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ATTITUDE DETERMINATION FOR THREE-AXIS STABILIZED SATELLITE

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Abstract This paper presents the on-board attitude determination algorithm for LEO(Low Earth Orbit) three-axis stabilized spacecraft. Two advanced star trackers and a three-axis Inertial Reference Unit(IRU) are assumed to be attitude sensors.
The gyro in the IRU provides a direct measurement of the attitude rates. However, the attitude estimation error increases with time due to the gyro drift and noise. An update filter with measurements of star trackers and/or sun sensor is designed to update these gyro drift bias and to compensate the attitude error. Kalman Filter is adapted for the on-board update filter algorithm. Simulation results will be presented to investigate the attitude pointing performance.

Keywords Attitude determination, Three-axis stabilized, Gyro drift, Star tracker, Kalman filter

1 Introduction

For many Earth oriented application satellites, high pointing accuracy is key element for the success of mission. High pointing accuracy requires precision attitude control and determination system.

Many communication satellites in geo - stationary orbit employ the method ofmomentum biased three-axis control system. And, near Earth application satellites for Earth resources, weather mapping and defence employ zero momentum control system.[7] For LEO satellites, Inertial Reference Unit (IRU) is employed for the attitude determination system because it can provide an excellent attitude information. However, the gyro drift deteriorates the gyro output in long term operation. Star trackers or sun sensor measurements are used to update satellites' attitude errors and gyro biases for that reason. Kalman Filter is used to combine the two measurement data from different sources with a statistically optimum manner.[1][2]

Related with the Kalman Filtering, there are three ways to represent the attitude of satellite. Early strapdown systems adapted Direction-Cosine Matrix (DCM) but it has not been widely used recently for large computational time. The three-parameter Euler-Angle representation was used in several early applications of Kalman Filtering for attitude estimation.[1] However, the kinematic equations for Euler angles involve nonlinear and computationally expensive trigonometric functions, and the angles become undefined for some rotations. (the gimbal-lock situation) From early 80's, Quaternion, the global nonsingular four parameter was introduced in the attitude determination application. It is more efficient for computation of rotating spacecraft than DCM. Quaternion is widely used to determine attitude of LEO satellites. [3][4][5][6]

2 Satellite Kinematics

• Kinematic Integration Module

The attitude computation are contained in two modules, kinematic integration and attitude estimation. The kinematic equation for updating a quaternion is

$$q(t_{n+1}) = \left\{ \cos(\frac{\omega T}{2})I + \frac{1}{\omega}\sin(\frac{\omega T}{2}) \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix} \right\} q(t_n) \quad (1)$$

where

- ω_i ; Angular rate of the spacecraft in the body frame w.r.t. the ECI frame
- ω ; $(\omega_x{}^2 + \omega_y{}^2 + \omega_z{}^2)^{1/2}$
- T ; Time interval

Since the gyro output consists of three angles $\theta_x, \theta_y, \theta_z$ the following substitutions can be made;

$$\theta_x = \omega_x T \quad : \quad \theta_y = \omega_y T \quad : \quad \theta_z = \omega_z T$$

Then, **Eq. 1** becomes

$$q(t_{n+1}) = \left\{ \cos(\frac{\theta}{2})I + \frac{1}{\theta}\sin(\frac{\theta}{2}) \begin{bmatrix} 0 & \theta_z & -\theta_y & \theta_x \\ -\theta_z & 0 & \theta_x & \theta_y \\ \theta_y & -\theta_x & 0 & \theta_z \\ -\theta_x & -\theta_y & -\theta_z & 0 \end{bmatrix} \right\} q(t_n) \quad (2)$$

where

$$\theta = (\theta_x{}^2 + \theta_x{}^2 + \theta_x{}^2)^{1/2}$$

Since the error quaternion consists of four parameters, the number of estimation variables are four. However, for slow rotation, coupling effects among these four parameters (i.e. the norm must equal to unity), are negligible. Consequently, only vector part of the attitude error quaternion is sufficient for the estimation.

3 Attitude Error Model

- Gyro Measurement

Gyro dynamic model can be expressed with the following equation.

$$\begin{aligned} \dot{\vec{\theta}} &= \vec{\omega} + \vec{b}_o + \vec{b} - \vec{n}_v \\ \dot{\vec{b}} &= \vec{n}_u \end{aligned}$$

where

- $\dot{\vec{\theta}}$; Gyro rate measurement
- $\vec{\omega}$; True spacecraft rate
- $\vec{b}_o$; Gyro bias error
- $\vec{b}$; Gyro random walk error
- $\vec{n}_v$; Float torque noise (Gaussian white)
- $\vec{n}_u$; Float torque derivative noise (Gaussian white)

- Gyro Drift Error

$$\begin{aligned} \dot{\vec{e}} &= \vec{\omega} - \dot{\vec{\theta}} \\ \dot{\vec{e}} &= -\vec{b}_o - \vec{b} + \vec{n}_v \end{aligned}$$

If the gyro bias error is assumed to be known, it can be corrected.

- Attitude Error

$$\dot{\Psi} + \vec{\omega} \times \Psi = \dot{\vec{e}} \quad (3)$$

- Attitude Error Dynamic Model

$$\begin{aligned} \dot{\Psi} &= -\vec{\omega} \times \Psi - \vec{b} + \vec{n}_v \\ \dot{\vec{b}} &= \vec{n}_u \end{aligned}$$

where

$\vec{\omega}$; A rotation vector of the body frame with respect tom the ECI frame is orbit rate.

- Linear State Space Formulation

$$\dot{\vec{X}}(t) = F\vec{X}(t) + \vec{W}(t) \quad (4)$$

$$\dot{\vec{X}}(t) = \begin{bmatrix} \dot{\Psi} \\ \dot{\vec{b}} \end{bmatrix}$$

$$= \begin{bmatrix} \Omega & -I_{3\times3} \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix} \begin{bmatrix} \Psi \\ \vec{b} \end{bmatrix} + \begin{bmatrix} \vec{n}_{v3\times1} \\ \vec{n}_{u3\times1} \end{bmatrix} \quad (5)$$

where

- Ω ; $\begin{bmatrix} 0 & \omega_z & -\omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{bmatrix}$
- Ψ ; Attitude error

- Discretized Linear State Space Formulation

$$\vec{X}(t_k) = \phi_k \vec{X}(t_{k-1}) + \vec{W}(t_k) \quad (6)$$

where

- $\phi_k = e^{AT_k}$ And $T_k = t_k - t_{k-1}$
- Mean, $E[\vec{W}(t)] = 0$
- Covariance,

$$E[\vec{W}(t)\vec{W}^T(t')] = \begin{bmatrix} \vec{n}_v\vec{n}_v^T & 0_{3\times3} \\ 0_{3\times3} & \vec{n}_u\vec{n}_u^T \end{bmatrix} \delta(t-t')$$

- There is no correlation between $\vec{n_v}$ and $\vec{n_u}$

- Discrete Dynamics Noise Covariance Matrix

Spectral Density Matrix, $Q(t) = E[\vec{W}(t)\vec{W}^T(t)]$
Covariance is $Q(t)\delta(t-t')$.
Discrete Dynamics Noise Covariance Matrix is

$$Q_k = \int_{t_{k-1}}^{t_k} \phi(t_k, t')Q(t')\phi^T(t_k, t')dt' \quad (7)$$

4 Observation Model

It is assumed that two star trackers are used.

- Definition of Z_k.

$$\vec{Z_k} = \vec{OS} - \vec{CS} \quad (8)$$

where
- $\vec{Z_k}$; Measurement residual
- $\vec{OS}$; Observed star unit vector
- $\vec{CS}$; Calculated star unit vector

- Observation Matrix.

$$H_k = \begin{bmatrix} (\vec{x} \times \vec{S_k}) & 0_{1\times3} \\ (\vec{y} \times \vec{S_k}) & 0_{1\times3} \end{bmatrix} \quad (9)$$

where
- $\vec{S_k}$; Observed star vector in the body frame
- $\vec{x}$; X axis of the star trackers in the body frame
- $\vec{y}$; Y axis of the star trackers in the body frame

- Observation Model.

$$\vec{Z_k} = H_k\vec{X} + \vec{V_k} \quad (10)$$

where
- $\vec{V_k}$; Sensor noise (Gaussian)
- Mean, $E[\vec{V_k}] = 0$
- Covariance,

$$E[\vec{V_k}\vec{V_k}^T] = \begin{bmatrix} R_{11} & 0_{3\times3} \\ 0_{3\times3} & R_{22} \end{bmatrix}$$

5 Update Filter

Eq. 6 and **Eq.10** can be rewriten as

$$\vec{x}(k+1) = \phi(k)\vec{x}(k) + \vec{w}(k) \quad (11)$$
$$\vec{z}(k) = H(k)\vec{x}(k) + \vec{v}(k) \quad (12)$$

where
- $v(k)$; measurement noise, covariance R(k)
- $w(k)$; process noise, covariance Q(k)
- $v(k)$ and $w(k)$ are independent with zero mean

The initial state vector x_0 is assumed to have zero mean with uncorrelated Gaussian. The state transition matrix is following,

$$\phi(k) = e^{[F(t_{k+1}-t_k)]}$$

and the update algorithm are the following equations. [8]

⟨ Kalman Filter Update Algorithm ⟩

- Prior State Covariance Matrix Propagation

$$P_k(-) = \phi_k P_{k-1}(+)\phi_k^T + Q_k \quad (13)$$

- Prior State Vector Propagation

$$\hat{\vec{x}}_k(-) = \phi_k\hat{\vec{x}}_{k-1}(+) \quad (14)$$

- Observation Model

$$\vec{z}_k = H_k\vec{x} + \vec{v}_k \quad (15)$$

- Kalman Gain Matrix Computation

$$K_k = P_k(-)H_k^T[H_kP_k(-)H_k^T + R_k]^{-1} \quad (16)$$

- Covariance Matrix Propagation

$$P_k(+) = [I - K_kH_k]P_k(-) \quad (17)$$

- State Vector Propagation

$$\hat{\vec{x}}_k(+) = \hat{\vec{x}}_k(-) + K_k(\vec{z}_k - H_k\hat{\vec{x}}_k(-)) \quad (18)$$

6 Conclusion

An attitude determination algorithm with two star trackers is studied. The system is constructed with an IRU and two star trackers. Attitude error model for IRU and observation model for star trackers are designed. They offers attitude data with different rate. The IRU produces output every 0.512sec and the star trackers produce output every 32.768sec for the case of normal mission mode. Kalman Filter is adapted to extract statistically optimum attitude data from the different sources.

Fig 1. shows a block diagram of a typical attitude determination system. Simulation is performed for the normal mission mode. Simulation parameters are listed in **Table 1.** Results are shown in **Fig 2.** to **Fig 7.** It shows that the above algorithm is good for many three-axis stabilized spacecraft in normal mission mode.

System with other sensors (such as sun sensors or combination of star tracker and sun sensor), and sensitivity analysis with other error sources (such as sensor misalignment, gyro scale factor error), are remained for further study.

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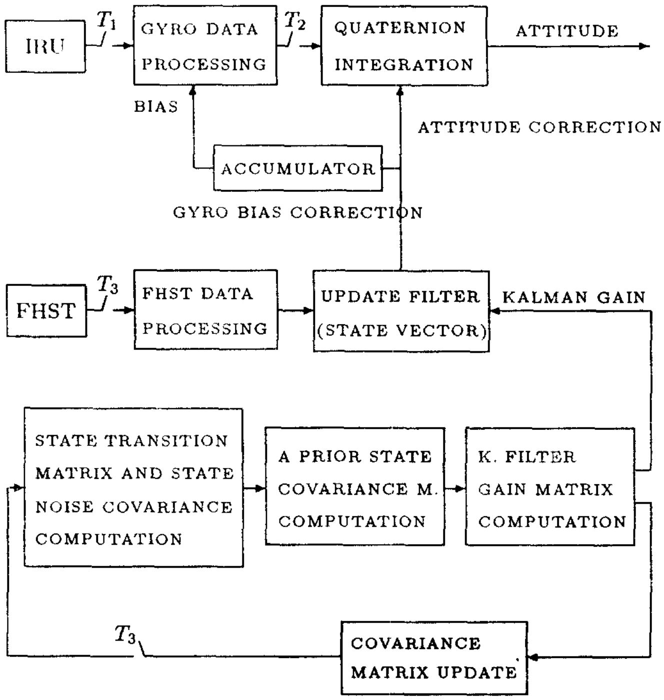

Figure 1: Function Block Diagram of Attitude Determination System

Table 1: SIMULATION PARAMETERS

Orbit Rate	
ω_x	0.0002 rad/sec
ω_y	0.0005 rad/sec
ω_z	0.0009 rad/sec
Sampling Time	
T_1	0.064 sec
T_2	0.512 sec
T_3	8.192 or 32.768 sec
Dynamic Noise	
White	4.2459E-2 arcsecs/sec$^{1/2}$ per axis
Random Walk	4.4413E-5 arcsecs/sec$^{3/2}$ per axis
Measurement Noise	32.3 arcsecs each
Initial Attitude Error	1800 arcsecs/axis
Initial Gyro Drift Error	0.5 arcsecs/sec/axis

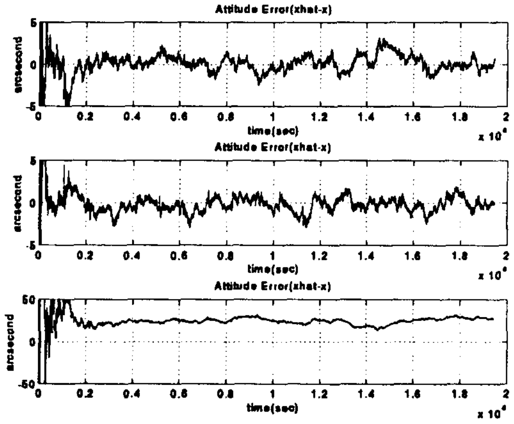

Figure 2: ATTITUDE ESTIMATION ERROR (observation data at every step = 0.512sec)

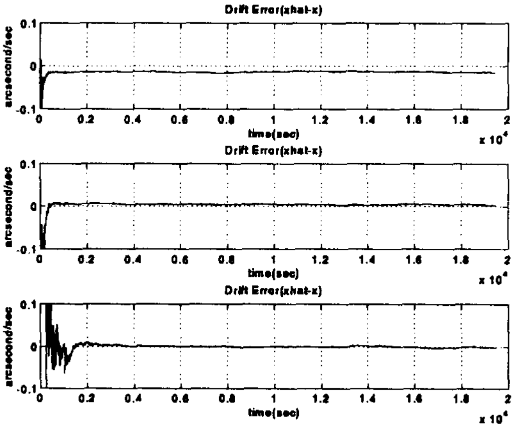

Figure 3: DRIFT ESTIMATION ERROR (observation data at every step = 0.512sec)

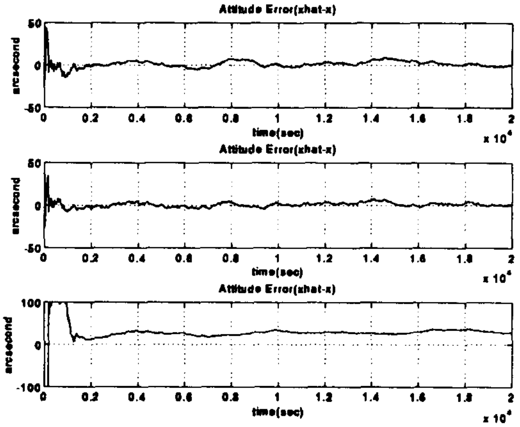

Figure 4: ATTITUDE ESTIMATION ERROR (observation data at every 16th step = 8.192sec)

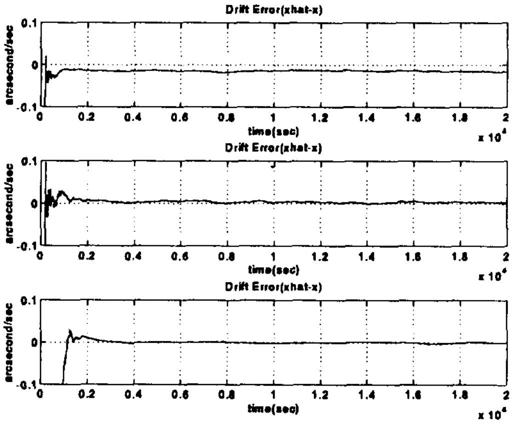

Figure 5: DRIFT ESTIMATION ERROR (observation data at every 16th step = 8.192sec)

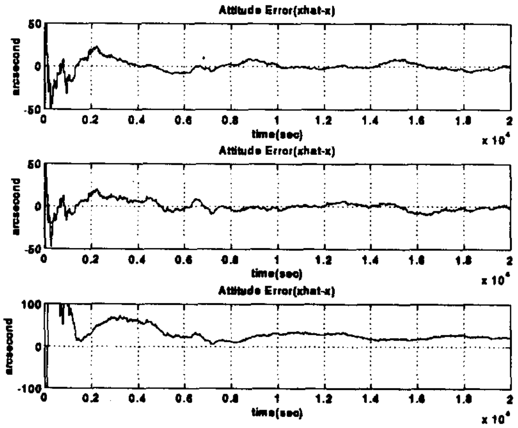

Figure 6: ATTITUDE ESTIMATION ERROR (observation data at every 64th step = 32.768sec)

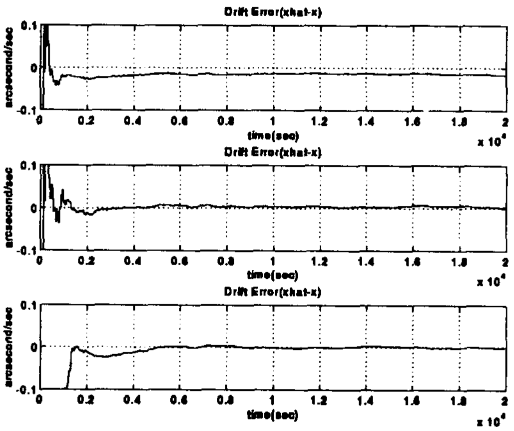

Figure 7: DRIFT ESTIMATION ERROR (observation data at every 64th step = 32.768sec)