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NONLINEAR OUTPUT VOLTAGE CONTROL OF PWM DC-DC CONVERTERS BY FEEDBACK LINEARIZATION

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Abstract : *New output voltage control technique based on the simple feedback linearization is proposed. The system states are first divided into fast states and slow states. Then, the control stage is composed of the fast inner current control loop and the slow outer voltage control loop. From the inner loop, the average control is derived by the sliding mode concept and it is inserted into the dynamic equations of the slow states in the outer loop. Applying the feedback linearization technique to the obtained large-signal models of the PWM dc-dc converters, linearized large-signal models are obtained for the slow states. With this technique, the output voltage controller of the PWM dc-dc converters can be designed easily in the global state space and its control performance can also be much improved.*

Key words : PWM dc-dc converter, Feedback Linearization

1. INTRODUCTION

Over the last decade numerous studies have been devoted to improve the output voltage control performance of the PWM dc-dc converters[1]-[7]. The market trend also shows the increasing demand for the dc-dc converters with high performance[8]. The basic dc-dc converters are largely classified into buck, boost, and buck-boost converters according to the obtainable output voltage range. The Cuk converter is often added to them due to its good properties such as wide conversion ratio, smooth .input and output currents etc. The output voltage dynamics of these basic converters except the buck-type are nonlinear, which are caused by the switched input inductor current as shown in Fig. 1. Therefore, the output voltage controller should be carefully designed to obtain the high control performance.

A well-known method to deal with these problems is the small-signal control approach[1]. In this technique, a nonlinear plant model is first linearized around the operating point and the output voltage controller is designed by using the linearized model. Such a system may be stable in the vicinity of the operating point, but may not be stable when the system undergoes a large perturbation. Therefore, the small-signal method cannot predict the stability when the system is subjected to large-signal perturbation or large parameter variation. Consequently, a large-signal control of the output voltage is essential to study the global dynamic characteristics of the switching converters and to design robust and high-performance switching power supply.

The large-signal control technique based on the power balance is presented in [2]. This method is, however, suitable only for the boost and Cuk converters. The reference [3] investigates the large-signal control using the state feedback linearization. In this method, the nonlinear plant model is first linearized by the state transformation. The state feedback control is, then, applied to the linearized model. As a result, the fast and slow states are not clearly separated and their dynamic characteristics are not fully used. Furthermore, it is very difficult to design the output voltage controller because the transformed states are complicated nonlinear function of the original system states such as the inductor current and output voltage etc.

In this paper, a simple linear large-signal control technique is newly proposed. The multi-loop controller is realized by the separating the fast and slow states. The outer voltage control loop is implemented by a linear controller such as the proportional-integral[PI] or integral-proportional[IP]·controller. The inner current control loop is also designed by the peak current-mode control scheme or average current-mode control scheme[4]. In addition to these control loops, the modulator to linearize the dynamic characteristic of the output voltage is combined as shown in Fig. 2.

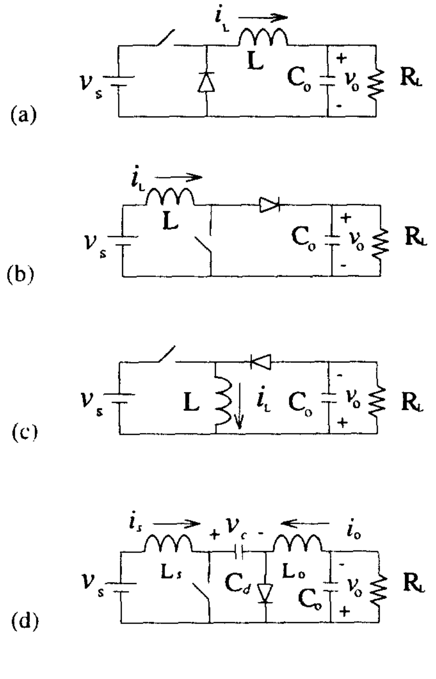


Fig. 1 Basic topologies of PWM dc-dc converters
(a) Buck (b) Boost (c) Buck-boost (d) Cuk

2. Linearization of PWM DC-DC Converters

Governing equations : Choosing the sate variables as the inductor current and voltage across the capacitor, the governing equations of the basic converters shown in Fig. 1 are derived as follows;

$$\dot{x} = A(x) + B(x)u + F \quad (1)$$

where

boost-type : $x = \begin{bmatrix} i_L & v_o \end{bmatrix}^T$

$$A(x) = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C_o} & -\frac{1}{R_L C_o} \end{bmatrix}, \quad B(x) = \begin{bmatrix} \frac{v_o}{L} \\ -\frac{i_L}{C_o} \end{bmatrix}, \quad F = \begin{bmatrix} \frac{v_s}{L} \\ 0 \end{bmatrix} \quad (2)$$

buck-boost-type : $x = \begin{bmatrix} i_L & v_o \end{bmatrix}^T$

$$A(x) = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C_o} & -\frac{1}{R_L C_o} \end{bmatrix}, \quad B(x) = \begin{bmatrix} -\frac{v_s + v_o}{L} \\ -\frac{i_L}{C_o} \end{bmatrix}, \quad F = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (3)$$

Cuk-type : $x = \begin{bmatrix} i_s & i_o & v_c & v_o \end{bmatrix}^T$

$$A(x) = \begin{bmatrix} 0 & 0 & -\frac{1}{L_s} & 0 \\ 0 & 0 & 0 & -\frac{1}{L_o} \\ \frac{1}{C_d} & 0 & 0 & 0 \\ 0 & \frac{1}{C_o} & 0 & -\frac{1}{R_L C_o} \end{bmatrix}, \quad B(x) = \begin{bmatrix} \frac{v_c}{L_s} \\ \frac{v_c}{L_o} \\ -\frac{i_s + i_o}{C_d} \\ 0 \end{bmatrix}, \quad F = \begin{bmatrix} \frac{v_s}{L_s} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4)$$

and u represents the switching state of the power switch and can be defined for the PWM operation as

$$u = \begin{cases} 1 & for \quad kT_s < t < kT_s + dT_s \\ 0 & for \quad dT_s + kT_s < t < (k+1)T_s \end{cases} \quad (5)$$

where d denotes the duty cycle determined by the PWM operation and T_s is the switching cycle.

Average model : For the PWM controlled dc-dc converters, the average model can be obtained by substituting the duty cycle d in place of the actual switching state u as[5]

$$z = A(z) + B(z)d + F \quad (6)$$

where z denotes the averaged state vector x. As can be seen from (1) through (6), the output voltage can be controlled by the *switching state* u or *duty cycle d*.

Separation of fast and slow states : If the states z are separated into the slow states and the fast states, the system dynamics of (6) can be modified as follows;

$$\begin{bmatrix} \dot{z}_f \\ \dot{z}_s \end{bmatrix} = \begin{bmatrix} f_f(z_f, z_s, d) \\ f_s(z_f, z_s, d) \end{bmatrix} \quad (7)$$

and the fast and slow states are determined as follows[6]-[7]:

boost, buck-boost-types

$$z_{fs} = \begin{bmatrix} i_{L.avg} \end{bmatrix}, \quad z_s = \begin{bmatrix} v_{o.avg} \end{bmatrix} \quad (9)$$

Cuk-type

$$z_f = \begin{bmatrix} i_{s.avg} & i_{o.avg} & v_{c.avg} \end{bmatrix}^T, z_s = \begin{bmatrix} v_{o.avg} \end{bmatrix} \quad (10)$$

In case of the Cuk converter, the circuit parameters should be properly selected for the separation in (10) to be successful. Consequently, the structure of a controller will reflect the system's structure in the multi-loop control. The inner loop will be designed to control the fast states and their commands will be generated from the outer slow states control loop. This separation of the design into two steps, each of a lower dimension than that of the original system, makes the design simpler. The control d is used to control the inner fast states and the slow output voltage is controlled by the fast states such as the inductor current etc. This concept can be easily applied to the buck-type converter because there exists no nonlinearity in the output voltage dynamics. However, its direct applications to the boost, buck-boost, and Cuk converters are not simple due to the cross coupled nonlinearity between the inductor current and the switching state in the voltage dynamics. This problem can be solved by introducing the sliding mode concept like in [1].

Large-signal model : Define the sliding surface σ_f as

$$\sigma_f = z_f - z_c \quad (11)$$

where z_c is the command vector to the fast states. Then, d_{avg} the average control input can be obtained from $\dot{\sigma}_f = 0$ as

$$d_{avg} = g(z_s, z_f). \quad (12)$$

If the dynamics of the fast states are much faster than those of the slow states, the average dynamics of the slow states in the large-signal sense can be determined from (7) and (12) as

$$\dot{z}_s = f_s(z_s, z_f, g(z_s, z_f)) \quad (13)$$

where z_f the fast states become the control inputs to z_s the slow states.

Linearization and modulator design : The slow state, or the output voltage dynamics of (13) are nonlinear in general. However, the equation (13) can be decomposed fortunately as follows

$$\dot{z}_s = -az_s + (m(z_s))^{-1} z_f \quad (14)$$

where a is determined by the output filter stage and $a = 1/(R_L C_o)$. This can be linearized simply by modulating z_c, the command to the fast states, generated from the outer controller as

$$z_c' = m(z_s) z_c. \quad (15)$$

where $m(z_s)$ is the output signal of the modulator as shown in Fig. 2 and calculated in Table 1 for each converter. Substituting (15) into (14) gives

$$\dot{z}_s = -az_s + z_c. \quad (16)$$

Therefore, the slow state or the output voltage can be linearly controlled by the control z_c in the global state space. Since the large-signal control is discussed in this article, the system constant a of (16) may be widely varied by the load value. Therefore, the feedforward term is added to z_c to minimize the load variation effect and maximize the benefits of the proposed control technique. Then,

$$\dot{z}_s = v \quad v \equiv -az_s + z_c \quad (17)$$

where az_s is the feedforward term and becomes the load current in the PWM dc-dc converter

3. Design Example

In this section, the proposed design method is applied to the output voltage control of the Cuk converter.

Linearization and modulator design : In the Cuk converter, the fast states are the input and output inductor current. The capacitor voltage across the decoupling capacitor Cd can also be a fast state. However, the design parameters should be selected properly so that the inductor currents and the voltage across the decoupling capacitor have the dynamic characteristics much faster than those of the output voltage. Then, as can be seen from (10), the fast states can be determined as the input inductor current, the output inductor current, and the voltage across the decoupling capacitor C_d. However, there is no need to control all the fast states individually. If we examine the internal operating characteristics of the Cuk converter, the output voltage can be controlled only by the input inductor current. From $\dot{\sigma}_f = 0$ and (7) through (10),

$$d_{avg} = 1 - \frac{v_s}{v_{c,avg}} = \frac{v_{o,avg}}{v_{c,avg}}, \quad (1-d_{avg}) i_{s,avg} = d_{avg} i_{o,avg} \tag{18}$$

or

$$v_s i_{s,avg} = v_{o,avg} i_{o,avg}. \tag{19}$$

The equation (19) exhibits the average power balance. From this relation, the conventional power balance equation can also be, therefore, derived as in the ***Remarks***. This can also be applied to the boost converter.

On the other hand, if the current command to the input inductor is chosen from (15) and (19) as

$$i_{s,ref} \equiv C_o \frac{v_{o,avg}}{v_s} z_c, \quad m(z_s) = C_o \frac{v_{o,avg}}{v_s} \tag{20}$$

the output voltage can be controlled by the input inductor current. In this case, $C_o z_c$ implies the command to the output inductor current. As a result, the Cuk converter can be controlled like the buck-converter. Of course, all the parameters such as the decoupling capacitor, input and output inductors should be properly designed to satisfy the condition (19). This is not included here and remained as a further work. This fact can also be obtained like in [1] or [2]. However, the proposed technique is relatively simple to implement.

The average output voltage dynamics are, then, written from (17), (19), and (20) as

$$\dot{v}_{o,avg} = \nu \tag{21}$$

Inner current control : There are two widely used current-mode control techniques, which are the peak current-mode control (PCMC) and the average current-mode control (ACMC). In this paper, the PI-typed ACMC is used or

$$d = k_{pi}(z_c^* - z_f) + k_{ii}\int (z_c^* - z_f)dt, \quad z_f \equiv i_{s,avg}, \quad z_c^* \equiv i_{s,ref} \tag{22}$$

where $i_{s,ref}$ is the reference to the input inductor current and k_{pi} and k_{ii} are the proportional and integral gain, respectively. Then, the average value of the actual current follows the current command exactly. The control gains should be determined not to generate the subharmonic oscillations.

Outer voltage control : As an output voltage controller, the IP action is employed instead of the PI action because it is relatively easy to evaluate the proposed model. The IP action can be described as

$$\nu = -k_{pv} z_s + k_{iv}\int (z_s^* - z_s)dt, \quad z_s \equiv v_{o,avg}, \quad z_s^* \equiv v_{ref} \tag{23}$$

where v_{ref} is the reference to the output voltage and k_{pv} and k_{iv} are the proportional and integral gain, respectively.

Remarks : The conventional power valance approach can also be derived from the proposed technique. In other words, the multiplication of the state $v_{o,avg}$ on both sides of (21) gives

$$\frac{1}{2}\frac{d}{dt}\left(C_o v^2_{o,avg}\right) = \left(-\frac{v^2_{o,avg}}{R_L}\right) + v_s i_{s,avg}. \tag{24}$$

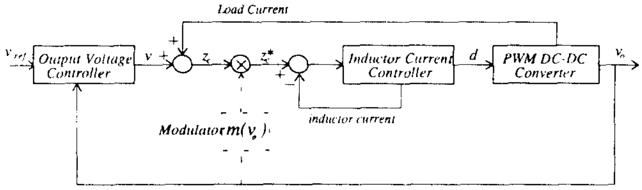


Fig. 2 Output voltage control system : small-signal approach without modulator and proposed method with modulator

Table 1 Modulator

	Buck	Boost	Buck-boost	Cuk
$m(v_o) =$	C_o	$C_o \frac{v_o}{v_s}$	$C_o\left(1+\frac{v_o}{v_s}\right)$	$C_o \frac{v_o}{v_s}$

4. SIMULATIONS

For the Cuk converter, the computer simulations are done to investigate the validity of the proposed technique with the following parameters:

$$v_s = 5V, \quad L_s = L_o = 50\mu H, \quad C_d = 10\mu F, \quad C_o = 100\mu F.$$

The switching frequency is chosen as *100kHz*. The following control gains are used:

$$k_{pv} = 1.0, \quad k_{iv} = 500, \quad k_{pi} = 0.5, \quad k_{ii} = 1000.$$

On the other hand, the small-signal model is also derived from (1), (6), and (19) for the comparative studies. In this model, the control gains are determined to have the output voltage dynamics similar to those by the proposed method as

$$k_{pv} = 1.0, \quad k_{iv} = 650, \quad k_{pi} = 0.5, \quad k_{ii} = 1000.$$

Fig. 3 shows the transient responses when the voltage command is step changed from *2V* to *15V*. The responses of Fig. 3(a) are expected by the wanted design equations (21) and (23), where it is assumed that the actual current follows the current command exactly. The responses when the average states of (21) through (23) are replaced by the actual states are obtained from (4), (5), (21) through (23) and simulated in Fig. 3(b). As can be seen Fig. 3, the average values of the simulated responses are well matched to the expected responses.

The responses when the load changes 5 to 15 and back to 5 are also simulated in Fig. 4. The responses with the feedforward compensation and the uncompensated responses are shown in the same figure. The former responses are obtained from (4), (5), (21), and (23), where the actual states are used instead of the average sates. The later responses are obtained from (14), (15), and (23), where ν is replaced by z_c. The output voltage is well regulated by the proposed control technique whether the load current is compensated or not.

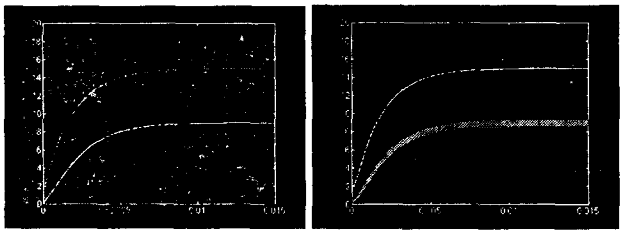
The source voltage variation can affect to the control performance because it is used in the modulator to calculate the current command as can be seen from (20). Therefore, the source voltage should be sensed to obtain the desired responses.

It provides, however, a relatively simple implementation to use the nominal value of the source voltage. In this case, since the real value of the source voltage is different from its nominal value used in the calculation of the current command, the output voltage responses are changed and the overall system may be unstable. In order to examine this problem, the output responses are simulated in Fig. 5 when the source voltage varies abruptly from *6V* to *4V* and back to *6V*($\pm$ *20%* variation from the nominal source voltage *5V*). In spite of the large variation of the source voltage, the output voltage is well regulated by the proposed controller.

On the other hand, the responses by the conventional small-signal approach and those by the proposed method are shown in Fig. 6(a) and (b), respectively. The proposed method predicts the output responses much closer to the expected responses than the small-signal approach does.

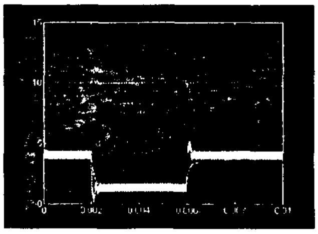
Consequently, the advantages of the proposed technique are summarized as follows:

(1) *Simplicity*: It is easy to design and simulate the controller.
(2) *Wide operating range* : Since this is the large-signal control, the control performances of the switching power supply can be evaluated in the wide operating range.
(3) *Wide application* : This technique can be applied widely to the basic topologies of Fig. 1 and also to the motor control etc.



(a) Expected (b) Simulated

Fig. 3 The simulated transient responses



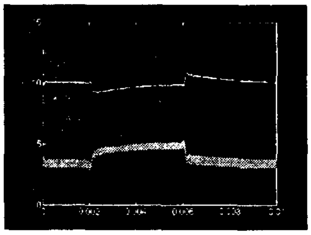
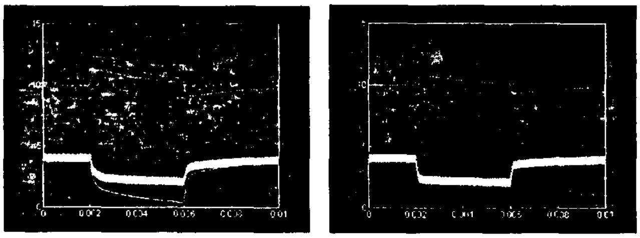


Fig. 4 The responses for the step-load change (Left)
Fig. 5 The responses for the step variation of the source voltage(Right)



(a) Small-signal approach (b) Proposed method (Without feedforward)

Fig. 6 The responses for the step load change
$(R_l = 5\Omega \leftrightarrow 10\Omega)$

Upper trace : output voltage
Lower trace : inductor urrent
(*Voltage = V / div, Current = A / div, Time = sec/ div*)

5. Conclusions

In this paper, a simple linear large-signal control technique is newly proposed. The multi-loop controller is realized by the separating the fast and slow states. The outer voltage control loop is implemented by a linear controller such as the PI or IP controller. The inner current control loop is also designed by the peak current-mode control scheme or average current-mode control scheme. In addition to these control loops, the modulator to linearize the dynamic characteristic of the output voltage is combined. The advantages of the proposed technique are the simplicity and the wide operating range. Furthermore, this can also be applicable to the motor control system etc. These are verified through the computer simulations. Further studies should be addressed to solve the operating range qualitatively and to examine the effects of the parastics such as the effective series resistances of the inductors and the capacitors etc.

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