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Robust Design Scheme of VS-MRC to Time–Varying Plant

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Abstracts In this paper, we propose a new rubust design scheme of a variable structure type model reference control(VS–MRC) which can be applied to linear time–varing plants. Our idea is started from the hypothesis that the plant consists of two parts, i.e., one has time–invariant parameters and the other has time–varying parameters. We consider the former the nominal part of the plant and the latter a kind of disturbance to the nominal one. In this design scheme, the ordinary VS–MRC is adopted to the nominal part and the signum function is introduced to eliminate the influence of the disturbance.

Keywords Time–Varying, Robust, Variable structure, Model reference control

1.INTRODUCTION

Most of reseraches on the adaptive control of linear time–varying(LTV) plants have been usually taken under some assumptions on the plant parameter variations([1]~[6]). In the case of arbitrarily fast time–varying plant, it seems difficult to guarantee the stability for conventional adaptive control methods. To resolve this problem, a non–adaptive new standard model reference control(NSMRC) method was proposed([7]). In this NSMRC scheme, it becomes possible to control the arbitrarily fast time–varying plants with unknown parameters using the concept of variable structure(VS) design. However, it is not obvious how to choose the design parameters β_0 and β_1 which guarantee the stability of the closed loop system.

In this paper, we propose a new robust design scheme of a variable–structure type model reference control(VS–MRC) which can be applied to LTV plants.

Our idea is started from the hypothesis that the plant consists of two parts, i.e., one has time–invariant parameters and the other has time–varying parameters. We consider the former the nominal part of the plant and the latter a kind of disturbance to the nominal one. In this design scheme, the ordinary VS–MRC is adopted to the nominal part and the signum function is introduced to eliminate the influence of the disturbance.

2.STATEMENT OF THE PROBLEM

Consider a single–input single–output(SISO) linear time–varying plant described as

$$A(s,t)y(t) = B(s,t)u(t) \tag{1}$$

where

$$\begin{aligned} A(s,t) &= A(s) + \Delta A(s,t) \\ B(s,t) &= B(s) + \Delta B(s,t) \\ A(s) &= s^n + \sum_{i=1}^{n} s^{n-i} a_i \\ B(s) &= \sum_{i=0}^{n-1} s^{n-1-i} b_i \\ \Delta A(s,t) &= \sum_{i=1}^{n} s^{n-i} \Delta a_i(t) \\ \Delta B(s,t) &= \sum_{i=1}^{n-1} s^{n-1-i} \Delta b_i(t) \end{aligned} \tag{2}$$

In (1), u(t) and y(t) are the input and the output of the plant, respectively. The following assumptions are made regarding the plant.

A1) The degree n is known.

A2) $|\Delta a_i(t)|$ and $|\Delta b_i(t)|$ are bounded and upper bounds of them are known.

A3) $A(s,t)$ and $B(s,t)$ are strongly right–coprime polynomial differential operators(PDO).

A4) $B^{-1}(s,t)$ is an exponentially stable polynomial integral operators(PIO).

A5) The sign of b_0 is known. Without loss of generallity the sign of b_0 is assumed positive for all $t \geq 0$.

Equation (1) can be rewritten as

$$A(s)y(t) = B(s)u(t) - \Delta A(s,t)y(t) + \Delta B(s,t)u(t) \tag{3}$$

Consider a reference model described by

$$A_M(s)y_M(t) = B_M(s)r(t) \tag{4}$$

where

$$A_M(s) = s^n + \sum_{i=1}^{n} a_{Mi} s^{n-i}$$
$$B_M(s) = \sum_{i=0}^{n-1} b_{Mi} s^{n-1-i} \quad (5)$$

In (4), $r(t)$ and $y_M(t)$ are bounded input and output of the reference model, respectively, and $A_M(s)$ is a Hurwitz polynomial.

The problem considered here is to construct MRCS where the plant output $y(t)$ will follow the model output $y_M(t)$ as closely as possible and consider the robustness of MRCS to variation of the plant parameters.

3.CONSTRUCTION OF MRCS

Introduce the following Hurwitz polynomials described as

$$P(s) = (s+\lambda)F(s)$$
$$F(s) = (s+f)^{n-1} = s^{n-1} + \sum_{i=1}^{n-1} f_i s^{n-1-i} \quad (6)$$

Using the above equations, (3) can be rewritten as

$$P(s)y(t) = B(s)u(t) + (P(s) - A(s))y(t) - \Delta A(s,t)y(t) + \Delta B(s,t)u(t) \quad (7)$$

Dividing both sides of (7) by F(s), we have

$$(s+\lambda)y(t) = b_0[u(t) + \overline{\theta}^T \overline{\xi}(t) + g(t)] \quad (8)$$

where

$$\overline{\xi}(t) = [\xi_1(t), \xi_2(t), \cdots, \xi_{2n-1}(t)]^T$$
$$\xi_i(t) = \frac{s^{n-i}}{F(s)} u(t), \xi_{n+i}(t) = \frac{s^{n-i}}{F(s)} y(t); \quad i = 1 \sim n-1 \quad (9)$$
$$\xi_n(t) = y(t)$$
$$g(t) = \frac{1}{b_0} \frac{1}{F(s)} [-\Delta A(s,t)y(t) + \Delta B(s,t)u(t)] \quad (10)$$

$\overline{\theta}$:unknown parameter vector.

Defining $e(t)$ as

$$e(t) \triangleq y(t) - y_M(t), \quad (11)$$

then from (8), we have

$$(s+\lambda)e(t) = b_0[u(t) + \theta^T \xi(t) + g(t)] \quad (12)$$

where

$$\theta = [\overline{\theta}^T, -\frac{1}{b_0}]^T$$
$$\xi(t) = [\overline{\xi}^T(t), y_{M\lambda}(t)]^T \quad (13)$$
$$y_{M\lambda}(t) = (s+\lambda)y_M(t)$$

Introduce an adjustable parameter vector $\hat{\theta(t)}$ defined as

$$\hat{\theta}(t) \triangleq [\hat{\theta}_1(t), \cdots, \hat{\theta}_{2n}(t)]^T \quad (14)$$

and synthesize the control input $u(t)$ as

$$u(t) = -\hat{\theta}(t)^T \xi(t) - g_s(t) \cdot agn(e(t)). \quad (15)$$

The adjustable parameter $\hat{\theta}_i(t)$ is adjusted by

$$\dot{\hat{\theta}}_i = \theta_{nrm} \cdot sgn(\xi_i(t)) \cdot sgn(e(t)); i = 1 \sim 2n \quad (16)$$

where

$$\theta_{nrm} > ||\theta|| . \quad (17)$$

The auxiliary signal $g_s(t)$ of (15) is synthesized as

$$g_s(t) = g_{s0}(t) + g_{s1}(t) + g_{s2}(t) \quad (18)$$

where

$$g_{s0}(t) = \delta_{s0}|y(t)| \quad (19)$$
$$\delta_{s0} = \sup_{\tau \le t} |\frac{\Delta a_1(\tau)}{b_0}| \quad (20)$$
$$g_{s1}(t) = \sum_{i=1}^{n-1} \frac{(s+f)^{n-1-i}}{F(s)} \delta_{s1}|y(t)| \quad (21)$$
$$\delta_{s1} = \sum_{i=1}^{n-1} \sup_{\tau \le t} |h_{1i}(\tau)| \quad (22)$$
$$g_{s2}(t) = \sum_{i=1}^{n-1} \frac{(s+f)^{n-1-i}}{F(s)} \delta_{s2}|u(t)| \quad (23)$$
$$\delta_{s2} = \sum_{i=1}^{n-1} \sup_{\tau \le t} |h_{2i}(\tau)| \quad (24)$$

4.ROBUSTNESS ANALYSIS

In this section, we consider the robustness of this design scheme. At first, (10) can be rewritten as

$$g(t) = g_0(t) + g_1(t) + g_2(t) \quad (25)$$

where

$$g_0(t) = -\frac{\Delta a_1(t)}{b_0} y(t) \quad (26)$$
$$g_1(t) = \sum_{i=1}^{n-1} \frac{s^{n-1-i}}{F(s)} (f_i \frac{\Delta a_i(t)}{b_0} - \frac{\Delta a_{i+1}(t)}{b_0}) y(t) \quad (27)$$
$$g_2(t) = \sum_{i=1}^{n-1} \frac{s^{n-1-i}}{F(s)} \frac{\Delta b_i(t)}{b_0} u(t) \quad (28)$$

(27) can be expressed as the following state equation.

$$\dot{\boldsymbol{x}}_1(t) = \boldsymbol{F}_1 \boldsymbol{x}_1(t) + \boldsymbol{h}_1(t) y(t)$$
$$g_1(t) = \boldsymbol{C}_1^T \boldsymbol{x}_1(t) \quad (29)$$

where

$$F_1 = \begin{bmatrix} -f & 1 & 0 & \cdots & 0 \\ 0 & -f & 1 & & \vdots \\ 0 & 0 & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & 1 \\ 0 & 0 & \cdots & 0 & -f \end{bmatrix},$$

$$x_1(t) = \begin{bmatrix} x_{11}(t) \\ \vdots \\ x_{1n-1}(t) \end{bmatrix}, \qquad (30)$$

$$h_1(t) = \begin{bmatrix} h_{11}(t) \\ \vdots \\ h_{1n-1}(t) \end{bmatrix}, C_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

(28) can be expressed as

$$\begin{aligned} \dot{x}_2(t) &= F_1 x_2(t) + h_2(t)u(t) \\ y_2(t) &= C_1^T x_2(t) \end{aligned} \qquad (31)$$

where

$$x_2(t) = \begin{bmatrix} x_{21}(t) \\ \vdots \\ x_{2n-1}(t) \end{bmatrix}, h_2(t) = \begin{bmatrix} h_{21}(t) \\ \vdots \\ h_{2n-1}(t) \end{bmatrix} \qquad (32)$$

On the other hand, (21) can be rewritten as

$$\begin{aligned} \dot{\eta}_1(t) &= F_1 \eta_1(t) + \overline{h}_1 \delta_{s1} |y(t)| \\ g_{s1}(t) &= C_1^T \eta_1(t) \end{aligned} \qquad (33)$$

where

$$\eta_1(t) = \begin{bmatrix} \eta_{11}(t) \\ \vdots \\ \eta_{1n-1}(t) \end{bmatrix}, \overline{h}_1 = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}, \qquad (34)$$

(23) can be also expressed as

$$\begin{aligned} \dot{\eta}_2(t) &= F_1 \eta_2(t) + \overline{h}_1 \delta_{s2} |u(t)| \\ g_{s2}(t) &= C_1^T \eta_2(t) \end{aligned} \qquad (35)$$

where

$$\eta_1(t) = \begin{bmatrix} \eta_{21}(t) \\ \vdots \\ \eta_{2n-1}(t) \end{bmatrix} \qquad (36)$$

The following lemma will state a condition on the auxiliary signal $g_s(t)$ so that this design scheme is kept robust in the presense of a disturbance $g(t)$.

[Lemma] With regard to $g(t)$ of (10) and $g_s(t)$ of (18), if $|x_{1i}(0)| \leq \eta_{1i}(0)$ and $|x_{2i}(0)| \leq \eta_{2i}(0) (i = 1 \sim n-1)$, the following inequality is formed.

$$|g(t)| \leq g_s(t) \qquad (37)$$

[Proof] See Appendix.

Under the above preparations, we have the following theorem.

[Theorem] In the above mentioned design scheme of MRCS, boundness of the internal signals and $e(t) \to 0$ as $t \to \infty$ are guaranteed.

[Proof] From (12) and (15), we have

$$\begin{aligned} \dot{e}(t) &= -\lambda e(t) + b_0[\theta^T \xi(t) - \hat{\theta}^T(t)\xi(t) \\ &\quad + g(t) - g_s(t) \cdot sgn(e(t))] \\ &= -\lambda e(t) + b_0 \sum_{i=1}^{2n} (\theta_i - \hat{\theta}_i(t))\xi_i(t) \\ &\quad + b_0[g(t) - g_s(t) \cdot sgn(e(t))] \end{aligned} \qquad (38)$$

Consider the positive definite function defined as

$$V(t) = \frac{1}{2} e^2(t) \qquad (39)$$

Take the time derivative of $V(t)$ and substitute (16) and (38) in it, then we have

$$\begin{aligned} \dot{V}(t) &\leq -\lambda e^2(t) - b_0 \sum_{i=1}^{2n} \theta_{nrm} |\xi_i(t)||e(t)| \\ &\quad + b_0 ||\theta|| \sum_{i=1}^{2n} |\xi_i(t)||e(t)| + b_0 |g(t)|(t)| \\ &\quad - b_0 g_s(t)|e(t)| \\ &\leq -\lambda e^2(t) - b_0(g_s(t) - |g(t)|)|e(t)| \end{aligned} \qquad (40)$$

where

$$\theta_{nrm} \geq ||\theta|| \qquad (41)$$

Using (37) in (40), then we have

$$\dot{V}(t) \leq -\lambda e^2(t) \qquad (42)$$

From (39) and (42), it follows that $V(t)$ is a monotonic non-increasing function. Therefore, if $V(0)$ is bounded, i.e., $e(0)$ is bounded, then boundedness of $e(t)$ is guaranteed. From the assumption A4) and (1), the boundedness of $u(t)$ is assured. From (9) and (13), we obtain that $\xi(t)$ is also bounded. Therefore, boundedness of the internal signals is guaranteed. Furthermore, we can lead the boundedness of $\dot{e}(t)$ in (38), then we conclude that $e(t)$ tends to 0 asymptotically.

5.CONCLUSION

In this note, a new design scheme of VS type MRCS which is robust for the time-varying plant has been proposed.

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< APPENDIX > Proof of Lemma

At first, from (25), we have that

$$|g(t)| \leq |g_0(t)| + |g_1(t)| + |g_2(t)| \qquad \text{(A1)}$$

From (26) and (19)

$$\begin{aligned} |g_0(t)| &\leq \sup_{\tau\leq t} \left|\frac{\Delta a_1(t)}{b_0}\right| |y(t)| \\ &\leq g_{s0}(t) \end{aligned} \qquad \text{(A2)}$$

The solution of (29) can be written as

$$\begin{aligned} g_1(t) &= \boldsymbol{C}_1^T e^{\boldsymbol{F}_1 t}\boldsymbol{x}_1(0) \\ &+ \int_0^t \boldsymbol{C}_1^T e^{\boldsymbol{F}_1(t-t')}\boldsymbol{h}_1(t')y(t')dt' \end{aligned} \qquad \text{(A3)}$$

where

$$e^{\boldsymbol{F}_1 t} = \begin{bmatrix} e^{-ft} & te^{-ft} & \cdots & \frac{t^{\mu-1}}{\mu!}e^{-ft} \\ 0 & e^{-ft} & & \\ \vdots & & \ddots & \\ 0 & \cdots & & e^{-ft} \end{bmatrix} \qquad \text{(A4)}$$

From (A3), we have that

$$\begin{aligned} |g_1(t)| &\leq \boldsymbol{C}_1^T e^{\boldsymbol{F}_1 t}\overline{\boldsymbol{x}}_1(0) \\ &+ \sup_{\tau\leq t}\|\boldsymbol{h}_1(\tau)\| \int_0^t \left\|\boldsymbol{C}_1^T e^{\boldsymbol{F}_1(t-t')}\right\| |y(t')|dt' \end{aligned} \qquad \text{(A5)}$$

where

$$\begin{aligned} \boldsymbol{C}_1^T e^{\boldsymbol{F}_1(t-t')} &= [e^{-f(t-t')}, (t-t')e^{-f(t-t')}, \\ &\quad \cdots, \frac{(t-t')^{\mu-1}}{\mu!}e^{-f(t-t')}]^T \\ \overline{\boldsymbol{x}}_1(0) &= [|x_{11}(0)|, \cdots, |x_{1n-1}(0)|]^T \end{aligned} \qquad \text{(A6)}$$

On the other hand, the solution of (33) can be expressed as

$$\begin{aligned} g_{s1}(t) &= \boldsymbol{C}_1^T e^{\boldsymbol{F}_1 t}\boldsymbol{\eta}_1(0) \\ &+ \delta_{s1}\int_0^t \boldsymbol{C}_1^T e^{\boldsymbol{F}_1(t-t')}\overline{\boldsymbol{h}}_1|y(t')|dt' \end{aligned} \qquad \text{(A7)}$$

where

$$\begin{aligned} \boldsymbol{C}_1^T e^{\boldsymbol{F}_1(t-t')}\overline{\boldsymbol{h}}_1 &= e^{-f(t-t')} + (t-t')e^{-f(t-t')} + \\ &\quad \cdots + \frac{(t-t')^{\mu-1}}{\mu!}e^{-f(t-t')} \end{aligned} \qquad \text{(A8)}$$

The following relationship can be easily derived.

$$\left\|\boldsymbol{C}_1^T e^{\boldsymbol{F}_1(t-t')}\right\| \leq \boldsymbol{C}_1^T e^{\boldsymbol{F}_1(t-t')}\overline{\boldsymbol{h}}_1 \qquad \text{(A9)}$$

Furthermore, from (22), we have that

$$\delta_{s1} = \sum_{i=1}^{n-1}\sup_{\tau\leq t}|h_{1i}(\tau)| \geq \sup_{\tau\leq t}\|\boldsymbol{h}_1(\tau)\| \qquad \text{(A10)}$$

Therefore, under the conditions of $|x_{1i}(0)| \leq \eta_{1i}(0)$ ($i = 1 \sim n-1$), the following inequality can be derived from (A5) and (A7).

$$|g_1(t)| \leq g_{s1}(t) \qquad \text{(A11)}$$

Similary, the solusion of (31) can be expressed as

$$\begin{aligned} g_2(t) &= \boldsymbol{C}_1^T e^{\boldsymbol{F}_1 t}\boldsymbol{x}_2(0) \\ &+ \int_0^t \boldsymbol{C}_1^T e^{\boldsymbol{F}_1(t-t')}\boldsymbol{h}_2(t')u(t')dt' \end{aligned} \qquad \text{(A12)}$$

The solution of (33) can be written as

$$\begin{aligned} g_{s_2}(t) &= \boldsymbol{C}_1^T e^{\boldsymbol{F}_1 t}\boldsymbol{\eta}_2(0) \\ &+ \int_0^t \boldsymbol{C}_1^T e^{\boldsymbol{F}_1(t-t')}\overline{\boldsymbol{h}}_1\delta_{s_2}u(t')dt' \end{aligned} \qquad \text{(A13)}$$

Therefore, in the same way of the above discussion, under the conditions of $|x_{2i}(0)| \leq \eta_{2i}(0)$ ($i = 1 \sim n-1$), the following inequality can be derived from (A12) and (A13).

$$|g_2(t)| \leq g_{s2}(t) \qquad \text{(A14)}$$

Substituting (A2), (A11) and (A14) into (A1), then we conclude that

$$\begin{aligned} |g(t)| &\leq g_{s0}(t) + g_{s1}(t) + g_{s2}(t) \\ &= g_s(t) \end{aligned} \qquad \text{(A15)}$$