A New Kinematic Formulation of Closed-Chain Mechanisms with Redundancy and Its Applications to Kinematic Analysis

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Abstract This paper presents a new formulation of the kinematics of closed-chain mechanisms and its applications to obtaining the kinematic solutions and analyzing the singularities. Closed-chain mechanisms under consideration may have the redundancy in the number of joints. A closed-chain mechanism can be treated as the parallel connection of two open-chains with respect to a point of interest. The kinematics of a closed-chain mechanism is then obtained by imposing the kinematic constraints of the closed-chain on the kinematics of the two open-chains. First, we formulate the kinematics of a closed-chain mechanism using the kinematic constraint between the controllable active joints and the rest of joints, instead of the kinematic constraint between the two open-chains. The kinematic formulation presented in this paper is valid for closed-chain mechanisms with and without the redundancy. Next, based on the derived kinematics of a closed-chain mechanism, we provide the kinematic solutions which are more physically meaningful and less sensitive to numerical instability, and also suggest an effective way to analyze the singularities. Finally, the computational cost associated with the kinematic formulation is analyzed.

Keywords Closed-Chain, Open-chain, Kinematics, Redundancy, Singularity

1. INTRODUCTION

Closed-chain mechanisms have drawn attention from research society due to their advantage in rigidity and accuracy over open-chain mechanisms [8]. One example of closed-chain mechanisms can be found in the finger structure of UCSB hand, which is a five bar closed linkage [5]. Another example is a dual arm system which is a closed-chain formed by two open-chain or serial manipulator [6]. While open-chain mecahnisms consist of active joints (joints with actuators) only, closed-chain mechanisms may contain passive joints (joints without actuators), as well. The existence of passive joints makes it possible to be built to be lightweight [1], and furthermore, helps increasing motion flexibility at the cost of load capacity.

To improve the kinematic performance, one may introduce the redundancy in the number of active joints to a closed-chain mechanism. Such redundancy can be introduced either by putting additional active joints (Type I redundancy) or by replacing the passive joints with active ones (Type II redundancy). It is shown that Type I redundancy increases motion flexibility at the expense of load capacity, and Type II redundancy increases load capacity at the expense of motion flexibility [7]. Type I and II redundancies and their combination may be useful to design closed-chain mechanisms for applications demanding high performance. It should be noted that Type II redundancy does not increase the number of active joints which can be controlled independently, although it provides us with the multiple choices in selecting the set of controllable active joints.

A closed-chain mechanism is allowed to have limited numbers of active and passive joints to maintain mobility and controllability in task execution. Let n_a be the number of controllable active joints and n_p be the number of uncontrollable active and passive joints. For an m d.o.f. closed-chain mechanism ($m=3$ for a planar mechanism and $m=6$ for a spatial mechanism), the mobility condition states that the number of whole joints, n_a+n_p, should be greater than or equal to $2m$, that is, $n_a+n_p \geq 2m$; the controllability condition states that the number of uncontrollable joints, n_p, should be equal to m, that is, $n_p=m$. In the case of a 6 d.o.f. dual-arm system, which is an extreme of closed-chain mechanisms having twelve active joints with no passive one, only six active joints are controllable, such that $n_a=n_p=6$, satifying both mobility and controllability conditions.

It is convenient to treat a closed-chain mechanism as the parallel connection of two open-chains, each of which may contain passive and/or active joints, with respect to a point of interest. There may be two different approaches to formulating the kinematics of a closed-chain mechanism from the kinematics of the two open-chains. The first approach is based on the kinematic constraint existing between the two open-chains interacting each other [2]. The second approach, to be dealt with in this paper, is based on the kinematic constraint existing between the controllable active joints and the rest of joints.

The kinematic analysis of closed-chain mechanisms with redundancies mentioned above is essential for the design and implementation of advanced closed-chain mechanisms. It is a major task to develop the kinematic solutions, which is physically meaningful, numerically stable, as well as computationally efficient. One such effort would be to reduce the number of matrix inversions involved in the kinematic solutions. Next, it is also important to provide an effective way to detect the singularities, at which a closed-chain mechanism cannot resist certain external forces or maintain its structure. The singularities of a closed-chain mechanism may cause serious problems during operation, for instance, the damage of a closed-chain mechanism and an object under manipulation.

This paper presents a new formulation of the kinematics of a closed-chain mechanism possibly the redundancy in the number of joints, and demonstrates its power for the kinematic analysis. This paper is organized as follows: In Section 2, we formulate the kinematics of a closed-chain mecahnism based on the kinematic constraint between the two open-chains. Problems related to the derived kinematics in the kinematic analysis is discussed. In Section 3, we reformulate the kinematics based on the kinematic constraint between the controllable active joints and the rest of joints The derived kinematics is shown to be effective in obtaining the kinematic solutions and analyzing the singularities. In Section 4, we analyze and compare the computational cost associated with the kinematic formulations in Section 2 and 3. Finally, conclusions are made in Section 5.

In this paper, a closed-chain mechanism is assumed to satisfy both mobility and controllablity conditions. The controllable active joints are referred to as active joints and the rest of joints are referred to as passive joints.

2. PROBLEM STATEMENT

This section formulates the kinematics of a closed-chain mecahnism based on the kinematic constraint between the two open-chains, and describes problems related to the derived kinematics in terms of physical meaning, computatinal requirement, and singularity analysis. In what follows, the two open-chains are denoted by limb 1 and limb 2.

Let $\boldsymbol{\theta}_{ia}$ and $\boldsymbol{\theta}_{ip}$, $i=1,2$, be the active and the passive joints of limb i. The Cartesian velocity at a task point, TP, of limb i, $\dot{\mathbf{x}}_i$, is given by

$$\dot{\mathbf{x}}_i = \mathbf{J}_{ia}\,\dot{\boldsymbol{\theta}}_{ia} + \mathbf{J}_{ip}\,\dot{\boldsymbol{\theta}}_{ip}, \quad i=1,2 \tag{1}$$

where $\mathbf{J}_{i\alpha}$, $i=1,2$, $\alpha=a,p$. represents the Jacobian of limb i corresponding to $\dot{\boldsymbol{\theta}}_{i\alpha}$. With

$$\dot{\boldsymbol{\theta}}_i^{\,t} = [\ \dot{\boldsymbol{\theta}}_{ia}^{\,t}\ \ \dot{\boldsymbol{\theta}}_{ip}^{\,t}\ ]^{t}, \quad i=1,2 \tag{2}$$

$$\mathbf{J}_i = [\ \mathbf{J}_{ia}\ \ \mathbf{J}_{ip}\ ], \quad i=1,2 \tag{3}$$

(1) can be written as

$$\dot{\mathbf{x}}_i = \mathbf{J}_i\,\dot{\boldsymbol{\theta}}_i, \quad i=1,2 \tag{4}$$

The Cartesian velocity at TP of a closed-chain mechanism, $\dot{\mathbf{x}}_o$, can be expressed as

$$\dot{\mathbf{x}}_o = \dot{\mathbf{x}}_1 = \dot{\mathbf{x}}_2 \tag{5}$$

(5) represents the kinematic constraint existing between two limbs, indicating that two limbs are constrained by each other in generating motions at TP.

For a given $\dot{\mathbf{x}}_o$, from (4) and (5),

$$\dot{\boldsymbol{\theta}}_i = \mathbf{Q}_i\,\dot{\mathbf{x}}_o, \quad i=1,2 \tag{6}$$

where

$$\mathbf{Q}_i = \mathbf{J}_i^{\,t}\,(\,\mathbf{J}_i\,\mathbf{J}_i^{\,t}\,)^{-1}, \quad i=1,2 \tag{7}$$

which is the minimum norm solution of $\dot{\boldsymbol{\theta}}_i$. From (6),

$$\begin{bmatrix} \dot{\boldsymbol{\theta}}_{1a} \\ \dot{\boldsymbol{\theta}}_{2a} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_{1a} \\ \mathbf{Q}_{2a} \end{bmatrix} \dot{\mathbf{x}}_o \tag{8}$$

where $\mathbf{Q}_{ia}$, $i=1,2$, represents the submatrix of $\mathbf{Q}_i$ corresponding to $\dot{\boldsymbol{\theta}}_{ia}$. With

$$\dot{\boldsymbol{\theta}}_a^{\,t} = [\ \dot{\boldsymbol{\theta}}_{1a}^{\,t}\ \ \dot{\boldsymbol{\theta}}_{2a}^{\,t}\ ]^{t} \tag{9}$$

$$\mathbf{Q}_a^{\,t} = [\ \mathbf{Q}_{1a}^{\,t}\ \ \mathbf{Q}_{2a}^{\,t}\ ]^{t} \tag{10}$$

(8) can be written as

$$\dot{\boldsymbol{\theta}}_a = \mathbf{Q}_a\,\dot{\mathbf{x}}_o \tag{11}$$

(11) represents the inverse kinematics of a closed-chain mechanism, relating the Cartesian velocity $\dot{\mathbf{x}}_o$ to the active joint velocity $\dot{\boldsymbol{\theta}}_a$. From (11),

$$\dot{\mathbf{x}}_o = (\,\mathbf{Q}_a^{\,t}\,\mathbf{Q}_a\,)^{-1}\,\mathbf{Q}_a^{\,t}\,\dot{\boldsymbol{\theta}}_a \tag{12}$$

(12) represents the forward kinematics of a closed-chain mechanism, relating $\dot{\boldsymbol{\theta}}_a$ to $\dot{\mathbf{x}}_o$.

Let $\boldsymbol{\tau}_a$ be the joint torque corresponding to $\dot{\boldsymbol{\theta}}_a$. And, let $\mathbf{f}_o$ be the Cartesian force at TP of a closed-chain mechanism. Using the priciple of virtual work, from (11),

$$\mathbf{f}_o = \mathbf{Q}_a^{\,t}\,\boldsymbol{\tau}_a \tag{13}$$

(13) can be used for the singularity analysis of a closed-chain mechanism.

Related to the kinematic formulation of a closed-chain mecahnism, given above, the following discussions can be made:

1) The inverse kinematic solution $\dot{\boldsymbol{\theta}}_a$, given by (11), consists of subvectors of two solutions $\dot{\boldsymbol{\theta}}_1$ and $\dot{\boldsymbol{\theta}}_2$, which are obtained subject to min $\|\,\dot{\boldsymbol{\theta}}_1\,\|^2$ and min $\|\,\dot{\boldsymbol{\theta}}_2\,\|^2$, respectively. Refer to (6). Note that $\|\,\dot{\boldsymbol{\theta}}_i\,\|^2 = \|\,\dot{\boldsymbol{\theta}}_{ia}\,\|^2 + \|\,\dot{\boldsymbol{\theta}}_{ip}\,\|^2$, $i=1,2$. It seems physically meaningless to minimize the norm of the mixture of active and passive ones.

2) Seen from (11) and (12), the kinematic formulation requires the computation of $\mathbf{Q}_a$ consisting of submatrices of $\mathbf{Q}_1$ and $\mathbf{Q}_2$, which involve the inversion of $\mathbf{J}_1$ and $\mathbf{J}_2$. Refer to (7). The two matrix inversion involved in the kinematic formulation may be adverse to numerical stability as well as computational efficiency.

3) Based on (13), a closed-chain mechanism is said to be singular if there exists a nonzero solution $\boldsymbol{\tau}_a$ satisfying

$$\mathbf{0} = \mathbf{Q}_a^{\,t}\,\boldsymbol{\tau}_a \tag{14}$$

(14) tells that the singularity of a closed-chain mechanism occurs within the joint torque space, specified by the nullspace of $\mathbf{Q}_a^{\,t}$. The singularity

of a closed-chain mechanism can be detected by checking the dimension of $\mathbf{Q}_a{}'$, which is not readily obtained, as mentioned in 2).

3. NEW KINEMATIC FORMULATION

This section develops a new formulation of the kinematics of a closed-chain mecahnism based on the kinematic constraint between the active joints and the passive joints. The derived kinematics is examined to show its effectiveness in the kinematic analysis, compared to the kinematics obtained in Section 2.

The passive joint velocity of limb i, $\dot{\boldsymbol{\theta}}_{ip}$, $i=1,2$, is uniquely determined according to the active joint velocities of limb 1 and limb 2, $\dot{\boldsymbol{\theta}}_{1a}$ and $\dot{\boldsymbol{\theta}}_{2a}$:

$$\dot{\boldsymbol{\theta}}_{ip} = \mathbf{G}_{i1}\ \dot{\boldsymbol{\theta}}_{1a} + \mathbf{G}_{i2}\ \dot{\boldsymbol{\theta}}_{2a},\quad i=1,2 \tag{15}$$

where

$$\mathbf{G}_{ij} = \frac{\partial\ \boldsymbol{\theta}_{ip}}{\partial\ \boldsymbol{\theta}_{ja}},\quad i,j=1,2 \tag{16}$$

With

$$\dot{\boldsymbol{\theta}}_p{}^t = [\ \dot{\boldsymbol{\theta}}_{1p}{}^t\ \ \dot{\boldsymbol{\theta}}_{2p}{}^t\ ]^t \tag{17}$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix} \tag{18}$$

(15) can be written as

$$\dot{\boldsymbol{\theta}}_p = \mathbf{G}\ \dot{\boldsymbol{\theta}}_a \tag{19}$$

(19) represents the kinematic constraint existing between the active and the passive joints.

Plugging (15) into (1),

$$\begin{aligned}\dot{\mathbf{x}}_o &= [\ \mathbf{J}_{1a}+\mathbf{J}_{1p}\mathbf{G}_{11}\quad \mathbf{J}_{1p}\mathbf{G}_{12}\ ]\begin{bmatrix}\dot{\boldsymbol{\theta}}_{1a}\\ \dot{\boldsymbol{\theta}}_{2a}\end{bmatrix} \\ &= [\ \mathbf{J}_{2p}\mathbf{G}_{21}\quad \mathbf{J}_{2a}+\mathbf{J}_{2p}\mathbf{G}_{22}\ ]\begin{bmatrix}\dot{\boldsymbol{\theta}}_{1a}\\ \dot{\boldsymbol{\theta}}_{2a}\end{bmatrix}\end{aligned} \tag{20}$$

With

$$\mathbf{J}_o = [\ \mathbf{J}_1\quad \mathbf{J}_2\ ] \tag{21}$$

where

$$\begin{aligned}\mathbf{J}_1 &= \mathbf{J}_{1a}+\mathbf{J}_{1p}\mathbf{G}_{11} = \mathbf{J}_{2p}\mathbf{G}_{21}\\ \mathbf{J}_2 &= \mathbf{J}_{1p}\mathbf{G}_{12} = \mathbf{J}_{2a}+\mathbf{J}_{2p}\mathbf{G}_{22}\end{aligned} \tag{22}$$

(20) can be written as

$$\dot{\mathbf{x}}_o = \mathbf{J}_o\ \dot{\boldsymbol{\theta}}_a \tag{23}$$

and thus

$$\dot{\boldsymbol{\theta}}_a = \mathbf{J}_o{}^t\ (\ \mathbf{J}_o\ \mathbf{J}_o{}^t\ )^{-1}\ \dot{\mathbf{x}}_o \tag{24}$$

(23) and (24) represent the forward and the inverse kinematics of a closed-chain mechanism. Note that the inverse kinematic solution, given by (24), is the minimum norm solution of $\dot{\boldsymbol{\theta}}_a$.

Referring to (16) and (18), the compuation of $\mathbf{G}$ seems to require the expression of $\boldsymbol{\theta}_{ip}$, $i=1,2$, as a function of $\boldsymbol{\theta}_{1a}$ and $\boldsymbol{\theta}_{2a}$, which is difficult to obtain in general. However, it is possible to compute $\mathbf{G}$ without the expression of $\boldsymbol{\theta}_{ip}$, $i=1,2$, as follows:

Rearranging (20),

$$\mathbf{J}_a\ \dot{\boldsymbol{\theta}}_a = \mathbf{J}_p\ \mathbf{G}\ \dot{\boldsymbol{\theta}}_a \tag{25}$$

with

$$\mathbf{J}_\alpha = [\ \mathbf{J}_{1\alpha}\quad -\mathbf{J}_{2\alpha}\ ],\quad \alpha=a,p \tag{26}$$

Since (25) holds for any $\dot{\boldsymbol{\theta}}_a$,

$$\mathbf{J}_a = \mathbf{J}_p\ \mathbf{G} \tag{27}$$

and thus

$$\mathbf{G} = \mathbf{J}_p{}^{-1}\ \mathbf{J}_a \tag{28}$$

(28) tells that $\mathbf{G}$ can be computed from $\mathbf{J}_a$ and $\mathbf{J}_p$, both of which are readily available. Note that $\mathbf{J}_p$ is always invertible.

For the singularity analysis, using (18), (25) can be rewritten as

$$\mathbf{J}_a\ \dot{\boldsymbol{\theta}}_a = \mathbf{J}_p\ \dot{\boldsymbol{\theta}}_p \tag{29}$$

Related to the kinematic formulation of a closed chain mechanism, given above, the following discussions can be made:

1) The inverse kinematic solution $\dot{\boldsymbol{\theta}}_a$, given by (24), is obtained subject to min $\|\ \dot{\boldsymbol{\theta}}_a\ \|^2$. Note that $\|\ \dot{\boldsymbol{\theta}}_a\ \|^2 = \|\ \dot{\boldsymbol{\theta}}_{1a}\ \|^2 + \|\ \dot{\boldsymbol{\theta}}_{2a}\ \|^2$. It would be physically meaningful to minimize the norm of the whole active joints, instead of the norm of the mixture of active and passive joints, as in Section 2.

2) Seen from (23) and (24), the kinematic formulation requires the computation of $\mathbf{J}_o$ or $\mathbf{G}$, which involves the inversion of $\mathbf{J}_p$. Refer to (28). The derived kinematics requires single matrix inversion only, and is less subject to numerical instability, than the one in Section 2.

3) Based on (29), a closed-chain mechanism is said to be singular if there exists a nonzero solution $\dot{\boldsymbol{\theta}}_p$ satisfying

$$\mathbf{0} = \mathbf{J}_p\ \dot{\boldsymbol{\theta}}_p \tag{30}$$

(30) tells that the singularity of a closed-chain mechanism occurs within the joint velocity space, specified by the nullspace of $\mathbf{J}_p$. Since $\mathbf{J}_p$ is readily available, it can be a more efficient means for detecting the singularity, than $\mathbf{Q}_a{}'$, introduced in Section 2. It should be noted that there is no relationship between the singularity of a closed-chain mecahnism and the singularity of its individual limbs.

4. COMPUTATIONAL COST ANALYSIS

This section analyzes and compares the computational cost associated with two kinematic formulations of a closed-chain mechanism, presented in Section 2 and 3. Both nonredundant and redundant closed-chain mechanisms are considered.

Without loss of generality, let us consider two 6 d.o.f. closed-chain mechanisms ($m=6$), including the one in which each limb has three controllable active joints and three uncontrollable active and passive joints, denoted by NCM, and the one in which each limb has four controllable active jonts and three uncontrollable active and passive joints, denoted by RCM. For NCM, which may have Type II redundancy, we have $n_a=n_p=6$, such that

TABLE 1. The computational cost required to compute $\mathbf{Q}_a$ and $\mathbf{J}_o$ for NCM and RCM.

	$\mathbf{Q}_a$	$\mathbf{J}_o$
NCM	$2\ C_i(6)$	$C_i(6)\ +\ C_m(6\times6\times6)\ +\ 2\ C_m(6\times3\times3)$
RCM	$2\ C_i(6)\ +\ 2\ C_m(6\times7\times6)\ +\ 2\ C_m(7\times6\times6)$	$C_i(6)\ +\ C_m(6\times6\times8)\ +\ 2\ C_m(6\times3\times4)$

$n_a+n_p=2m$ and $n_p=m$. And, for RCM, which has Type I redundancy, we have $n_a=8$ and $n_p=6$, such that $n_a+n_p>2m$ and $n_p=m$. Both closed-chain mechanisms satisfy the mobility and controllability conditions.

The computational cost associated the two kinematics, derived in Section 2 and 3, depends on the amount of computation required to compute $\mathbf{Q}_a$ and $\mathbf{J}_o$. Refer to (11) & (12) and (23) & (24). Here, we analyze the computational cost for $\mathbf{Q}_a$ and $\mathbf{J}_o$, for the comparison of the two kinematic formulations. The matrices and their dimension involved in computing $\mathbf{Q}_a$ and $\mathbf{J}_o$ are listed in the following:

For NCM, where $\mathbf{J}_{ia} \in \mathbf{R}^{6\times3}$, $i=1,2$, $a=a,p$,

$\mathbf{J}_i \in \mathbf{R}^{6\times6}$, $\mathbf{Q}_i\ (=\ \mathbf{J}_i^{-1}) \in \mathbf{R}^{6\times6}$, $i=1,2$

$\mathbf{Q}_{ia} \in \mathbf{R}^{3\times6}$, $i=1,2$; $\mathbf{Q}_a \in \mathbf{R}^{6\times6}$

and

$\mathbf{J}_a \in \mathbf{R}^{6\times6}$ $a=a,p$; $\mathbf{G}\ (=\ \mathbf{J}_p^{-1}\ \mathbf{J}_a) \in \mathbf{R}^{6\times6}$

$\mathbf{G}_{ij} \in \mathbf{R}^{3\times3}$, $i=1,2$, $j=1,2$

$\mathbf{J}_1\ (=\ \mathbf{J}_{2p}\ \mathbf{G}_{21})$, $\mathbf{J}_2\ (=\ \mathbf{J}_{1p}\ \mathbf{G}_{12}) \in \mathbf{R}^{6\times3}$

$\mathbf{J}_o \in \mathbf{R}^{6\times6}$

For RCM, where $\mathbf{J}_{ia} \in \mathbf{R}^{6\times4}$, $\mathbf{J}_{ip} \in \mathbf{R}^{6\times3}$, $i=1,2$,

$\mathbf{J}_i \in \mathbf{R}^{6\times7}$;

$\mathbf{Q}_i\ [\ =\ \mathbf{J}_i^{t}\ (\ \mathbf{J}_i\ \mathbf{J}_i^{t}\ )^{-1}\ ] \in \mathbf{R}^{7\times6}$, $i=1,2$

$\mathbf{Q}_{ia} \in \mathbf{R}^{4\times6}$, $i=1,2$; $\mathbf{Q}_a \in \mathbf{R}^{8\times6}$

and

$\mathbf{J}_a \in \mathbf{R}^{6\times8}$, $\mathbf{J}_p \in \mathbf{R}^{6\times6}$; $\mathbf{G} \in \mathbf{R}^{6\times8}$

$\mathbf{G}_{ij} \in \mathbf{R}^{3\times4}$, $i=1,2$, $j=1,2$;

$\mathbf{J}_1$, $\mathbf{J}_2 \in \mathbf{R}^{6\times4}$; $\mathbf{J}_o \in \mathbf{R}^{6\times8}$

For the notational convenience, let us define

$C_m(l\times m\times n)$ = the cost for multiplying an $l\times m$ matrix and an $m\times n$ matrix

$C_i(n)$ = the cost for inverting an $n\times n$ matrix

TABLE 1 summarizes the computational cost required to compute $\mathbf{Q}_a$ and $\mathbf{J}_o$ for NCM and RCM. From TABLE 1, we observe that 1) the kinematic formulation in Section 3 is computationally more efficient than that in Section 2, for RCM, 2) the opposite is true for NCM. However, it should be noted that the kinematic solutions derived in Section 3 is more physically meaningful and less sensitive to numerical instability, compared to those derived in Section 2.

5. CONCLUSIONS

In this paper, we presented a new kinematic formulation of a closed-chain mechanism and demonstrated its effectiveness in the kinematic analysis. The kinematic formulation presented in this paper is valid for closed-chain mechanisms with and without the redundancy in the number of joints. The kinematics of a closed-chain mechanism was formulated based on the kinematic constraint existing between the controllable active joints and the rest of joints, instead of the kinematic constraint existing between the two open-chains. The derived kinematics of a closed-chain mechanism was shown to be effective in the development of the kinematic solutions and the analysis of the singularities. The analysis of the computational cost associated with the kinematic formulation was made.

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