z in order to avoid using the first derivative of u_m. That is, the second equation in Eq. (4) and Eq. (5) are rewritten as follows:

$$u_p^* = K_x^* x_m + K_u^* u_m + K_v^* v \tag{8a}$$
$$K_x^* = S_{21}, \; K_u^* = S_{22}, \; K_v^* = \Omega_{21} \tag{8b}$$

$$\begin{cases} \dot{z}^* = v^* & (9a)\\ v^* = \Omega_{11}^{-1} z^* - \Omega_{11}^{-1} S_{12} u_m . & (9b)\end{cases}$$

3. INVERSE DYNAMICS

In Eq. (9) or Eq. (5), Ω_{11}^{-1} is required to be a stable matrix. By the matrix inversion lemma, the submatrix Ω_{11} in Eq. (6) can be expanded as follows:

$$\begin{aligned}\Omega_{11} &= \left(A_p - B_p D_p^{-1} C_p\right)^{-1} \\ &= A_p^{-1} + A_p^{-1} B_p\left(D_p - C_p A_p^{-1} B_p\right)^{-1} C_p A_p^{-1}.\end{aligned} \quad (10)$$

The determinant can be represented as

$$|\Omega_{11}| = \left|A_p^{-1}\right| \cdot \left|I_m + \left(D_p - C_p A_p^{-1} B_p\right)^{-1} C_p A_p^{-1} B_p\right|. \quad (11)$$

This implies that if $D_p = 0$, then $|\Omega_{11}| = 0$; Ω_{11} becomes singular.

Next, the invariant zeros of the plant are defined by the following system equation.

$$\begin{aligned}&\begin{vmatrix} -(sI - A_p) & B_p \\ C_p & D_p \end{vmatrix} \\ &= (-1)^n |sI - A_p| \left| D_p + C_p (sI - A_p)^{-1} B_p \right| = 0.\end{aligned} \quad (12)$$

Also, the characteristic equation of Ω_{11}^{-1} is expanded as follows:

$$\begin{aligned}&\left|sI - \Omega_{11}^{-1}\right| = \\ &|sI - A_p| \left| D_p + C_p (sI - A_p)^{-1} B_p \right| \left| D_p^{-1} \right| = 0.\end{aligned} \quad (13)$$

These two equations imply that the eigenvalues of Ω_{11}^{-1} are equal to the invariant zeros of the plant, i.e., the plant is required to be a minimum phase system.

4. ADAPTIVE CGT DESIGN FOR LFSS CONTROL

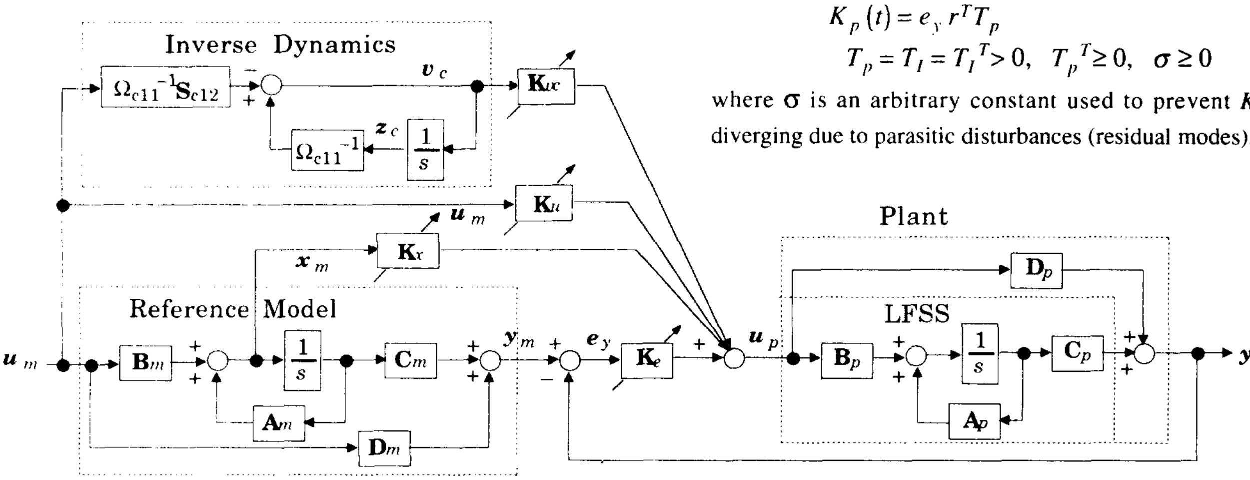

Fig. 1. Block diagram of adaptive CGT.

As described previously, the SAC neglects the third term in Eq. (8) and the existence of Eq. (9) by restricting the command signal to a constant signal [1-3]. However, even if the step input is constant during the control, at the instant when the step signal is switched from zero to one, the signal v^* should give some transient effects to the plant in order to track the reference model perfectly. Therefore, neglect of this transient effect makes it impossible for the SAC to achieve perfect model tracking [5].

In this section, we attempt to increase the tracking accuracy by partially reviving the inverse dynamics.

4.1 Adaptive control system design

Let us define a control input by the following:

$$u_p = K(t) r. \quad (14a)$$

Here $K(t)$ and r are as follows:

$$K(t) = [K_e(t)\; K_x(t)\; K_u(t)\; K_{vc}(t)] \quad (14b)$$

$$r = [e_y\; x_m\; u_m\; v_c]^T. \quad (14c)$$

This differs from the case of SAC in that the fourth elements are added in Eqs. (14b-c). Here v_c is generated by

$$\begin{cases} \dot{z}_c = v_c, \quad z_c \in R^{n_c} & (15a)\\ v_c = \Omega_{c11}^{-1} z_c - \Omega_{c11}^{-1} S_{c12} u_m & (15b)\end{cases}$$

Eq. (15) is a reduced-order reverse dynamics, and Ω_{c11} and S_{c12} are determined by the following equations which are similar to Eqs. (6) and (7).

$$\Omega_c = \begin{bmatrix} \Omega_{c11} & \Omega_{c12} \\ \Omega_{c21} & \Omega_{c22} \end{bmatrix} = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix}^{-1} \quad (16)$$

$$\begin{bmatrix} S_{c11} & S_{c12} \\ S_{c21} & S_{c22} \end{bmatrix} = \begin{bmatrix} \Omega_{c11} & \Omega_{c12} \\ \Omega_{c21} & \Omega_{c22} \end{bmatrix} \begin{bmatrix} S_{c11} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_m & B_m \\ C_m & D_m \end{bmatrix} \quad (17)$$

where A_c, B_c, C_c, D_c are nominal values of a reduced plant of order $n_c < n$, which consists of controlled modes only.

The adaptive gain matrix, $K(t)$, in Eq. (14) is adjusted by the following adaptive algorithm.

$$K(t) = K_I(t) + K_p(t) \quad (18a)$$

$$\dot{K}_I(t) = -\sigma K_I(t) + e_y r^T T_I \quad (18b)$$

$$K_p(t) = e_y r^T T_p \quad (18c)$$

$$T_p = T_I = T_I^T > 0, \quad T_p^T \geq 0, \quad \sigma \geq 0$$

where σ is an arbitrary constant used to prevent $K_I(t)$ from diverging due to parasitic disturbances (residual modes).

4.2 Stability analysis of adaptive control system

Let us define the error signals from the ideal values for the output, the state and the input as follows:

$$\begin{aligned} e_y &= y_m - y_p = y_p^* - y_p \\ e_x &= x_p^* - x_p \\ e_u &= u_p^* - u_p . \end{aligned} \quad (19)$$

By these definitions, $\dot{e}_x$ and e_y can be described in the following form using Eqs. (1,3,8,14)

$$\dot{e}_x = \tilde{A}_p e_x + \tilde{B}_p \Delta K r + f_x \quad (20a)$$
$$e_y = \tilde{C}_p e_x + \tilde{D}_p \Delta K r + f_y \quad (20b)$$
$$\Delta K = K^* - K(t) \quad (20c)$$
$$f_x = \tilde{B}_p f_u \quad (20d)$$
$$f_y = \tilde{D}_p f_u \quad (20e)$$
$$f_u = K_{vc}^* (v_c^* - v_c) + K_{vr}^* v_r^* \quad (20f)$$

Here v_c^*, v_r^*, corresponding to the controlled modes and the residual modes, respectively, are the ideal outputs of the inverse dynamics of a nominal plant, and K_{vc}^*, K_{vr}^* are the ideal gains corresponding to them.

Also, $(\tilde{A}_p, \tilde{B}_p, \tilde{C}_p, \tilde{D}_p)$ in Eqs. (20a,b) is a closed-loop system obtained by applying feedback with an arbitrary constant gain K_e^* to the plant (A_p, B_p, C_p, D_p), and is described by the following:

$$\begin{cases} \tilde{A}_p = A_p - B_p K_e^* (1 + D_p K_e^*)^{-1} C_p \\ \tilde{B}_p = B_p \{1 - K_e^* (1 + D_p K_e^*)^{-1} D_p\} \\ \tilde{C}_p = (1 + D_p K_e^*)^{-1} C_p \\ \tilde{D}_p = (1 + D_p K_e^*)^{-1} D_p . \end{cases} \quad (21)$$

In order to prove the stability, let us consider the following equation as a candidate for a Lyapunov function.

$$V = \frac{1}{2} e_x^T P e_x + \frac{1}{2} tr[\{K^* - K_I(t)\} T_I^{-1} \{K^* - K_I(t)\}^T] \quad (22)$$

where K^* is composed of the ideal gains

$$K^* = [K_e^* \; K_x^* \; K_u^* \; K_v^*]. \quad (23)$$

Differentiating V with respect to time and using Eqs. (18) and (20) we obtain

$$\begin{aligned} \dot{V} = & \frac{1}{2} e_x^T (\tilde{A}_p{}^T P + P \tilde{A}_p) e_x + e_x^T (P \tilde{B}_p - \tilde{C}_p{}^T) \Delta K r \\ & - (\Delta K r)^T \tilde{D}_p{}^T (\Delta K r) - tr [e_y r^T T_p r e_y^T] \\ & - \sigma tr [(K^* - K_I(t)) (K^* - K_I(t))^T] \\ & + \sigma tr [(K^* - K_I(t)) K^{*T}] - f_y{}^T \Delta K r + f_x{}^T P e_x . \end{aligned} \quad (24)$$

Here, if the closed-loop system $(\tilde{A}_p, \tilde{B}_p, \tilde{C}_p, \tilde{D}_p)$ satisfies the strictly positive real (SPR) conditions, the following Kalman-Yakubovich lemma should hold.

$$\begin{cases} \tilde{A}_p{}^T P + P \tilde{A}_p = -L L^T - Q \\ P \tilde{B}_p - \tilde{C}_p{}^T = -L W \\ \tilde{D}_p{}^T + \tilde{D}_p = W^T W \end{cases} \quad (25)$$

Thus, $\dot{V}$ can be rewritten as

$$\begin{aligned} \dot{V} = & -\frac{1}{2} (L^T e_x + W \Delta K r)^T (L^T e_x + W \Delta K r) \\ & - tr [e_y r^T T_p r e_y^T] - \frac{1}{2} e_x^T Q e_x \\ & - \sigma tr [(K^* - K_I(t)) T_I^{-1} (K^* - K_I(t))^T] \\ & + \sigma tr [(K^* - K_I(t)) T_I^{-1} K^{*T}] \\ & - f_u{}^T \tilde{D}_p^T \Delta K r + e_x{}^T P \tilde{B}_p f_u . \end{aligned} \quad (26)$$

From the above equation, $\dot{V} < 0$ can be guaranteed in a region where $\|e_x\|, \|e_y\|, \|\Delta K r\|, \|\Delta K\|$ are large enough. Thus it is shown that if a plant satisfies the SPR conditions with an output feedback, the present adaptive control system is stable.

5. NUMERICAL SIMULATION

5.1 Flexible spacecraft model

As a controlled plant, a flexible spacecraft (6 modes in total) composed of a rigid body and flexible components (assumed to include elastic vibration modes up to the 5th mode) was considered. In order to satisfy the almost strict positive real (ASPR) condition [3], a position and velocity sensor were assumed to be collocated with an actuator at the central rigid body.

$$\dot{x}_p = \begin{bmatrix} 0_6 & I_6 \\ -\Omega^2 & -2\zeta\Omega^2 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ \Phi^T b \end{bmatrix} u_p \quad (27a)$$

$$y_p = [1 \;\; \alpha] \begin{bmatrix} b^T \Phi & 0 \\ 0 & b^T \Phi \end{bmatrix} x_p + D_p u_p \quad (27b)$$

$\Omega = \mathrm{diag}\,[0,\ 0.1794,\ 1.854,\ 5.970,\ 12.45,\ 21.28]$
$b^T \Phi = [0.0162, -0.0224, -0.0035, -0.0014, -0.0008, -0.0006]$
$\zeta = 0.005$, $\alpha = 100$, $D_p = 0.5$

5.2 Reference model

The following 2-order model was adopted:

$$\dot{x}_m = \begin{bmatrix} 0 & 0 \\ -0.04^2 & -2 \times 0.9 \times 0.04^2 \end{bmatrix} x_m + \begin{bmatrix} 0 \\ 0.04^2 \end{bmatrix} u_m \quad (28a)$$

$$y_m = [1 \;\; 100]\, x_m \quad (28b)$$

5.3 Simulation results

A slewing maneuver of three degrees was performed. In the simulation, all modal frequencies and damping ratios were uniformly changed by 1/2 and 1/10 of their normal values, respectively. The coefficient matrices adopted in the adjustment rule were $T_I = T_p = I$ in both Figs. 3 and 4.

Figs. 3 and 4 are comparisons of the tracking property between SAC and the present adaptive CGT. As shown in Fig. 3, it was difficult for the SAC to follow the reference model. However, as shown in Fig. 4, model tracking was achieved by using the signal v

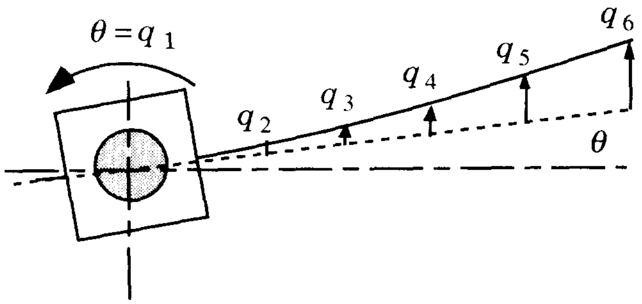

Fig. 2. Rigid central body with flexible appendages.

generated by the added inverse dynamics. Also the attitude angle of the central rigid body and the displacement at the tip of the flexible appendages, which were the original objectives of the control, were well damped.

6. SUMMARY

The SAC owes its simplicity to abandoning accuracy of perfect model tracking. For better model tracking, the inverse dynamics is necessary.

In this paper, an extended SAC was presented which can improve tracking properties of the SAC. The proposed system is obtained by adding the reduced-order inverse dynamics corresponding to the invariant zeros of the plant to the SAC. The effect of this addition was shown in the simulation for a LFSS attitude control.

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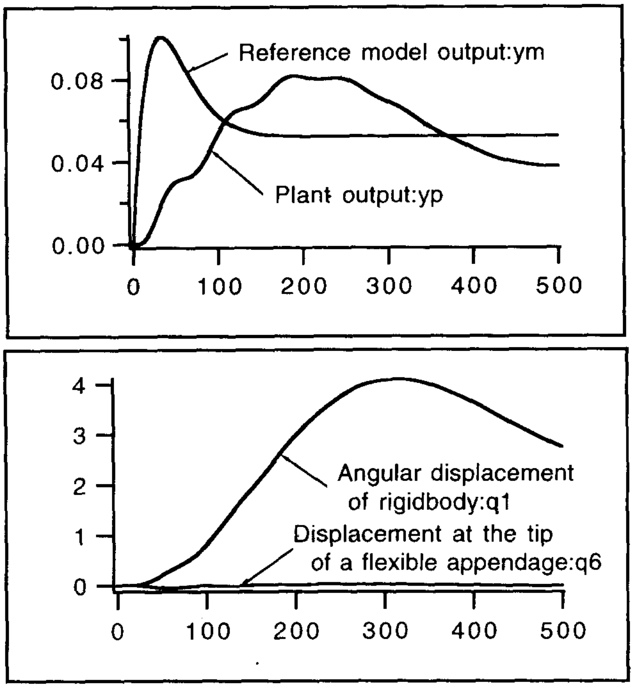

Fig. 3. SAC step responses.

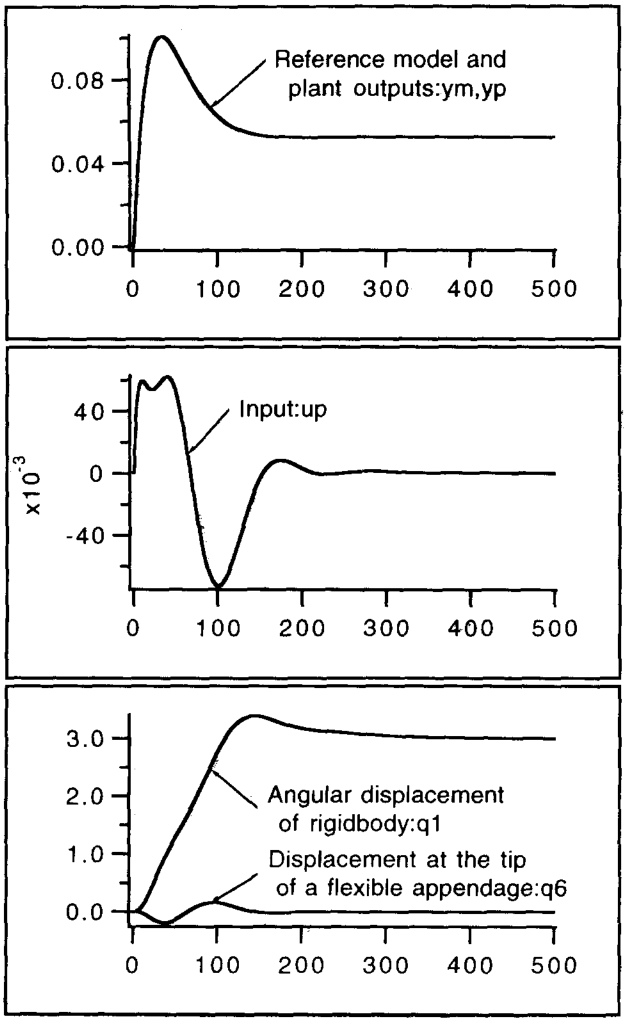

Fig. 4. Adaptive CGT step responses.