'95 KACC (1995. 10. 23 ~ 25)

Robust Stability Evaluation of Multi-Loop Control Systems based on Experimental Data of Frequency Response

Hong CHEN, Yoshifumi OKUYAMA and Fumiaki TAKEMORI

Department of Information and Knowledge Engineering,
Faculty of Engineering, Tottori University,
4-101, Koyama-cho Minami, Tottori 680, Japan
Phone +81-857-31-5213, Fax +81-857-31-0879
E-mail: chen@ike.tottori-u.ac.jp

Abstract: In this paper, we describe the composition of frequency response bands based on experimental data of plants (controlled systems) with uncertainty and nonlinearity, and the robust stability evaluation of feedback control systems. Analysis and design of control systems using the upper and lower bounds of such experimental data would be effective as a practicable method which is not heavily dependent upon mathematical models such as the transfer function. First, we present a method to composite gain characteristic bands of frequency response of cascade connected plants with uncertainty and a recurrent inequality for the composition. Next, evaluation methods of the robust stability of multi-loop control systems obtained through feedback from the output terminals and multi-loop control systems obtained through feedback into the input terminals are described.

In actual control systems, experimental data of frequency responses often depends on the amplitude of input. Therefore, we present the evaluation method of the nominal value and the width of the frequency response band in such a case, and finally give numerical examples based on virtual experimental data.

1 Introduction

Analysis and design of control systems using the upper and lower bounds of experimental data of frequency responses would be effective as a practicable method which is not heavily dependent upon mathematical models such as the transfer function. This paper presents the composition method of frequency response bands based on experimental data of plants and control elements with uncertainty and nonlinearity, and the robust stability evaluation of feedback control systems. The composition of frequency response bands is represented sequentially by a recurrent inequality, and the evaluation of the robust stability is expressed by a series of inequalities provided in the reduction process of the block diagram.

In actual control systems, experimental data of frequency responses often depends on the amplitude of input. Therefore, we present the evaluation method of the nominal value and the width of the frequency response band in such a case, and give numerical examples based on virtual experimental data.

2 Cascade Connection

The frequency responses given from experimental data are written as nominal systems and their multiplicative perturbations as follows:

$$G_k^*(j\omega) = G_k(j\omega)(1 + \Delta_k(j\omega)), \quad (k = 1, 2, \cdots, N), \tag{1}$$

where G_k is a parameterized nominal model of a controlled system and Δ_k is an unstructured uncertainty. They are represented only as a band where the data exists, because of the uncertainty in the high frequency range and the nonlinearity on the input terminal. That is, the upper bound of the uncertain term of frequency responses can be written as

$$|\Delta_k(j\omega)| \leq \rho_k(\omega), \tag{2}$$

where $\rho_k(\omega)$ is a frequency-dependent radius. The following discussions can be applied to any case even though the perturbation $\Delta_k(j\omega)$ changes in Eq. (1) according to how the nominal system $G_k(j\omega)$ is considered.

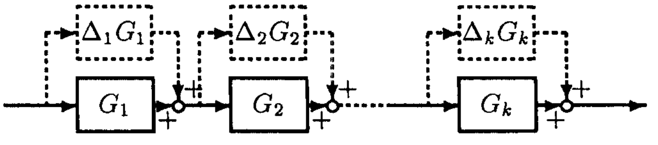

Fig. 1 Cascade-connected system.

the uncertainties when Eq. (9) is satisfied.

On the other side, for the case where the k-th nominal feedback subsystem is unstable, we can see that the k-th feedback control system is unstable regardless of the existence of the uncertainties when Eq. (9) is satified.

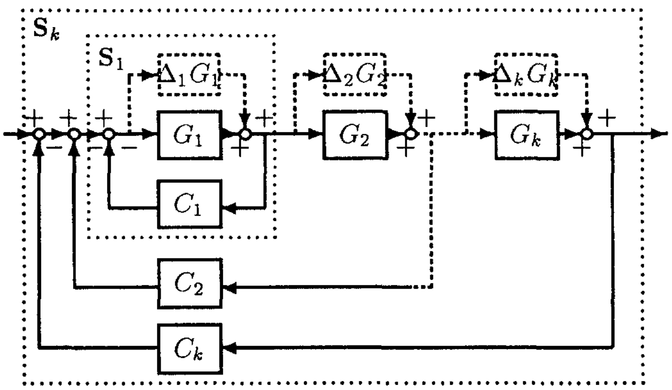

Fig. 4 Multi-loop control system-2.

As for a multi-loop control system as shown in Fig. 4, the robust stability condition or the invariant condition of the dynamic characteristic is written by the same form as Eq. (6), i.e.,

$$|(1 + C_1(s)G_1(s))^{-1}C_1(s)G_1(s)| < 1/\rho_1(s), \quad (11)$$

as to

$$|\Delta_1(s)| \le \rho_1(s), \quad s \in \Gamma.$$

A series of inequalities for the robustness condition of the dynamic characteristic corresponding to Eq. (9) is obtained by sequential reduction from the subsystem S_1 as follows:

$$|(1 + C_k(s)\mathcal{G}_k(s))^{-1}C_k(s)\mathcal{G}_k(s)| < 1/|\mathcal{D}_k(s)|, \quad s \in \Gamma, \quad (12)$$

where we assume

$$\mathcal{G}_k(s) = G_k(s)(1+C_{k-1}(s)\mathcal{G}_{k-1}(s))^{-1}\mathcal{G}_{k-1}(s), \quad (13)$$

$$\mathcal{G}_0(s) \equiv 1, \quad C_0(s) \equiv 0.$$

The upper bound of the absolute value of transfer characteristic $\mathcal{D}_k(s)$ is the same as that in Eq. (10).

Inequality conditions Eqs. (11) and (12) are same form as those of the multi-loop control system-1, that is, Eqs. (6) and (9). However, if we handle variables and transfer functions are assumed to be a vector and a matrix, the order of the matrix expression will be important.

4 Amplitude Dependency[3]

The experimental data of those frequency responses often depends on the amplitude of input in an actual system. In such a case, the frequency response given from experimental data is written as a nominal system and its multiplicative perturbation as follows:

$$G_k^*(a_k, s) = G_k(a_k, s)(1 + \Delta_k(a_k, s)), \quad s \in \Gamma. \quad (14)$$

By considering that the frequency response characteristic varies with the amplitude parameter a_k, the analysis and design of the control system using mathematical model becomes more realistic.

When feedback takes place for such systems, it is necessary to examine not only the condition of a mere robust stability but also the condition of invariance (robustness condition) of the dynamic characteristic. That is, it means a condition that the characteristic root which exists on the left half plane will not move to the right half plane, and vice versa, the characteristic root which exists on the right half plane will not move to the left half plane.

In this paper, we also consider the gain characteristics band which depends on the amplitude and the movement of the band which corresponds to a traditional describing function. When feedback control for such systems is taken place, it is necessary to examine the stability problem of the closed loop system. we consider the first feedback subsystem as shown in Fig. 3.

The robustness condition which corresponds to Eq. (8) is represented as

$$|\mathcal{G}_k(a_k, s)C_k(s)(1+\mathcal{G}_k(a_k, s)C_k(s))^{-1}| < 1/|\mathcal{D}_k(a_k, s)|, \quad (15)$$

where $\mathcal{G}_k(a_k, s)$ and $|\mathcal{D}_k(a_k, s)|$ are defined by the same form as Eq. (8) and (10) (or Eq. (13) and (14)), respectively.

We consider a control system which is unstable regardless of the existence of the uncertainty when the amplitude parameter a_k is small, and becomes stable (robust stable) regardless of the existence of the uncertainty when the amplitude parameter a_k is large. This case always occurs in an actual control system due to the saturation of each element. For time response in this case, the sustained oscillation (periodic or pseudo-periodic oscillation) with a limited amplitude is estimated, independent of whether it clearly becomes a limit cycle.

5 Numerical Examples

The Bode diagram of the band of gain characteristics (frequency response bands) of control systems whose uncertainty is given in the form of multiplicative perturbation is shown.

[Example 1] Consider a cascade connection $G^*(s) = G_1^*(s)G_2^*(s)$. The calculation result of the cascade connection of the frequency response bands G_1^* and G_2^* is shown in Fig. 5, where we assume that G_1^* and G_2^* contain saturation characteristics and first order lag uncertainties. Their 'modified' or estimated nominal transfer functions are as follows:

$$G_1(s) = \frac{10(1 + 0.01s)}{(4 + 2s + s^2)(1 + 0.02s)},$$

As for frequency response of the nominal system, the frequency transfer function is usually estimated from the amplitude ratio, that is, gain characteristic curves of those experimental data. However, for the minimum phase system, it is known that the phase characteristic can be provided from the gain characteristic.

Frequency response of a subsystem in which such control elements are cascade connected is given by the following recurrent inequality[1]:

$$|\mathcal{D}_k(j\omega)| \leq |1+\Delta_k(j\omega)|\cdot|\mathcal{D}_{k-1}(j\omega)|+|\Delta_k(j\omega)|, \quad (3)$$

$$\mathcal{D}_0(j\omega) \equiv 0,$$

where $\mathcal{D}_k$ is the uncertain term composited to the k-th term. When the upper bound of the absolute value of frequency response $|\Delta_k(j\omega)|$ and $|1+\Delta_k(j\omega)|$ can be determined experimentally, this recurrent inequality is applicable.

When only the upper bound $|\Delta_k(j\omega)|$ is known in Eq. (2), the following inequality can be applied:

$$|\mathcal{D}_k(j\omega)| \leq (1+\rho_k(\omega))\cdot|\mathcal{D}_{k-1}(j\omega)|+\rho_k(\omega). \quad (4)$$

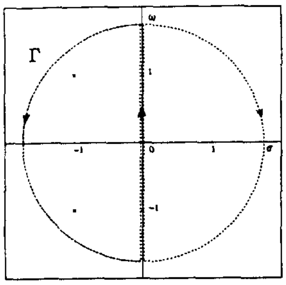

Fig. 2 s-plane contour Γ.

3 Robust Stability

When the cascade-connected system with the frequency response band is feedback connected, it is necessary to examine the problem of the robust stability. For the subsystem $\mathbf{S}_1$ of a multi-loop control system as shown in Fig. 3, the robust stability condition is given by

$$\|\Delta_1(s)G_1(s)C_1(s)(1+G_1(s)C_1(s))^{-1}\|_\infty < 1. \quad (5)$$

The condition Eq. (5) for H_∞ norm is equivalently rewritten as follows:

$$|G_1(s)C_1(s)(1+G_1(s)C_1(s))^{-1}| < 1/\rho_1(s), \quad (6)$$

as to

$$|\Delta_1(s)| \leq \rho_1(s), \quad s \in \Gamma.$$

Here, we consider the variable s on the closed curve Γ which encircles the left (or right) half plane including imaginary axis of the complex plane as shown in Fig. 2. By considering so, we can treat the relation between the frequency transfer characteristics of the nominal system $G_1(s)$ and the upper bound of an uncertain part $\Delta_1(s)$ provided from the experimental data on the s-plane.

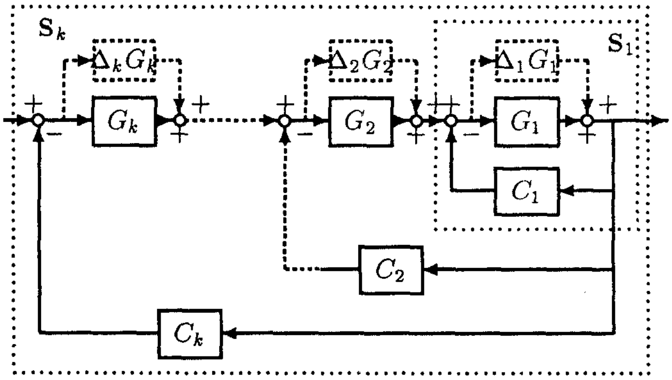

Fig. 3 Multi-loop control system-1.

For the case where nominal feedback subsystem is stable, that is, the closed loop transfer function of the subsystem $\mathbf{S}_1$ has no pole in the right half s-plane, we can see that the subsystem is stable regardless of the existence of the uncertainty when Eq. (6) is satisfied.

On the other hand, for the case where nominal feedback subsystem is unstable, that is, the closed loop transfer function of the subsystem $\mathbf{S}_1$ has some poles in the right half s-plane, we can see that the subsystem is unstable regardless of the existence of the uncertainty when Eq. (6) is satified.

Therefore, we can generally refer to the robust stability condition Eq. (6) as the condition of invariance (robustness condition) of the dynamic characteristic.

For the overall multi-loop control system shown in Fig. 3, the robust stability condition is represented as a series of inequalities by[2]

$$\|\mathcal{D}_k(s)\mathcal{G}_k(s)C_k(s)(1+\mathcal{G}_k(s)C_k(s))^{-1}\|_\infty < 1, \quad (7)$$

Here,

$$\mathcal{G}_k(s) = \mathcal{G}_{k-1}(s)(1+\mathcal{G}_{k-1}(s)C_{k-1}(s))^{-1}G_k(s), \quad (8)$$

$$\mathcal{G}_0(s) \equiv 1, \quad C_0(s) \equiv 0.$$

The criteria Eq. (7) based on H_∞ norm is rewritten as the robustness condition as follows:

$$|\mathcal{G}_k(s)C_k(s)(1+\mathcal{G}_k(s)C_k(s))^{-1}| < 1/|\mathcal{D}_k(s)|, \quad s \in \Gamma \quad (9)$$

where the upper bound of the absolute value of complex frequency transfer characteristic $\mathcal{D}_k(s)$ is given by

$$|\mathcal{D}_k(s)| \leq (1+\rho_k(s))\cdot|\mathcal{D}_{k-1}(s)|+\rho_k(s), \quad (10)$$

from Eq. (4).

For the case where the k-th nominal feedback subsystem is stable, we can see that the k-th feedback control system is stable regardless of the existence of

$$G_2(s) = 0.4 \cdot \frac{(1+5.0s)(1+0.01s)}{(1+0.5s)(1+0.02s)}.$$

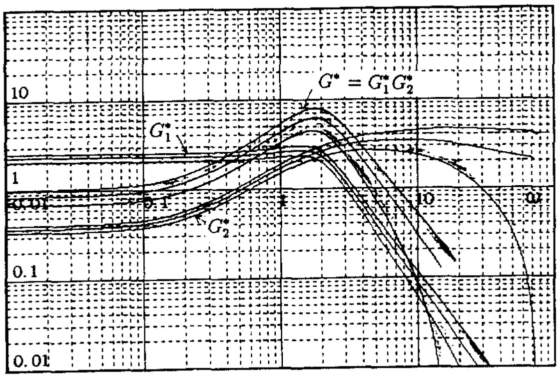

Fig. 5 Cascade connection.

[Example 2] Consider feedback control systems in which the plant is written as

$$G(s) = \frac{1}{s(2+2s+s^2)}.$$

Fig. 6 shows the frequency response band of a control system when feedback $K = 2.3$ takes place, as well as the open loop one.

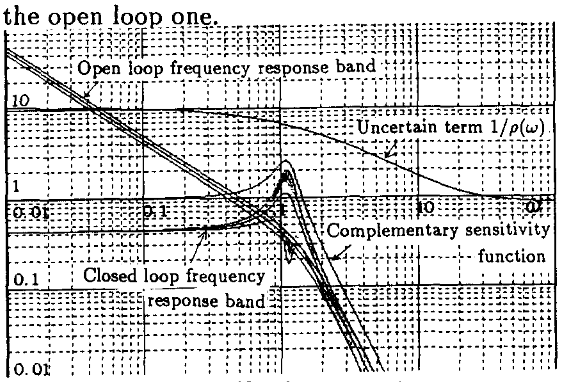

Fig. 6 Feedback connection.

Fig. 7 is the case where a feedback compensator

$$C(s) = 2.3 \cdot \frac{1+s}{1+0.2s}$$

is used. As is obvious from the figure, the robust stability is improved.

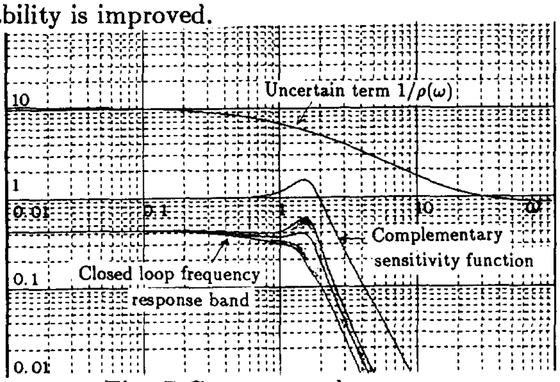

Fig. 7 Compensated system.

Fig. 8 is the case where the nominal system is unstable. In this case, the invariant condition of the dynamic characteristic, that is, the robustness ('robust instability') condition holds. When the gain K decreases depending on an increase of the amplitude parameter a_k and increases depending on a decrease of the amplitude parameter, the sustained oscillation, for instance, a limit cycle will be estimated.

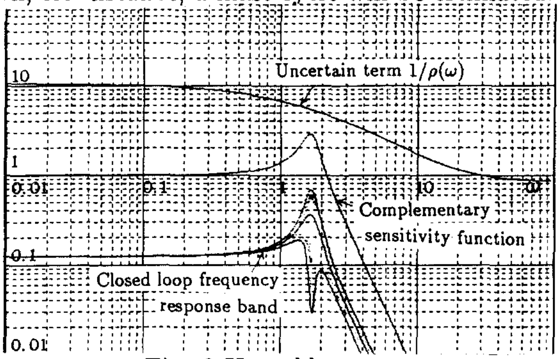

Fig. 8 Unstable system.

6 Conclusions

In this paper, we have described the composition method of the frequency response band by experimental data of frequency responses of the controlled systems with uncertainty and nonlinearity, and the evaluation method of the robust stability condition or the invariant condition of the dynamic characteristic of multi-loop control systems. The concept is applicable to a realistic case in which the frequency response band depends on the amplitude of input. The shifting of the frequency response band corresponds to the generalization of a traditional describing function method. With the application of such a concept, it will be possible to elucidate the existence of the periodic oscillation of a general multi-loop control system.

References

[1] H. Chen, Y. Okuyama and F. Takemori, "Composition of Frequency Responses for Control System with Model Uncertainty and its Graphical Representation," Proc. of the 2nd Asia-Pacific Conference on Control and Measurement, Wuhan-Chongqing, P.R.China, pp. 43-46, 1995.

[2] H. Chen, Y. Okuyama and F. Takemori, "Composition of Frequency Responses and Robust Stability for Multi-Loop Control System with Model Uncertainty," Proc. of the 34th SICE Annual Conference, International Session, pp. 1395-1398, 1995.

[3] Y. Okuyama, H. Chen and F. Takemori, "Robust Stability Evaluation of Control Systems Composed by Amplitude Dependent Frequency Response Characteristics," The 38th JACC of Japan, 1995 (to appear).