'95 KACC (1995. 10. 23 ~ 25)

INPUT GROUPING OF LOGICAL CIRCUIT BY USE OF M-SEQUENCE CORRELATION

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Abstract: A new method for grouping of relevant and equivalent inputs of a logical circuit was proposed by the authors by making use of pseudorandom M-sequence correlation. The authors show in this paper that it is possible to estimate the input grouping from a part of correlation functions when we admit small percentage of error, whereas it is impossible to reduce the data necessary to estimate the grouping by use of the truth table method. For example in case of 30-input logic circuit, the number of correlation functions necessary to calculate can be reducible from 1.07×10^9 to 465.

Key words: M-sequence, correlation function, truth table, logical circuit, fault diagnosis

1. INTRODUCTION

One of the authors has developed an efficient method of fault detection by use of M-sequence correlation functions. (We call this method as MSEC method)[1]. This method has a characteristic that the undetected fault ratio is extremely small, whereas the fault position in the circuit can not be estimated.

The authors have proposed a new method for grouping of relevant and equivalent inputs of a logical circuit by making use of input-output correlation functions obtained by MSEC method[3]. This input grouping gives us the structure of a logical circuit, and it is possible to estimate the faulty part by use of this method.

In this study, the authors show that the grouping method we proposed is far advantageous over the truth table method when we admit small percentage of error. Especially in this paper, the investigation was made on the estimation error of input grouping by use of a part of correlation functions.

2. M-SEQUENCE CORRELATION FUNCTION

Fig. 1 shows block diagram of this input grouping method. Let any M-sequence signal be a basic signal $m_0(\tau)$, who's order isn't smaller than the number of inputs of a logical circuit under test, and let us denote delayed signal from the basic signal as $m_a(\tau)$, where a is the delay digit. M-sequence signals $m_0, m_1, \cdots$ are applied to a logical circuit and their outputs are observed. Crosscorrelation function C_i between the delayed signal m_i and the output Y is written as

$$C_i = \sum_{k=1}^{L} \{1 - 2 \times m_i(k) \oplus Y(k)\}$$

where $\oplus$ denotes an exclusive OR[2]. Let the order of M-sequence be r, and M-sequence period be L. Delayed signals from m_r to m_{L-1} can be made from m_0 to m_{r-1} by use of shift and add property, so let's call delayed signals from m_0 to m_{r-1} as independent signals and remainder signals from m_r to m_{L-1} as dependent signals. In this method, independent signals are applied to a logical circuit under test.

Let us call input signals which are relevant to output as "relevant signals", remainder as "irrelevant signals", and let's call input signals as "equivalent signals" when these input signals are changed each other, the output signal doesn't change.

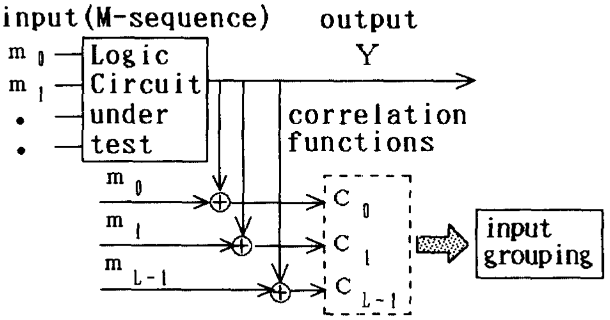

Fig. 1 Logic diagram of input grouping method

3. RELATION BETWEEN RELEVANT INPUT AND M-SEQUENCE CORRELATION

The authors showed that "input signal m_a is an irrelevant signal" is equivalent to " $C_{a\_i} = 2a_0 - 1$"[3]. Here C_{a_i} is a correlation function between the delayed signal m_{a_i} ($= m_a \oplus m_i$, m_i denotes arbitrary delayed signal except m_a, and let's assume m_{a_i} includes m_a) and the output Y. a_0 is an output value when all input values are zero. Especially, a_0 is used through this paper in this sense.

So, the relevant signals can be determined as follows: find the delayed signals who's correlation functions with the output are not equal to $2a_0-1$, divide them into dependent signals, then the resulting dependent signals are the relevant signals.

4. RELATION BETWEEN EQUIVALENT INPUTS AND M-SEQUENCE CORRELATION

The authors showed that "input signals m_a and m_b are an equivalent set" is equal to "correlation function $C_{a\_i} = C_{b\_i}$ "[3]. So, it is possible to examine whether input signals m_a and m_b are an equivalent set or not as follows. If $C_{a\_i} = C_{b\_i}$ is hold for any delay value i except a or b, then m_a and m_b are an equivalent set.

5. SIMULATION RESULT OF ESTIMATION ERROR RATIO OF INPUT GROUPING BY USE OF A PART OF CORRELATION FUNCTIONS

From the method descrived in section 3, 4, the input grouping can be estimated from correlation functions. These input grouping can also be estimated by the inspection of the truth table, because the truth table shows relation between input and output. But in the truth table method, whole data in the truth table must be used to calculate grouping even when we admit small percentage of error. On the other hand, in the grouping method by use of correlation functions, it is possible to calculate grouping from a part of correlation functions when we admit small percentage of error. This is due to the fact that a correlation function contains all input-output relations.

Let's call the case as case P when a part of correlation functions are used to calculate a input grouping, and let's consider two cases P1 and P2 as case P. P1 means a case that correlation functions concerned with input signals only are used, and P2 means that correlation functions concerned with delayed signals consisting of 1 or 2 input signals are used. And, let us call the case as case T when all correlation functiuons are used to calculate grouping.

Table 1 shows simulation results of estimation error ratio of input grouping by use of a part of correlation functions. n denotes number of input of a logic circuit which is as same as M-sequence order, so 'period' becomes 2^n-1. ratio denotes estimation error ratio of input grouping calculated by above conditions P1 and P2, N1 and N2 denote number of n-input random generated logic functions which are used to calculate the estimation error ratio. 'no.' is a number of correlation functions used to calculate input grouping which is equal to $n+{}_nC_2$ in case of P2. 'No.' of P1 is omitted, because that is equal to n. We did not carry out the simulation over the range of $n \geqq 18$ of P1, $n \geqq 10$ of P2, due to our computer limitation.

For example, in case of n=9, the number of all correlation function is 512, but we can estimate a grouping with about 0% error by use of only 45 correlation functions. On the other hand, the truth table method necessary all data in the truth table, that is equal to 'period' in Table 1.

Table 1 Estimation error of input grouping by use of a part of correlation functions

condition		P1		P2		
n	period	ratio	N1	ratio	no.	N2
2	3	0.13	16	0	3	16
3	7	0.48	256	7.8e-03	6	256
4	15	0.85	65536	5.4e-02	10	65536
5	31	0.96	50001	5.9e-02	15	50001
6	63	0.96	50001	1.1e-02	21	1000001
7	127	0.95	50001	5.2e-04	28	1000001
8	255	0.93	50001	1.6e-05	36	1000001
9	512	0.90	50001	0	45	1000001
10	1023	0.86	5001		55	
12	4095	0.75	5001		78	
14	16383	0.60	5001		105	
16	65535	0.43	3001		136	
18	262143	0.32	3001		171	

6. THEORETICAL STUDY OF ESTIMATION ERROR RATIO OF INPUT GROUPING

6.1 Estimation error of relevant input

Estimation error of relevant input in case P occurs in the following conditions.

$$C_{a\_i} = 2a_0 - 1 \quad (C_{a\_i} \in \alpha)$$
$$C_{a\_i} \neq 2a_0 - 1 \quad (C_{a\_i} \in \beta)$$

Here $\{\alpha\}$ denotes correlation functions used to calculate input grouping, $\{\beta\}$ denotes remainder correlation functions. The estimation error is only the case that the irrelevant input is wrongly estimated as relevant input. So, estimation error ratio of relevant input ε_{re} can be written as follows.

$$\varepsilon_{re} = (R_A + R_B) / Q$$
$$= (R_{Ap} - R_{At} + R_B) / Q$$

Here, R_A $(= R_{Ap} - R_{At})$ denotes the number of logic functions wrongly estimated that there is some irrelevant input, although there is not such an input. R_{Ap}, R_{At} denotes the number of logic functions having irrelevant input in case P1 and in case T respectively. R_B denotes the number of logic functions that the number of irrelevant input is wrongly estimated as greater than true case. Q denotes number of all logic functions of n-input and is written as Eq. 1.

$$Q = 2^{2^n} \tag{1}$$

In the following, let's consider the case P1 only as case P for simplicity.

Evaluation of R_{At}, R_{Ap}

R_{At} is written as Eq. 2 when we omit terms greater than G(3). Here, G(i) means the number of n-input logic functions who have k-relevant inputs in case T.

$$R_{At} = \binom{n}{1} G(1) - \binom{n}{2} G(2) + \binom{n}{2} G(3) \tag{2}$$

$$G(k) = 2^{2^{n-k}} \tag{3}$$

Similarly R_{Ap} can be written as Eq. 4

$$R_{Ap} = \binom{n}{1} E(1) - \binom{n}{2} E(2) + \binom{n}{2} E(3) \tag{4}$$

Here E(k) denotes number of n-input logical functions having k-relevant inputs in case P1. E(k) can be determined from the relation between the input signals and the appearence frequancies of the correlation functions in a M-sequence period. Stirling's formula is applied to these results then Eq. 5 are derived. Eq. 5 is hold in case of n $\geqq$ 7.

$$\left.\begin{aligned} E(1) &\approx 2^{2^n} \frac{1}{\sqrt{\pi 2^{n-1}}} \\ E(2) &\approx 2^{2^n} \frac{1}{\pi 2^{n-2}} \\ E(3) &\approx 2^{2^n} \frac{1}{2}\left(\frac{1}{\sqrt{\pi 2^{n-3}}}\right)^3 \end{aligned}\right\} \tag{5}$$

Evaluation of R_B

It is difficult to estimate R_B correctly because many cases must be considered. So let's approximate R_B as δR_{At} for R_B is contained in R_{At}, here δ is a coefficient of $\delta \leqq 1$.

Estimation error ratio of relevant input ε_{re}

In case of n $\geqq$ 7, R_{At} is substatially smaller than R_{Ap} to omit R_{At} in Eq. 1, then Eq. 6 is derived.

$$\varepsilon_{re} \approx \binom{n}{1} \frac{1}{\sqrt{\pi 2^{n-1}}} - \binom{n}{2} \frac{1}{\pi 2^{n-2}} + \binom{n}{3}\left(\frac{1}{\sqrt{\pi 2^{n-3}}}\right)^3 \tag{6}$$

6. 2 Estimation error of equivalent inputs

Estimation error of equivalent input in case P occurs in the following conditions.

$C_{a\_i} = C_{b\_i}$ (for arbitrary i)
$C_{a\_j} \neq C_{b\_j}$ (for some j)

Here C_{a_i}, $C_{b\_i} \in \alpha$, C_{a_j}, $C_{b\_j} \in \beta$, $\{\alpha\}\{\beta\}$ means the same sense of section 6. 1. So, estimation error of equivalent input is only the case that equivalent input a, b are wrongly estimated as not equivalent. Estimation error ratio of equivalent input ε_{eq-re} which is omitted the estimation error ratio concerned with relevant input, is given as follows.

$$\begin{aligned} \varepsilon_{eq-re} &= (E_A - E_{Are} + E_B) / Q \\ &= (E_{Ap} - E_{At} - E_{Are} + E_B) / Q \end{aligned}$$

Here, E_A denotes number of logic functions wrongly estimated as there are equivalent inputs although there are not such inputs. E_{Ap}, E_{At} means the same sense of E_A in case of case P and cast T respectively. E_{Are} denotes number of functions in E_A that the correlation functions concerned with equivalent inputs are $2a_0 - 1$. E_B denotes the number of logic circuit having equivalent inputs in case T, but who's equivalent inputs differ from the case P, and who's correlation functions concerned with equivalent inputs are not $2a_0 - 1$.

Estimation of E_{At}, E_{Ap}, E_{Are}

Let the number of n-input logic functions who have k-equivalent input be H(k), then H(k) becomes

$$H(k) = 2^{(k+1) \cdot 2^{n-k}}$$

But the number of logic functions having equivalent input E_{At} can not be approximated like Eq. 2 or Eq. 4 by use of H(k) only, because there are many kind of combinations of equivalent input, and the kinds of combinations are varied with the number of input n. So, let's approximate E_{At} as Eq. 7 by use of H(2) only.

$$\begin{aligned} E_{At} &= \binom{n}{2} H(2) \\ &= 2^{3 \cdot 2^{n-2}} \end{aligned} \tag{7}$$

Similarly E_{Ap}, E_{Are} are approximated like E_{At} by use of the relation of 2 input only.

Estimation of E_B

It is difficult to estimate E_B as like as R_B. So let's approximate E_B as ξE_{At}, because E_B is contained in E_{At}, here ξ is a coefficient of $\xi \leq 1$.

Estimation error ratio of relevant input ε_{eq-re}

Stirling criteria is used to above results and smaller terms are omitted, then Eq. 8 is derived. Eq. 8 is hold in case of $n \geq 7$.

$$\varepsilon_{eq-re} \approx \binom{n}{2} \frac{1}{\sqrt{\pi 2^{n-2}}} \tag{8}$$

6.3 Calculating result of estimation error ratio

From above study, estimation error ratio of input grouping is shown by Eq. 9.

$$\varepsilon = \binom{n}{1} \frac{1}{\sqrt{\pi 2^{n-1}}} - \binom{n}{2} \frac{1}{\pi 2^{n-2}} + \binom{n}{3} \left(\frac{1}{\sqrt{\pi 2^{n-3}}}\right)^3 + \binom{n}{2} \frac{1}{\sqrt{\pi 2^{n-2}}} \tag{9}$$

Calculating result of Eq. 9 concerned with $2 \leq n \leq 40$ is shown in Fig. 2 as a solid line. In Fig. 2 simulation result concened with $2 \leq n \leq 18$ is shown simultaineously as a dotted line.

In Fig. 2, the reason that the estimation error ratio ε calculated by Eq. 9 is far greater than simulation result at the position around about n=8 is due to the approximation error by use of only 2-input relations in Eq. 7, 8. So, this calculating line shows the maximum value of error ratio. From Table1, Fig. 2, if we admit 1% error about the estimation error of input grouping, the number of correlation functions necessary to calculate grouping is follows.

2^n-1	(all data)	when $n \leq 6$
$n(n+1)/2$	(G=2)	when $7 \leq n \leq 31$
n	(G=1)	when $n \geq 32$

Here, ( ) shows the kind of selection of correlation functions. For example, in case of n=30, the number of data which must be memorized is reduceable from 1.07×10^9 to 465 by use of correlation functions.

7. CONCLUSION

Estimation error ratio of input grouping was investigated when a part of correlation functions are used to calculate input grouping. We take up a part of correlation functions as 2 types: one type P1 is that correlation functions concerned with input signals are used to calculate input grouping, and the other type P2 is the correlation functions concerned with delayed signals consist of 1 or 2 input signals are used. From thoretical study we derived Eq. 9 which shows the maximum estimation error ratio of input grouping in case of P1. Simulations of the estimation error ratio in case of P1, P2were carried out, and from the simulation results and the theoretical study, we show that when we admit 1% of estimation error, the number of correlation functions necessary to calculate input grouping are 2^n-1 ($n \leq 6$), $n(n+1)/2$ ($7 \leq n \leq 31$), $n(n \geq 32)$. For example, in case of n=30, the number of correlation functions necessary to calculate is reducible from 1.07×10^9 to 465. It is seen that the grouping method by use of correlation function is far advantageous over the truth table method when we admit small percentage of error.

This input grouping method would be very useful in a fault diagnosis of logic boards.

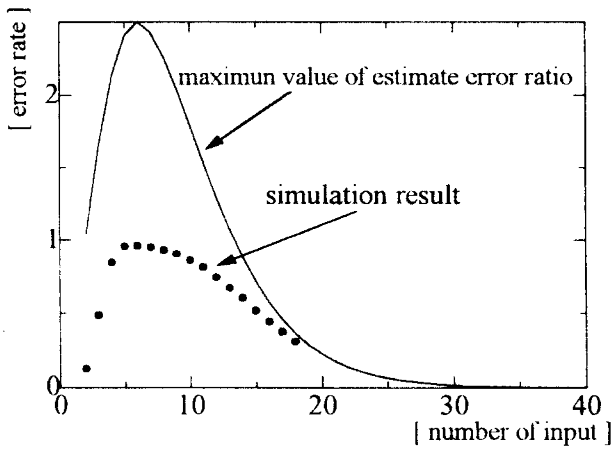

Fig. 2 Estimation error ratio by Eq. 9 and simulation result(G=1)

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