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Performance Analysis and Quality Control of Serial Production Lines

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Abstract In this paper, a model of an asymptotically reliable serial production line with quality control devices is introduced and analyzed. By an asymptotic technique and Taylor series expansion, its average production rate is approximated in a closed form. The results are applied to a case study of a surface mount system.

Keywords Serial production lines, Performance analysis, Quality control, Surface mount system

1. Introduction

The high quality and the low cost are the most important factors in manufacturing systems. Quality control satisfies both of them and brings the effects of *fewer rework labor hours, less materials waste* and *high–quality of final goods*[1]. To accomplish such a quality control, the high–performance manufacturing machine has a quality control device(QCD) or a function for on–line test and parts verification, automatic parts recovery[10],[11].

The serial production lines have been received widespread attentions of researches: modeling and approximate analysis by a decomposition technique[2],[4],[5],[9] or by an asymptotic technique[3] or by a recursive method[8],[11] or by a simulation based method[6]. Using the asymptotic technique, D. Jacobs *et al.* introduced asymptotically reliable serial production lines with QCDs, but they only dealt with 2 machines/1 buffer system[7].

In this paper, using the asymptotic technique a model is introduced and a performance analysis method is suggested for an asymptotically reliable serial production line with QCDs which consists of M machines, $M-1$ buffers and C QCDs. Finally the model and the method will be validated by a case study.

Consider a serial production line defined by the following assumptions[3][7].

(i) The system consists of M machines, m_i, $i = 1, \ldots, M$, and buffer B_i, $i = 1, \ldots, M-1$, separating each consecutive pair of machines, m_i and m_{i+1}.

(ii) The machines have identical cycle time T. The time axis is slotted with the slot duration T. Machines begin their operation at the beginning of each time slot.

(iii) Each buffer is characterized by its capacity, n_i, $i = 1, \ldots, M-1$, where n_i is a positive integer.

(iv) Machine m_i is starved during a time slot if buffer B_{i-1} is empty at the beginning of this slot; m_i is blocked during a time slot if at the beginning of this time slot buffer B_i is full and machine m_{i+1} either fails or is blocked.

(v) Machine m_i, being neither blocked nor starved, produces a material during a time slot with probability $1-p_i$ and fails to do so with probability p_i, $i = 1, \ldots, M$, where $0 < p_i \ll 1$.

A serial production line defined by (i)–(v) is called the *asymptotically reliable serial production line.*

In general, a machine can make a defective part even though it is in good order. And the existence of defective parts in a system is significant of *lower good quality productivity.* Therefore, a quality control method is required to improve the good quality production rate.

In order to account the quality control, the following assumptions should be added.(see Fig. 1).

(vi) Machine m_i introduces a defect to its product with probability r_i and doesn't introduce a defect with probability $1-r_i$ for each cycle time T where $0 \leqslant r_i \ll 1$, $i = 1, \ldots, M$. And the defects introduced by m_i cannot be removed by $m_{i+1}, \ldots, m_M$. The machine cycle time T includes a QCD cycle time provided that the machine has a QCD.

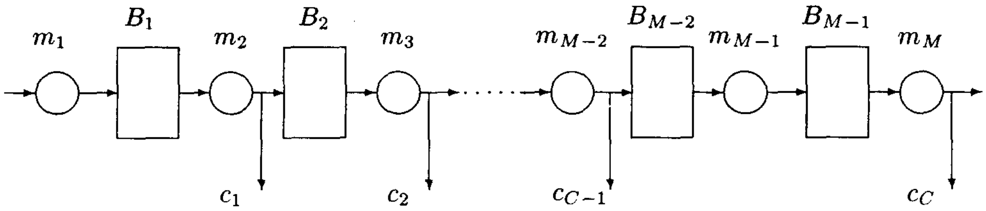

Fig. 1: An open-loop line

(vii) The system has C QCDs, c_i, $i = 1, \dots, C$ where $C \leqslant M$ and each QCD immediately follows a machine. QCD c_j of machine m_i perfectly detects any defective workpiece. The defective workpiece is removed from the system as soon as it is detected by QCD.

When the given system is satisfied by (i)–(vii), it is referred to as the *asymptotically reliable serial production line with QCDs* or shortly the *open-loop line*. It is clear that the production rate function of an open-loop line is analytic for $0 \leqslant p_i < 1$, $1 \leqslant i \leqslant M$ and $0 \leqslant r_i < 1$, $1 \leqslant i \leqslant M$.

2. Performance analysis

Consider an asymptotically reliable serial production line (m_1, B_1, m_2), its production rate is[3]

$$PR(p_1, k, p_2) \triangleq F(p_1, k, p_2) = (1 - B(0))(1 - p_2) \quad (1)$$

where $B(0) = \frac{1}{1 + \frac{1}{p_2}\sum_{i=1}^{k}\left(\frac{p_2}{p_1}\frac{1-p_1}{1-p_2}\right)^i}$, $B(0)$ is the buffer empty probability of B_1 and k is the capacity of B_1.

In case an asymptotically reliable serial production line has some perfect machines, it can be represented by a reduced system. For example, consider $(m_1, B_1, m_2, B_2, m_3)$ where m_2 is the perfect machine, by its steady state equations and a simple algebraic manipulation we obtain

$$PR(p_1, n_1, 0, n_2, p_3) = (1 - B_e(0))(1 - p_3) \quad (2)$$

where $B_e(0) = \frac{1}{1 + \frac{1-p_1}{1-p_3} + \frac{1-p_1}{p_3(1-p_3)}\sum_{i=1}^{n_1+n_2-1}\left(\frac{p_3}{p_1}\frac{1-p_1}{1-p_3}\right)^i}$. For Eq. 1, replace p_2 and k to p_3 and $n_1 + n_2 - 1$ respectively. Then the error $B_e(0) - B(0)$ is

$$B_e(0) - B(0) = \frac{\alpha^n}{(\sum_{i=1}^{n}\alpha^i)^2}p_3^2 + \mathcal{O}(\epsilon^3) \quad (3)$$

where $\alpha = \frac{p_3}{p_1}$, and $n = n_1 + n_2 - 1$. Therefore, it is clear that

$$PR(p_1, n_1, 0, n_2, p_3) = F(p_1, n_1 + n_2 - 1, p_3) + \mathcal{O}(\epsilon^2). \quad (4)$$

Now consider the general case.

Theorem 1 *Under assumption (i)-(v), let $p_i = 0$, $1 \leqslant i \leqslant M$, $i \neq j$, $i \neq \xi$, then the production rate is*

$$PR(0, n_1, 0, \dots, p_j, n_j, \dots, p_\xi, n_\xi, \dots, n_{M-1}, 0) = F(p_j, n_e(j, \xi - 1), p_\xi) + \mathcal{O}(\epsilon^2) \quad (5)$$

where $n_e(\zeta, \nu) = \sum_{i=\zeta}^{\nu} n_i + \zeta - \nu.$

Proof : *Available from the authors upon request.*

It should be mentioned that an open–loop line can be transferred to the system which all machines have QCDs without breaking a flow conservation raw. In the transform if m_i had no QCD originally, it will get the new defective probability $r_i' = 0$. Also if m_i had QCD c_j in the original system, its new defective probability r_i' should be

$$r_i' = 1 - \prod_{w=\xi+1}^{i}(1 - r_w) \quad (6)$$

where $2 \leqslant i \leqslant M$ and ξ is the location of the machine that had QCD c_{j-1} in the original system.

We will stick to the convention that each machine has a QCD, i.e., $C = M$ for notational convenience hereafter. Therefore, for an open–loop line, the general production rate equation should be what follows

$$PR(p_1, r_1, n_1, \dots, p_i, r_i, n_i, \dots, p_M, r_M). \quad (7)$$

In case an open–loop line has some 1 size buffers, the 1 size buffers can be eliminated by the sense of $\mathcal{O}(\epsilon^2)$ error bound.

Theorem 2 *Under assumption (i)-(vii) with $C = M$, suppose $n_i = 1$, $j \leqslant i \leqslant \xi$. Then the production rate is*

$$PR(p_1, r_1, n_1, \dots, p_j, r_j, 1, \dots, 1, p_\xi, r_\xi, 1, \dots, p_M, r_M) = PR(p_1, r_1, n_1, \dots, n_{j-1}, p_a, r_a, n_{\xi+1}, p_{\xi+2}, \dots, p_M, r_M) + \mathcal{O}(\epsilon^2) \quad (8)$$

where $p_a = 1 - \prod_{i=j}^{\xi+1}(1 - p_i)$, $r_a = 1 - \prod_{i=j}^{\xi+1}(1 - r_i)$.

Proof : *Available from the authors upon request.*

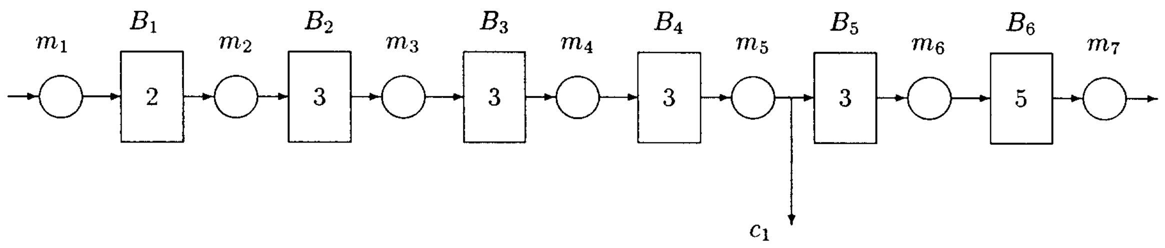

Fig. 2: A model of SMS

Now, consider the performance analysis of a general open-loop line.

Theorem 3 *Under assumption (i)-(vii) with $C = M$, suppose $n_i > 1$ for all i and let $(1-p_j)\prod_{w=j}^{M}(1-r_w) \leqslant (1-p_i)\prod_{w=i}^{M}(1-r_w)$, $i \neq j$. Then the production rate is*

$$\begin{aligned} &PR(p_1, r_1, n_1, \ldots, n_{M-1}, p_M, r_M) \\ &= \sum_{\substack{\xi=1 \\ j>1}}^{j-1} F(p_e(\xi,\xi), n_e(\xi, j-1), p_j) \prod_{w=j}^{M}(1-r_w) \\ &+ \sum_{\substack{\xi=j+1 \\ j<M}}^{M} F(p_e(j,\xi-1), n_e(j,\xi-1), p_\xi) \prod_{w=\xi}^{M}(1-r_w) \\ &- (M-2)(1-p_e(j,M)) + \mathcal{O}(\epsilon^2) \qquad (9) \end{aligned}$$

where $p_e(\xi,\nu) = 1-(1-p_\xi)\prod_{w=\xi}^{\nu}(1-r_w)$.

Proof : *Available from the authors upon request.*

3. Case study: Surface mount system

Surface Mount System(SMS) places and attaches the active and passive electrical components directly to the prepared surface of a Printed Circuit Board(PCB). The system is modeled by an open-loop line. It consists of 7 machines and 6 buffers, 1 QCD at machine m_5. In Fig. 2, its diagram is shown where a circle denotes a machine while a square denotes a buffer and the inside number of the square means the maximum buffer level. QCD c_1 is called as a visual checker in SMS. In ideal situation, it means no failure and no defect, the given system has the production rate 300 PCBs/hr for a specific PCB. The aim of this case study is to calculate the production rate of the given system and find the best location of c_1 which maximizes the production rate.

Table 1 shows the role, the defective probability and the failure probability for each machine. The failure of machines includes single or cycle stop, PCB trouble, bond count up stop, PCB recognition error, take up miss, height miss, mount miss, carrier trouble, take out error, machine trouble, etc. On the other hand, the defects include loose or inverted components, components miss, improper soldering, PCB defects, etc. The components defect is excluded because it was eliminated by incoming inspection and on-line test before it is mounted.

Both stockers, m_1 and m_7 scarcely ever introduce a defect to their products, so that r_1 and r_7 are assumed as zero. Even though m_2 and m_3 contribute defects to the parts, r_2 and r_3 cannot be directly checked by the given QCD. But r_2 and r_3 are propagated by r_4 and r_5, r_6. For example, an inadequate adhesive application will cause a component missing defect. Therefore, they are assumed zero too. Finally, the defect probability r_6 is assumed zero because the data of defects introduced by m_6 are not available in the given configuration.

Consider two configurations, the first is the given configuration and the second is that m_4 has c_1. For two configurations, the simulation and calculation results appear in Table 2. The simulation duration times are 30000 machine cycle times and the error is from

$$\text{error} = \frac{PR_{\text{simulation}} - PR_{\text{calculation}}}{PR_{\text{simulation}}} \times 100(\%).$$

By Theorem 3, we know that the production rate of the first configuration can't be larger than $(1-p_5)(1-r_4)(1-r_5)$ and for the second configuration, it can't be larger than $(1-p_4)(1-r_4)(1-r_5)$. So, if n_3, n_4, n_5 are large enough, then the performance of the second configuration is better than the first configuration. Both simulation and calculation results show that the fact is still held despite of the small buffer sizes.

4. Conclusions

A model was suggested for an asymptotically reliable serial production line with a quality control device and the production rate was approximated in a closed form formula within $\mathcal{O}(\epsilon^2)$ error bound. And the model and the method were validated by a case study.

Table 1: Description of machines

machine	name	role	p_i	r_i
m_1	stocker	automatic PCB load	0.01	0
m_2	screen printer	solder application	0.024	0
m_3	adhesive dispenser	adhesive application	0.032	0
m_4	high–speed mounter	chip mount	0.082	0.018
m_5	multi–functional mounter	irregular component mount	0.095	0.021
m_6	air reflow oven	curing and reflow	0.01	0
m_7	stocker	automatic PCB unload	0.01	0

Table 2: Simulation and calculation results

case	configuration	simulation	calculation	error(%)
1	c_1 is at m_5	0.8482	0.8505	-0.2670
2	c_1 is at m_4	0.8550	0.8541	0.1018

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