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NUMERICAL SOLUTION OF A DYNAMIC SHAPE CONTROL PROBLEM

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Abstract

In this paper, we consider a dynamic shape control problem with an example of controlling a flexible beam shape. Mathematical formulations are obtained by employing the Green's function approach. Necessary conditions for optimality are derived by considering the quadratic performance criteria. Numerical results for both of the dynamic and the static cases are obtained and compared.

Keywords dynamic shape control, Green's function

1. INTRODUCTION

Problem of shape control is especially important in the field of large space antenna. The size of space antenna such as antenna for space station tends to be large. As a consequence, in order to minimize the mass, the structure of large space antenna becomes flexible and the antenna manufactured in stowed form to facilitate deployment in space. Hence it is essential to maintain the required shape for proper electromagnetic performance. Thermal shock or vibration excitation of the antenna also contributes the local surface distorsion which results in the degradation of the electromagnetic performance.

There have been much researches in the area of shape control of flexible space structure [1, 3, 4, 5]. In [7], Weeks did extensive studies for the static control problems of the large space structures.

In this paper, as an extension of the static case, we consider a dynamic shape control problem with an example of a flexible beam. We did mathematical formulations by using Green's function approach and the quadratic cost functional. Numerical results are obtained for the case of dynamic shape control of a flexible beam, and the results are compared with the corresponding static shape control problem.

2. DYNAMIC SHAPE CONTROL PROBLEM

As a model of dynamic shape control problem, the following dynamic system (eq. 1) can be employed. Let x be a point on the model. Then $s(t,x)$ represents the shape at time t induced by the forcing function $f(t,x)$. For example, the shape of a dish type space antenna maintained by applying the forcing term $f(t,x)$ is denoted by $s(t,x)$

$$\frac{\partial^2 s(t,x)}{\partial t^2} + Ls(t,x) = f(t,x), \quad 0<t<T, \quad x \in \Omega \subseteq R^n$$

$$B_i s(t,x) = 0\ ;\ 1 \le i \le k_0 \quad \text{for } x \in \partial\Omega \tag{1}$$

$$s(0,x) = c(x)$$

$$\frac{\partial s(0,x)}{\partial t} = d(x)$$

L : Linear partial differential operator
B_i : Boundary operators
k_0 : Number of boundary conditions

The idea of dynamic shape control is that we subdivide the time horizon into several intervals and apply optimal forces for each interval at discrete points. Then the forcing term takes the form of

$$f(t,x)=\sum_{i=1}^{m} f_i(t)\delta(x-x_i).$$

where (2)

$$f_i(t)=\sum_{k=1}^{T_i} u_i^k\left[H(t-t_i^{k-1})-H(t-t_i^k)\right]$$

$H(t-\tau)$: Heavyside function

By employing the Green's function approach[6], the solution of eq. (1) can be represented as

$$s(t,x)=\int_0^T\int_\Omega g(x,t;\xi,\tau)f(\tau,\xi)d\tau d\xi \ . \tag{3}$$

$g(x,t;\xi,\tau)$ is a Green's function which is the impulse response at (x,t) for force applied at (ξ,τ). The Green's function, $g(x,t;\xi,\tau)$, satisfies

$$\frac{\partial^2 g}{\partial t^2}+Lg=\delta(x-\xi)\delta(t-\tau)$$

where (4)

$$g(x,t;\xi,\tau)=0 \ \text{ for } \ \tau>t$$

As a cost functional, we take a typical quadratic form in order to determine the best shape and the optimal control forces.

$$\begin{aligned} J(f,s)&=\frac{1}{2}\sum_{i=1}^{m}\int_0^T f_i^2(t)r_i(t)dt \\ &+\frac{1}{2}\int_0^T\int_\Omega[s_d(x)-s(t,x)]^2dxdt \\ &+\frac{1}{2}\int_\Omega[s_d(x)-s(T,x)]^2dx \end{aligned} \tag{5}$$

where

$s_d(x)$: desired shape

Necessary condition for optimality, i.e. $\dfrac{\partial J}{\partial u_j^l}=0$ yields the following system of equations.

$$\sum_{i=1}^{m}\sum_{k=1}^{T_i}A_{jl,ik}u_i^k+\hat{A}_{jl}u_j^l=B_j^l \ ; \tag{6}$$

for $j=1,\ldots,m$; $l=1,\ldots,T_i$

where

$$A_{jl,ik}=\int_0^T\int_\Omega\{\int_{t_i^{k-1}}^{t_i^k}g(t,x;\tau,x_i)d\tau\int_{t_j^{l-1}}^{t_j^l}g(t,x;\gamma,x_j)d\gamma\}dxdt$$

$$+\int_\Omega\{\int_{t_i^{k-1}}^{t_i^k}g(T,x;\tau,x_i)d\tau\int_{t_j^{l-1}}^{t_j^l}g(T,x;\gamma,x_j)d\gamma\}dx$$

$$\hat{A}_{jl}=\begin{cases}\int_{t_j^{l-1}}^{t_j^l}r_j(t)dt, & \text{if } j=l\\ 0, & \text{otherwise}\end{cases}$$

$$B_{jl}=\int_0^T\int_\Omega\varphi(x)\int_{t_j^{l-1}}^{t_j^l}g(t,x;\tau,x_j)d\tau dxdt$$

$$+\int_\Omega\varphi(x)\int_{t_j^{l-1}}^{t_j^l}g(T,x;\tau,x_j)d\tau dx$$

As a physical example, we consider a simply supported beam which is governed by the following differential equation[8].

$$m\frac{\partial^2 s}{\partial t^2}+EI\frac{\partial^4 s}{\partial x^4}=f(t,x) \tag{7}$$

$$s(t,0)=s(t,l)=0$$

$$\frac{\partial^2 s(t,0)}{\partial x^2}=\frac{\partial^2 s(t,l)}{\partial x^2}=0$$

By employing the eigenfunction expansion, and using the Laplace transformation, the following Green's function can be obtained from equations (1) and (4).

$$g(t,x;\tau,\xi)=$$

$$H(t-\tau)\frac{2l}{\pi^2\sqrt{E\,\mathrm{Im}}}\sum_{n=1}^{\infty}\left\{\frac{1}{n^2}\sin\frac{n\pi x}{l}\sin\frac{n\pi\xi}{l}\sin\left(\sqrt{\frac{EI}{m}}\frac{n^2\pi^2}{l^2}(t-\tau)\right)\right\} \tag{8}$$

For numerical experiments, we take the beam parameters as $m=E=I=1$, $l=10$.

3. STATIC SHAPE CONTROL PROBLEM[7]

The static shape control problem of an elastic beam can be summarized as follows. Shape of the beam shall be maintained by applying the transverse force f_i at discrete positions x_i, $1\le i\le m$, where $0<x_1<x_2\cdots<x_m<l$. The dynamic system representing the shape of beam is characterized by the following differential equation.

$$\frac{d^4\tilde{s}}{dx^4}=\sum_{i=1}^{m}\tilde{f}_i\delta(x-x_i) \tag{9}$$

where

$$\tilde{s}(0)=\tilde{s}(l)=0,\quad \frac{\partial^2\tilde{s}(0)}{\partial x^2}=\frac{\partial^2\tilde{s}(l)}{\partial x^2}=0$$

Taking into account of the cost functional for the dynamic case, the one for the static problem shall take the following form.

$$J(F,\tilde{s}) = \frac{1}{2}\sum_{i=1}^{m}\tilde{f}_i^{\,2} r_i + \int_{\Omega}[s_d(x) - \tilde{s}(x)]^2 dx \qquad (10)$$

where

$$F = (f_1 \cdots f_m)^T$$

Similary as the dynamic case, the Green's function for the static case taks the following form.

$$\tilde{g}(x,\zeta) = \begin{cases} \dfrac{(\zeta - l)x}{6l}(x^2 - 2l\zeta + \zeta^2) & 0 \le x \le \zeta \\ \dfrac{(x-l)\zeta}{6l}(x^2 - 2lx + \zeta^2) & \zeta < x \le l \end{cases} \qquad (12)$$

Optimal forces for the static case can be obtained from the system of equations below.

$$\sum_{i=1}^{m}(a_{ij} + r_{ij}) f_j = b_j, \; j = 1,\ldots,m$$

where

$$a_{ij} = \int_{\Omega} \tilde{g}(x,x_i)\tilde{g}(x,x_j)dx \;\; ; \;\; i,j = 1,\ldots,m \qquad (13)$$

$$b_i = \int_{\Omega} \varphi(x)\tilde{g}(x,x_i)dx \;\; ; \;\; i = 1,\ldots,m$$

$$r_{ij} = diag(r_1 \cdots r_m)$$

4. NUMERICAL RESULTS AND DISCUSSIONS

Figure 1 shows the dynamic shapes developed at various time intervals. Figure 3 is the final shape achieved for time horizon (T=13). From this figure, we can see that the dynamic case yields better shape than static one at the final time. From Table 1, we can also see that the dynamic shape control gives better results in term of cost comparisons. From the same table, it can be observed that having a bigger time horizon produces better results, i.e. smaller costs for the dynamic case. But in term of the final shape obtained(Fig. 4), time horizon of 13 gives the best final shape. This may imply the existence of optimal final time. Figure 3 shows the variation of optimal forces for each time interval. From this figure, we can see that the direction of forces are changed for the first two time intervals. This may happened to keep the dynamic shape close to the desired one. Even though bigger forces are required for the case of the dynamic shape control, it gives better results in term of cost(Tab. 2) and shape achieved(Fig. 2). We also examined the effect of number of actuators used. From the table 3, it is clear that employing more number of actuators yields better results in the sense of overall cost. This is also evident from Fig 2 and Fig 5. Although it may depend on the desired shape, Tab. 2 suggests the existence of optimal location of actuators.

5. CONCLUSIONS

A dynamic shape control problem with an example of controlling a flexible beam has been studied. We have done a mathematical formulation by employing the Green's function approach. Optimal forces for controlling the shape of a flexible beam are obtained by solving the necessary conditions for optomality. Comparisons of numerical results for dynamic case and static case show that the former one produces better shapes with less costs than the latter one.

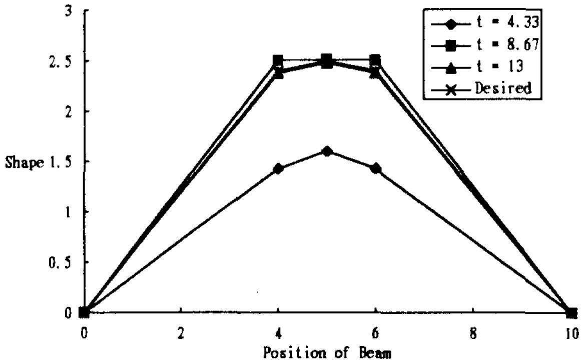

Fig. 1. Evolution of Dynamic Shape.

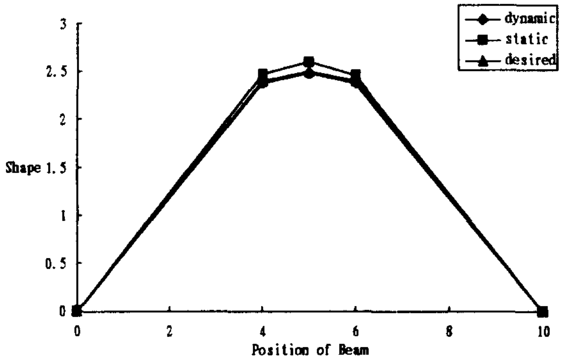

Fig. 2. Shape for T = 13.

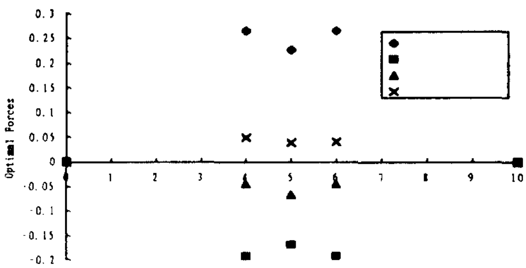

Fig. 3. Optimal Forces for each time interval.

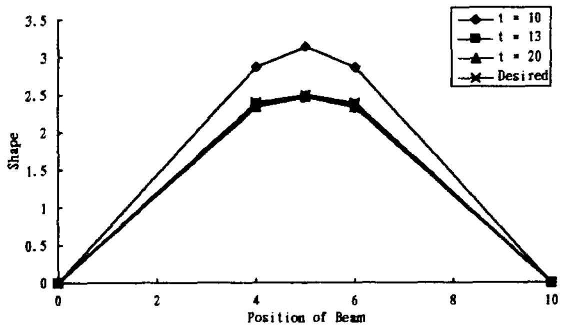

Fig. 4. Effect of Time Horizon on Final Shape.

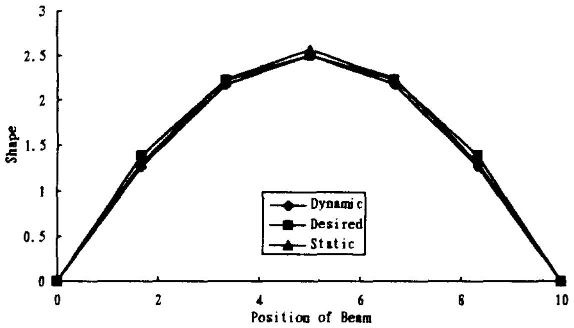

Fig. 5. Shape obtained using 5 Actuators.

TABLE 1. Comparison of Costs for Various Final Times.

Final Time	Cost	
	Dynamic	Static
10 13 20	0.03575 0.02613 0.02582	0.08754 " "

TABLE 2. Effect of Location Actuators.

Location of Actuators	Dynamic Cost
x = 4, 5, 6	0.02613
x = 2.5, 5, 7.5	0.02582

TABLE 3. Effect of # of Actuators.

# of Actuators	Dynamic Cost
3 5	0.02582 0.00916

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