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Nonlinear H_∞ Control to Semi-Active Suspension

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Abstracts: Recently H_∞ control theory for nonlinear systems based on the Hamilton-Jacobi inequality has been developed. In this paper, we apply the state feedback controller solved via Riccati equation to a semi-active suspension model, two degree of freedom vehicle model, and show that it is effective for vibration control.

Key Words: Nonlinear H_∞ Control, Semi-active Suspension, Hamilton-Jacobi Inequality

1. Introduction

Recently, many researcher investigate nonlinear H_∞ control theory, and its application is demanded. We controlled semi-active suspension with nonlinearity by nonlinear H_∞ control based on the Hamilton-Jacobi inequality in this report. Semi-active suspension control reduces the vibration by varying its damping coefficient, while active suspension by actuator. Since the plant is bilinear in general, the design of the controller is not easy.

We designed this controller by solving a linear Riccati equation, and verified the effectiveness of the method by simulations.

2. Plant

Suspension model used in this paper is two degree of freedom model shown in Fig.1. This is the one connected the body model and the wheel model by the spring and the variable shock absorber. The dynamic equations of the model are as follows.

$$m_1\ddot{y}_1 = -k_2(y_1 - y_2) - c(\dot{y}_1 - \dot{y}_2) + k_1(y_0 - y_1) \quad (1)$$

$$m_2\ddot{y}_2 = k_2(y_1 - y_2) + c(\dot{y}_1 - \dot{y}_2), \quad (2)$$

where $c = c_p + \Delta c$. c_p is the passive damping coefficient and Δc is the variation.

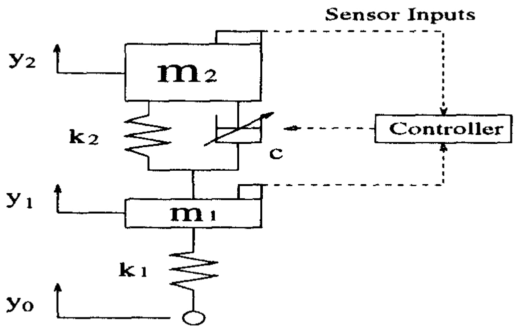

Figure 1: Suspension model

In this paper, we designed the controller for this model, and examined the performance about its riding comfort.

3. State space description of the model

Generally, riding comfort and wheel traction is important as the performance of suspension. The estimated quantity of riding comfort is car body's acceleration, and that of wheel traction is its relative displacement to the road surface. So, we make this model's state equation, having these value as the estimated output and x_p as the state as follows.

$$x_p = \begin{bmatrix} y_0 - y_1 \\ y_1 - y_2 \\ \dot{y}_1 \\ \dot{y}_2 \end{bmatrix}, \quad z_p = \begin{bmatrix} y_0 - y_1 \\ y_1 - y_2 \\ \ddot{y}_2 \end{bmatrix}$$

Now, we set $\dot{y}_0$ as the disturbance from the road surface w, and Δc as the control input u. Then its state space description is as follows.

It is known that the disturbance from the road surface $w = \dot{y}_0$ is a white noise 1).

$$\begin{array}{rcll} \dot{x}_p & = & A_p x_p + B_{p1} w & + B_{p2}(x) u \\ z_p & = & C_{p1} x_p & + D_{p12}(x) u, \end{array} \quad (3)$$

where

$$B_{p2}(x) = \begin{bmatrix} 0 \\ 0 \\ -\frac{x_3 - x_4}{m_1} \\ \frac{x_3 - x_4}{m_2} \end{bmatrix}, \quad D_{p12}(x) = \begin{bmatrix} 0 \\ 0 \\ \frac{x_3 - x_4}{m_2} \end{bmatrix}$$

Since this is a bilinear system, we cannot apply linear control theories to this system. In this report, we controlled this plant by nonlinear H_∞ state feedback.

4. Design of the control system

4.1. Nonlinear H_∞ control problem

Consider the following nonlinear system.

$$\begin{aligned} \dot{x} &= f(x) + g_1(x)w \quad + g_2(x)u \\ z &= h_1(x) \quad + j_{12}(x)u, \end{aligned} \tag{4}$$

where x, w, u, and z are state, exogenous disturbance input, control input, and output vector, respectively. Suppose this system satisfies $z = S_{zw}w$ and

$$h_1{}^T j_{12} = o, \quad j_{12}{}^T j_{12} = I. \tag{5}$$

Then nonlinear H_∞ control problem and conditions for its solubility of state feedback are as follows 2).

Nonlinear H_∞ control problem For the system by (4), find the state feedback controller $u = k(x)$ which satisfies the following; The closed loop system S_{zw} is internally (asymptotically) stable. In addition,

$$\|S_{zw}\|_{L_{2c}} = \sup_{w \in L_2/\{o\}} \frac{\|S_{zw}w\|_2}{\|w\|_2} \leq \gamma .$$

Conditions for solubility The nonlinear H_∞ control problem is solvable, if and only if there exist two positive definite function, $\phi(x)$ and $\rho(x)$, which satisfy the following two conditions.

$$\begin{aligned} &\frac{\partial \phi}{\partial x^T} f + \frac{1}{4\gamma^2}\frac{\partial \phi}{\partial x^T} g_1 g_1{}^T \frac{\partial \phi}{\partial x} + h_1{}^T h_1 + \rho \\ &- \frac{1}{4}\frac{\partial \phi}{\partial x^T} g_2 g_2{}^T \frac{\partial \phi}{\partial x} \leq 0 \end{aligned} \tag{6}$$

$$\lim_{\|x\| \to 0} \|g_1{}^T \frac{\partial \phi}{\partial x}\|^2 / \rho < \infty \tag{7}$$

When there exist $\phi(x)$ and $\rho(x)$, one of nonlinear state feedback controllers can be given by

$$k(x) = -\frac{1}{2} g_2{}^T \frac{\partial \phi}{\partial x} \quad . \tag{8}$$

□

4.2. Generalized plant

4.2.1. Filtering

Generalized plant made from (3) cannot satisfy the above assumption (5) in nonlinear H_∞ control problem. So, we filtered the acceleration of car body with a low-pass filter which doesn't influence the design, and added $\ddot{y}_{2f}$ and u as factors of the estimated output:

$$z_f = [\ u \quad y_0 - y_1 \quad y_1 - y_2 \quad \ddot{y}_{2f}\ ]^T$$

Then the state space description of the model becomes as

$$\begin{aligned} \dot{x}_f &= A_f x_f + B_{f1}\omega \quad + B_{f2}(x)u \\ z_f &= C_{f1}x_f \quad + D_{f12}u \ . \end{aligned} \tag{9}$$

Generalized plant made from these equations satisfies the assumption (5). The block diagram is shown in Fig.2. $W(s)$ is a frequency weight function for the control object.

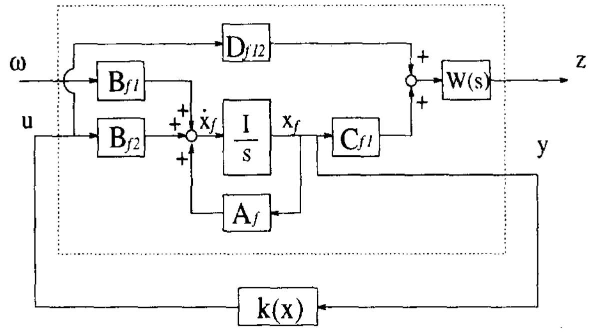

Figure 2: Generalized plant

4.2.2. Weight function

The weight function $W(s)$ to the output z_f forms as follows.

$$W(s) = \mathbf{diag}[1, W_2, W_3, W_1]$$

Here, the control object is

to reduce of car body's acceleration,

and only W_1 to car body's acceleration is determined, but not W_2, W_3.

4.2.3. State equations of generalized plant

The weight function $W_i(s)$ is

$$D_{W_i} + C_{W_i}(sI - A_{W_i})^{-1}B_{W_i},$$

and state vector of generalized plant is

$$x = [x_f \ x_{W_2} \ x_{W_3} \ x_{W_1}]^T.$$

Then state space description of generalized plant including the weight function is

$$\begin{aligned} \dot{x} &= Ax + B_1 w \quad + B_2(x)u \\ z &= C_1 x \quad + D_{12}u \\ y &= C_2 x \quad , \end{aligned} \tag{10}$$

where

$$A = \begin{bmatrix} A_f & o \\ B_W C_{f1} & A_W \end{bmatrix}, \ B_1 = \begin{bmatrix} B_{f1} \\ o \end{bmatrix}$$

$$B_2(x) = \begin{bmatrix} B_{f2}(x) \\ o \end{bmatrix}, \ C_1 = \begin{bmatrix} o & o \\ D_W C_{f1} & C_W \end{bmatrix}$$

$$C_2 = I, \quad D_{12} = D_{f12}$$

$$A_W = \mathbf{diag}(A_{W2}, A_{W2}, A_{W1})$$

$$B_W = \begin{bmatrix} 0 & B_{W2} & 0 & 0 \\ 0 & 0 & B_{W2} & 0 \\ 0 & 0 & 0 & B_{W1} \end{bmatrix}$$

$$C_W = \mathbf{diag}(C_{W2}, C_{W2}, C_{W1})$$

$$D_W = \begin{bmatrix} 0 & D_{W2} & 0 & 0 \\ 0 & 0 & D_{W2} & 0 \\ 0 & 0 & 0 & D_{W1} \end{bmatrix}$$

4.3. Application to bilinear system

Comparing generalized plant (10) and (4), it is clear that

$$f(\boldsymbol{x}) = \boldsymbol{A}\boldsymbol{x}, \quad g_1(\boldsymbol{x}) = \boldsymbol{B}_1, \quad g_2(\boldsymbol{x}) = \boldsymbol{B}_2(\boldsymbol{x}),$$
$$h_1(\boldsymbol{x}) = \boldsymbol{C}_1\boldsymbol{x}, \quad j_{12}(\boldsymbol{x}) = \boldsymbol{D}_{12}.$$

These satisfy the assumption (5). Substituting them for the inequality (6), leads to

$$\frac{\partial\phi}{\partial\boldsymbol{x}^T}\boldsymbol{A}\boldsymbol{x} + \frac{1}{4\gamma^2}\frac{\partial\phi}{\partial\boldsymbol{x}^T}\boldsymbol{B}_1{\boldsymbol{B}_1}^T\frac{\partial\phi}{\partial\boldsymbol{x}} + \boldsymbol{x}^T{\boldsymbol{C}_1}^T\boldsymbol{C}_1\boldsymbol{x} + \rho$$
$$- \frac{1}{4}\frac{\partial\phi}{\partial\boldsymbol{x}^T}\boldsymbol{B}_2(\boldsymbol{x}){\boldsymbol{B}_2}^T(\boldsymbol{x})\frac{\partial\phi}{\partial\boldsymbol{x}} \le 0 \ . \qquad (11)$$

Here, we choose

$$\phi(\boldsymbol{x}) = \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{x}, \quad \rho(\boldsymbol{x}) = \varepsilon\boldsymbol{x}^T\boldsymbol{x}$$

as a positive definite function $\phi(\boldsymbol{x})$ and $\rho(\boldsymbol{x})$, where $\boldsymbol{P}$ is a positive definite matrix and ε is a sufficiently small positive value. The function $\phi(\boldsymbol{x})$ satisfies

$$\frac{\partial\phi}{\partial\boldsymbol{x}} = \boldsymbol{P}\boldsymbol{x} + \boldsymbol{P}^T\boldsymbol{x}, \quad \frac{\partial\phi}{\partial\boldsymbol{x}^T} = \boldsymbol{x}^T\boldsymbol{P} + \boldsymbol{x}^T\boldsymbol{P}^T.$$

Substituting the above relatis to (11), we get

$$\boldsymbol{x}^T(\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{P} + \frac{1}{\gamma^2}\boldsymbol{P}\boldsymbol{B}_1{\boldsymbol{B}_1}^T\boldsymbol{P} + {\boldsymbol{C}_1}^T\boldsymbol{C}_1 + \varepsilon I)\boldsymbol{x}$$
$$- \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{B}_2(\boldsymbol{x}){\boldsymbol{B}_2}^T(\boldsymbol{x})\boldsymbol{P}\boldsymbol{x} \le 0 \ . \qquad (12)$$

Choosing the positive definite matrix $\boldsymbol{P}$ in order to satisfy the following Riccati equation,

$$\boldsymbol{P}\boldsymbol{A}+\boldsymbol{A}^T\boldsymbol{P}+\frac{1}{\gamma^2}\boldsymbol{P}\boldsymbol{B}_1{\boldsymbol{B}_1}^T\boldsymbol{P}+\boldsymbol{C}_1{\boldsymbol{C}_1}^T+\varepsilon I = \boldsymbol{O} \ , \quad (13)$$

the left-hand side of (12) becomes

$$-\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{B}_2(\boldsymbol{x}){\boldsymbol{B}_2}^T(\boldsymbol{x})\boldsymbol{P}\boldsymbol{x} \ ,$$

which is always less than zero. So, if $\boldsymbol{P}$ satisfies the Riccati equation (13), Hamilton-Jacobi inequality (6) is satisfied. Since $\phi(\boldsymbol{x})$ and $\rho(\boldsymbol{x})$ satisfy another condition for solubility (7), nonlinear H_∞ control problem has been solved. And, substituting them for (8), the feedback control law is as follows 3).

$$\boldsymbol{u} = k(\boldsymbol{x}) = -{\boldsymbol{B}_2}^T(\boldsymbol{x})\boldsymbol{P}\boldsymbol{x} \qquad (14)$$

5. Simulation

With the above method, we designed the controller of semi-active suspension and made some simulations.

This simulations are compared with active suspension's simulation results, which were designed by linear H_∞ control theory. And parameters are determined to optimize the passive response in simulations 4).

5.1. Parameters

Parameters used in simulation are shown in the following table.

Parameter	Value	Unit
m_1	0.5	(kg)
m_2	2.0	(kg)
k_1	1.97×10^3	(N/m)
k_2	40.0	(N/m)
c_p	12.49	$(N \cdot s/m)$

The low-pass filter to car body's acceleration $\ddot{y}_2$ is

$$filter = \frac{1}{1 + 0.002s} \ ,$$

and weight functions for the design are

$$W_1(s) = \frac{0.143}{(1 + 0.015s)^3}$$

and

$$W_2(s) = W_3(s) = \frac{400}{1 + 3000s}$$

In addition, the maximum and minimum value of the variable damper's damping coefficient $c = c_p + \Delta c$, is set as follows.

Maximum value	$c_{max} =$	50.0
Minimum value	$c_{min} =$	0.0

Finally, γ and ε in Riccati equation (13) are

$$\gamma = 1.0, \varepsilon = 1.0 \times 10^{-4} \ .$$

5.2. Result and examination

5.2.1. Time response

We gave sine wave with different amplitude as the disturbance from the road surface. The time response of $\ddot{y}_2$ are shown in Fig.3 and Fig.4. Semi-active suspension's vibration is bigger than that of active suspension, but smaller than the passive one. And the effectivenes of vibration control in semi-active suspension is remarkable as the amplitude of the disturbance becomes bigger. When the vibration is small, $\boldsymbol{u}$ is less effective, since $\boldsymbol{B}_2(\boldsymbol{x})$ in (14) is nearly equal to zero.

This characteristic is effective in terms that the control input doesn't become big when the vibration is small.

5.2.2. Frequency response

We compared semi-active control with active and passive control in frequency domain by using the M-series signal as the disturbance and calculating the spectrum

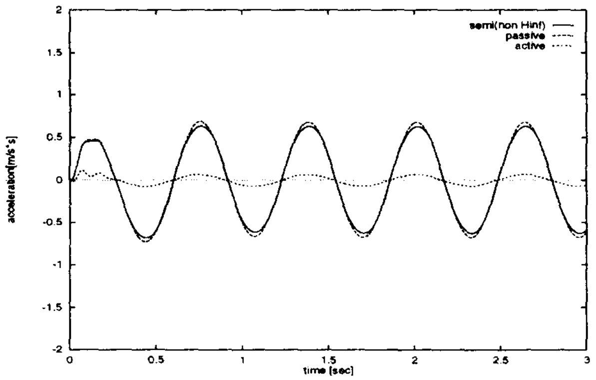

Figure 3: Time response of $\ddot{y}_2$ $(\dot{y}_0 = 0.1\sin 10t)$

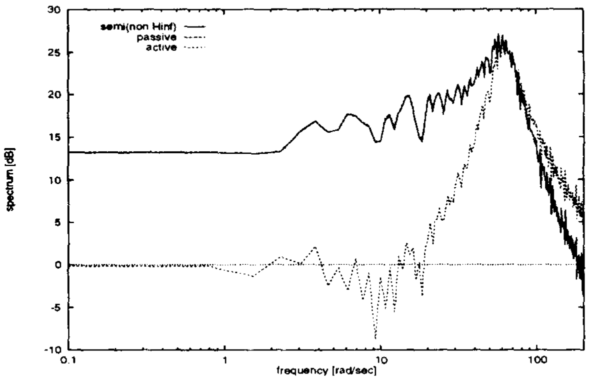

Figure 5: Spectrum Ratio (M-sequence : -36dB)

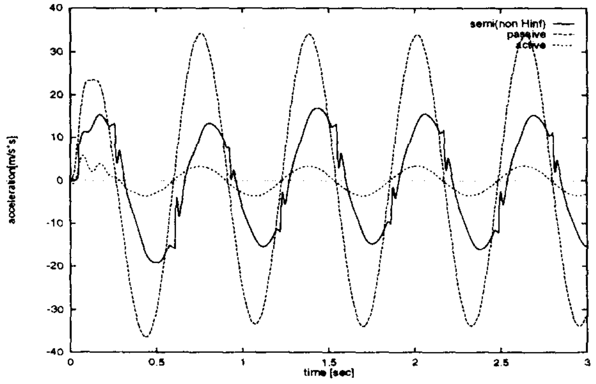

Figure 4: Time response of $\ddot{y}_2$ $(\dot{y}_0 = 5.0\sin 10t)$

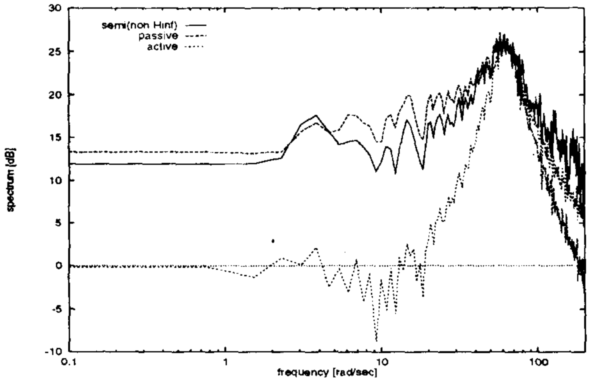

Figure 6: Spectrum Ratio (M-sequence : -16dB)

ratio between the output $\ddot{y}_2$ and the disturbance. The spectrum ratio G is given as

$$G = 20\log_{10}\sqrt{\frac{P_{out}}{P_{dis}}} \quad (dB) ,$$

where P_{out} is the power spectrum of $\ddot{y}_2$ and P_{dis} is that of the disturbance. Note that, G is equal to the gain (dB) in linear plants.

This results are shown in Fig.5 and Fig.6. It is clear that the spectrum ratio of semi-active suspension designed by nonlinear H_∞ control is smaller as the amplitude of the disturbance is bigger.

6. Conclusion

In this design of semi-active suspension with nonlinear H_∞ state feedback, we obtained the following conclusions.

- We can calculate the state feedback control law of nonlinear H_∞ control by solving linear Riccati equation.
- The method is not so effective in case the amplitude of the disturbance is big, but enough good when the small vibration can be neglected.
- The control object is achieved.

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