

# ROBOT MANIPULATOR'S CONTACT TASKS ON UNCERTAIN FLEXIBLE OBJECTS

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**Abstract** *The present paper studies a robot manipulator's contact tasks on the uncertain flexible objects. The flexible object's distributed parameter model is approximated into a lumped "position state-varying" model. By using the well-known nonlinear feedback compensation, the robot's control space is decomposed into the position control subspace and the object's torque control subspace. The optimal state feedback is designed for the position loop, and the robot's contact force is controlled through controlling the resultant torque on the object using model-reference simple adaptive control. Experiments of a PUMA robot interacting with an aluminum plate show the effectiveness of this control approach.*

**Keywords** Flexible object, Robot Manipulator, Position/force hybrid control, Simple adaptive control.

## 1. INTRODUCTION

The present paper studies on how to control a robot manipulator to perform the mechanical contact tasks on the uncertain flexible objects. The detailed examples are to cover a sheet, and to write on the flexible object. To perform these tasks successfully, the robot is required to move its end-effector on the flexible object while imposing a required contact force on its surface. Clearly, because of its flexibility, the object's dynamics will influence the robot's control system, and since it is usually a distributed parameter system, the object dynamics as seen from the robot's end-effector will change when the robot moves on its different positions. The problem becomes further complicated as it is difficult to decompose the robot's position and force control loops.

Hybrid position/force control usually considered the geometric constraints of the robot's environments. Few studies considered the influence of the environmental dynamics<sup>[1]</sup>. Recently, Yoshikawa and Umeino proposed an approach to consider the object dynamics in designing a robot's dynamic hybrid control. However, in their approach, the environment was modeled as a lumped time-invariant system. They didn't discuss about the environmental variation and decomposition difficulties<sup>[2],[3]</sup>. As we mentioned above, these two problems are the most fundamental difficulties that we should consider in specifying the position/force hybrid control on a flexible object. We will mainly study them in this paper.

In our approach, we assume that the flexible object is very stiff, which means that there is only small deflection on the object. Based on this assumption, we approximate the object's distributed parameter model into a lumped "position state-varying" model. Then, by using the well-known nonlinear

feedback compensation, we decompose the robot's control space into position control subspace and the object's torque control subspace. We design the optimal state feedback for the position control loop, and we control the robot's contact force through controlling the resultant torque on the object. Since the torque control loop is influenced by the object dynamics, and this dynamics varied with respect to the robot's contact position, we use model-reference adaptive control to control this "state-varying" torque loop. Experiments of a PUMA robot interacting with a aluminum plate show the effectiveness of our approach.

## 2. MODEL OF FLEXIBLE OBJECT AND ROBOT

### 2.1. Dynamic Equation of Flexible Object

When a robot manipulator is performing the contact task (e.g., to stick a long sheet) on its flexible object, as shown in Fig.1, the object dynamics will influence the robot's control system along the normal direction of its surface. Moreover, since the object is a distributed parameter system, its dynamics as seen from the robot will change from "hard" to "soft" when the robot moves from the object's fixed side to the free sides. In order to accurately control the contact force, formally, we should identify the flexible object's distributed parameter model --- a partial differential equation, and construct a distributed parameter control system. However, this approach makes the problem very complicated, and enables the robot to realize the real time high speed control.

In this section, we assume that the flexible object only makes a small deflection when the robot impose a contact force along its normal direction, and we approximate the object's distributed parameter model to a lumped parameter model.

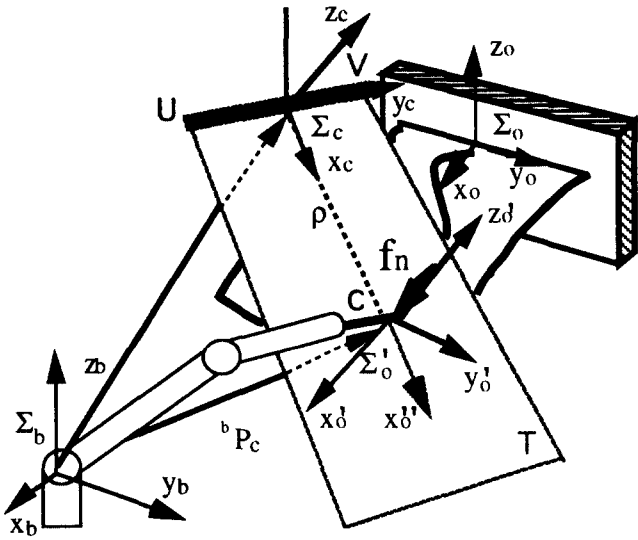


Fig.1 Robot's contact tasks on a flexible plate

As shown in Fig.1, we consider a flexible plate, one of its side is fixed and the other three sides are free. We set the world Cartesian coordinate as  $\Sigma_b$ , and the object Cartesian coordinate as  $\Sigma_o$ . The robot end-effector's position as seen from  $\Sigma_b$  and  $\Sigma_o$  are  ${}^bP_c = \mathbf{x} = [x, y, z]^T$  and  ${}^oP_c = \mathbf{x}_o = [x_o, y_o, z_o]^T$ , respectively. If there is no friction on the object surface, then the direction of the contact force  $F_n$  coincides with the normal  $n$  of the tangent surface  $T$  at the contact point  $C$ . The angles between the contact force  $F_n$  and the axes  $x_o, y_o$  and  $z_o$  of the object coordinate  $\Sigma_o$  are  $\alpha, \beta$  and  $\theta$ , respectively. We assume that both the positions  ${}^bP_c, {}^oP_c$  and the angles  $(\alpha, \beta, \theta)$  can be measured by using the robot's joint angle sensors and the force sensor. The length of the arc from the  $\Sigma_o$ 's origin to the contact point  $C$  is  $\rho$ .

Now, we translate the coordinate  $\Sigma_o$  to the contact point  $C$ , and then rotate it such that its  $z_o$  axis directs to the direction of the contact force  $F_n$ . The new coordinate becomes  $\Sigma'_o$  with its  $x'_o$  and  $y'_o$  axes are on the tangent surface  $T$ . We further rotate the  $\Sigma'_o$  along the axis  $z'_o$  with the angle  $\text{atan}(\cos\beta/\cos\alpha)$  and move it along the direction of  $-x''_o$  and the distance  $\rho$ . The final coordinate becomes  $\Sigma_c$ . The axis  $y_c$  is now the rotation axis of the tangent surface  $T$ , and the object dynamics at the contact point  $C$  as seen from the robot can be approximated into the following lumped "position-varying" model.

$$I(x_c, y_c)\ddot{\theta} + d(x_c, y_c)\dot{\theta} + k(x_c, y_c)\theta = F_n\rho \quad (1)$$

where  $\theta(t)$  is the approximated (or virtual) object's rotation angle,  $F_n$  is the contact force acting on its normal direction,  $x_c(t), y_c(t)$  are the robot end-effector's position on the constraint coordinate  $\Sigma_c$ .  $I(x_c, y_c)$ ,  $d(x_c, y_c)$  and  $k(x_c, y_c)$  are the object's inertia moment, viscosity and elasticity parameters, respectively. These parameters are unknown and varying with respect to the robot's position  $x_c$  and  $y_c$ , it is the first difficulty for the robot's hybrid control.

## 2.2. Dynamic Equation of the Manipulator

We now come to describe the robot manipulator's dynamic equation in the world coordinate  $\Sigma_b$  as

$$A(\mathbf{x})\ddot{\mathbf{x}} + p(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{F} - \mathbf{F}_n \quad (2)$$

where  $\mathbf{x} = [x, y, z]^T$  is the robot's end-effector position,  $A(\mathbf{x})$  is the robot's inertia matrix.  $p(\mathbf{x}, \dot{\mathbf{x}})$  is the coriolis and centrifugal force.  $\mathbf{F} = [f_x, f_y, f_z]^T$  is the robot's driving force and  $\mathbf{F}_n$  is the contact force from the object as seen in the world coordinate  $\Sigma_b$ .

Using the geometric relation between  $\mathbf{x} = [x, y, z]^T$  and  $[x_c, y_c, \theta]^T$  illustrated in Fig.1 and specifying the nonlinear feedback compensation

$$\mathbf{F} = A(\mathbf{x})\mathbf{J} \begin{pmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \end{pmatrix} + p(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{F}_n + A(\mathbf{x})\mathbf{J}[\mathbf{u} - \boldsymbol{\tau}] \quad (3)$$

to eq.(2), we then get

$$\ddot{x}_c = u_1 \quad (4)$$

$$\ddot{y}_c = u_2 \quad (5)$$

$$\ddot{\theta} = u_3 - f_n\rho ; \rho \approx \sqrt{x_c^2 + y_c^2} \quad (6)$$

where  $\mathbf{u} = [u_1, u_2, u_3]^T$  is the new control input vector, and  $\boldsymbol{\tau} = [0, 0, f_n\rho]^T$ .  $f_n$  is the contact force.

The eqs.(1) and (4) to (6) represent the simplified robot-object dynamics, and we should discuss on how to design the new control inputs  $\mathbf{u} = [u_1, u_2, u_3]^T$  which lead the robot to perform the required contact task successfully. Note that, although the position  $[x_c, y_c]$ 's control loop is independent from the contact force  $f_n$  in eqs.(4) and (5), the force  $f_n$ 's control loop is influenced by the contact position  $\rho \approx \sqrt{x_c^2 + y_c^2}$  in eq.(6). This is the second difficulty that we should overcome in the robot's hybrid control.

## 3. HYBRID CONTROL DESIGN

We now design the control input  $\mathbf{u} = [u_1, u_2, u_3]^T$  for the system eqs.(1), (4), (5) and (6), such that the robot makes the desired motion on the object surface while maintaining a constant contact force  $f_n$ .

### 3.1. Position Control Loop

As seen in the eqs.(4) and (5), because the position  $\mathbf{x}_p = [x_c, y_c]^T$  control loops are only controlled by  $\mathbf{u}_p = [u_1, u_2]^T$  and is independent from the contact force  $f_n$ , we first design these control loops using the optimal state feedback.

In detail, we design these position control inputs as

$$u_1 = \ddot{x}_c^d + K_d(\dot{x}_c^d - \dot{x}_c) + K_p(x_c^d - x_c) \quad (7)$$

$$u_2 = \ddot{y}_c^d + K_d(\dot{y}_c^d - \dot{y}_c) + K_p(y_c^d - y_c) \quad (8)$$

to optimize the performance index

$$E = \int_0^{\infty} \{ (\mathbf{x}_c^d - \mathbf{x}_c)^T Q (\mathbf{x}_c^d - \mathbf{x}_c) + \mathbf{u}_p^T R \mathbf{u}_p \} dt \quad (9)$$

where  $\mathbf{x}_c = [x_c, \dot{x}_c, y_c, \dot{y}_c]^T$  is the state vector and  $\mathbf{x}_c^d = [x_c^d, \dot{x}_c^d, y_c^d, \dot{y}_c^d]^T$  is the desired state vector.

### 3.2. Force Control Loop

After determining the robot's position response  $\mathbf{x}_c = [x_c, \dot{x}_c, y_c, \dot{y}_c]^T$ , the force control loop of eqs. (1) and (6) becomes a typical time-varying system. In this loop, although the relation from the control input  $u_3$  to the contact force  $f_n(t)$  is nonlinear, the relation between  $f_n$  and  $u_3$  is linear. Since  $\mathbf{x}_c = [x_c, \dot{x}_c, y_c, \dot{y}_c]^T$  is already defined, it is then straightforward to control the force  $f_n(t)$  implicitly through controlling the new variable  $f_n \rho(t)$  --- that is a torque imposed on the object.

Rewriting the eqs. (1) and (6) into the following state-space form

$$\dot{\mathbf{x}}_\theta(t) = A_\theta(t)\mathbf{x}_\theta(t) + b_\theta(t)u_3(t) \quad (10)$$

$$y_\theta(t) = c_\theta(t)\mathbf{x}_\theta(t) + d_\theta(t)u_3(t) \quad (11)$$

where  $\mathbf{x}_\theta = [\theta, \dot{\theta}]^T$ ,  $y_\theta = f_n \rho$

$$A_\theta = \begin{pmatrix} 0 & 1 \\ -k(t) & -d(t) \\ 1+I(t) & 1+I(t) \end{pmatrix}, \quad b_\theta = \begin{pmatrix} 0 \\ -1 \\ 1+I(t) \end{pmatrix}$$

$$c_\theta = \begin{pmatrix} k(t) & -d(t) \\ 1+I(t) & 1+I(t) \end{pmatrix}, \quad d_\theta = \begin{pmatrix} 0 \\ 1 \\ 1+I(t) \end{pmatrix}$$

The control objective is modified to control the system output  $f_n(t)\rho(t)$  to approach the  $f_n^d(t)\rho(t)$ . That is, the problem of designing the control  $u_3$  such as

$$\lim_{t \rightarrow 0} e_f(t) = \lim_{t \rightarrow 0} [f_n^d - f_n(t)] = 0 \quad (12)$$

equals to make

$$\lim_{t \rightarrow 0} e_y(t) = \lim_{t \rightarrow 0} [y_\theta(t) - y_m(t)]$$

$$= \lim_{t \rightarrow 0} \frac{1}{\rho(t)} e_f(t) = 0 \quad (13)$$

where  $y_m(t) = f_n^d \rho(t)$ . For this control design problem, we can select a reference model

$$\dot{\mathbf{x}}_m(t) = A_m(t)\mathbf{x}_m(t) + b_m(t)u_m(t) \quad (14)$$

$$y_m(t) = c_m(t)\mathbf{x}_m(t) \quad (15)$$

and apply model reference adaptive control such that the torque control system follows the output of the reference model, perfectly, where  $u_m = \rho^d f_n^d$  is the desired output of the reference model and  $y_m(t) = f_n^d \rho(t)$  is the model's real output. The problem in practice is how to select a proper reference model. Here, it is clear that the best reference model of the system (10) and (11) satisfies the relation that, its transfer function equals that of the robot's position control loop. That is

$$G_m(s) = \frac{y_m}{u_m} = c_m(sI - A_m)^{-1} b_m = G_{xp}(s) = \frac{x_c}{u_1} \quad (16)$$

Adaptive control algorithms have usually been proposed for a lumped time invariant system with parameter uncertainties, they generally require the slow adaptation rates. Recently, a simple adaptive control<sup>[4]</sup> has been presented which considered the control of a time-varying system, and it guarantees the system's robust stability at any rate of adaptation. The higher the adaptation rate, the faster the transient response and the smaller the steady-state error.

Moreover, this approach does not require to know about the plant's relative order. Motivated by these advantages, we will apply the simple adaptive control algorithm to our problem.

The control problem here is to design the control input  $u_3$  such that the time-varying system eqs. (10) and (11)'s output  $y_\theta(t)$  perfectly tracking  $y_m(t)$ , which is the output of the reference model. Using the **simple adaptive control algorithm**, we first derive a error system

$$\dot{\mathbf{e}}_x(t) = A_{\theta c}(t)\mathbf{e}_x(t) + b_{\theta c}(t)\phi^T(t)\omega(t) \quad (17)$$

$$e_y(t) = y_\theta(t) - y_m(t) = c_{\theta c}(t)\mathbf{e}_x(t) + d_{\theta c}(t)\phi^T(t)\omega(t) \quad (18)$$

where

$$A_{\theta c}(t) = A_\theta + b_\theta \frac{\tilde{k}_e}{\beta} c_\theta, \quad b_{\theta c}(t) = \frac{b_\theta}{\beta}, \quad c_{\theta c}(t) = \frac{c_\theta}{\beta}, \quad d_{\theta c}(t) = \frac{d_\theta}{\beta}$$

$$\beta = 1 - d_\theta \tilde{k}_e, \quad \phi = K(t) - \tilde{K}, \quad \omega = [e_y, x_m, u_m]$$

and  $K = [k_e, k_{x_m}, k_{u_m}]$ ,  $\tilde{K} = [\tilde{k}_e, \tilde{k}_{x_m}, \tilde{k}_{u_m}]$ . We then design the force control loop's control input as

$$u_3(t) = k_e(t)e_y(t) + k_{x_m}(t)x_m(t) + k_{u_m}(t)u_m(t) \quad (19)$$

where  $K = [k_e, k_{x_m}, k_{u_m}]$  is the **adaptive gains** which is adjusted according to the rule

$$\dot{K}(t) = -\Gamma e_y \omega(t) \quad (20)$$

where  $\Gamma = \text{diag}[\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4] > 0$  is a positive definite scaling matrix. This parameter adjustment rule together with the simple adaptive control eq.(19) guarantees the output error  $e_y(t)$  to approach zero, asymptotically. Therefore, from the eqs. (12) and (13), we get  $e_f(t) \rightarrow 0$ . The over all model matching simple adaptive control of the robot's contact force is shown in Fig. 2.

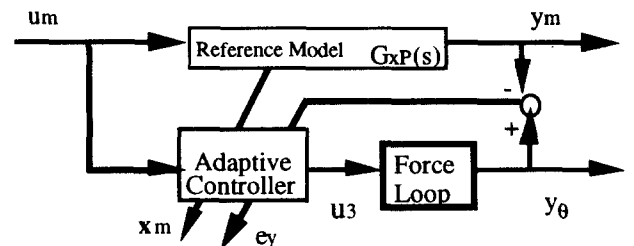


Fig. 2 Model matching simple adaptive control

## 4. EXPERIMENTAL RESULTS

We have succeeded in perform various contact tasks using a PUMA robot, such as to write the sine and circle curves on the surface of an aluminum plate, shown in Fig.3. The plate is 0.02[m] thick, 0.5[m] wide and 0.7[m] long. Figs. 4 to 8 show the experiment results when the robot writes the sine curve on the plate. Figs. 4 and 5 show the robot's position responses, and Fig.6 is the contact force response along the normal direction of the aluminum surface. In these three figures, the solid lines are our results and the dotted lines are the desired values. Figs.7 and 8 shows the parameter convergence of our control approach. The sampling time is 2[ms].

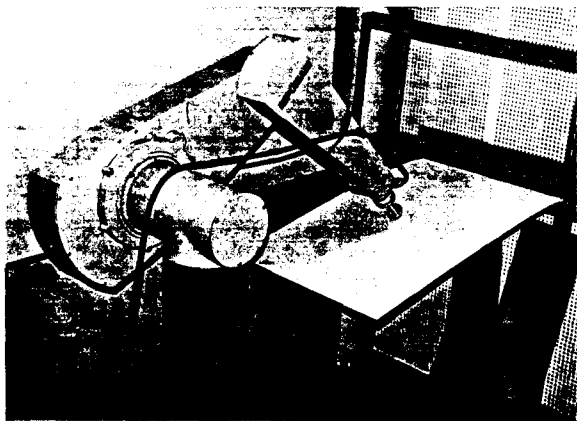


Fig.3 Contact tasks of a PUMA robot on the aluminum plate

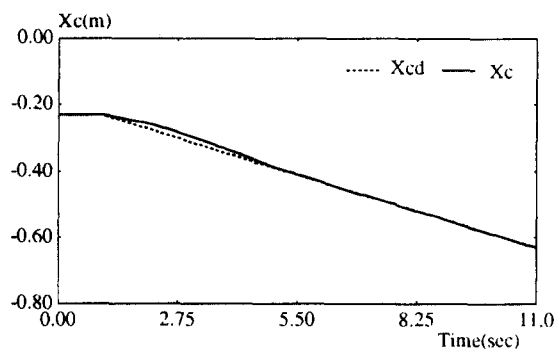


Fig. 4 Robot's Position responses  $x_c(t)$  when writing a sine curve on the plate

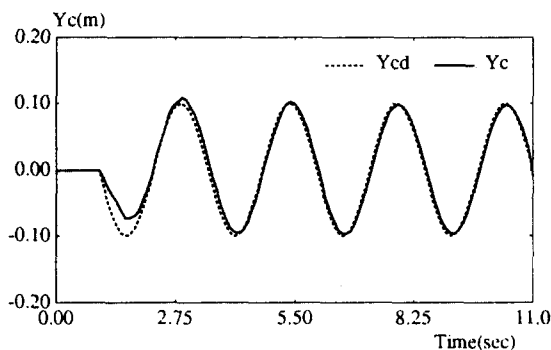


Fig. 5 Robot's Position responses  $y_c(t)$  when writing a sine curve on the plate

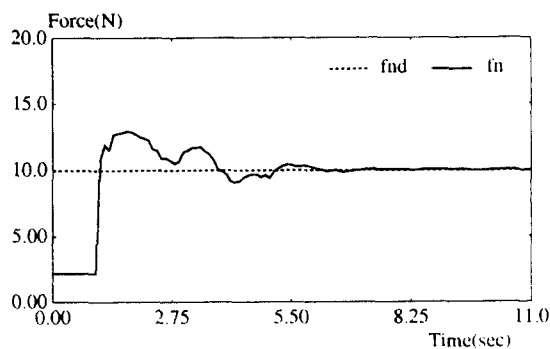


Fig. 6 Robot's contact force responses  $f_n(t)$  when writing a sine curve on the plate

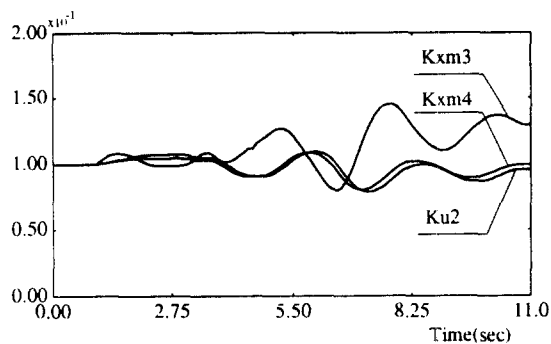


Fig.7 The parameter convergence of the control gains

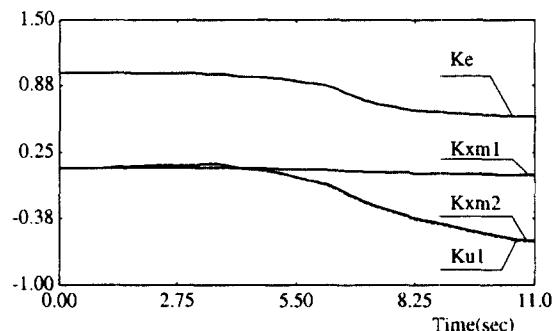


Fig.8 The parameter convergence of the control gains

## 5. CONCLUSION

This paper applied the simple adaptive control for a robot manipulator to perform the contact tasks on a flexible object. The object's distributed parameter model was approximated into a lumped "state-varying" model. The robot's contact force was controlled through controlling the resultant torque on the object using the simple adaptive control. The effectiveness of this control approach was shown by a PUMA robot's experiments.

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