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Design of Path Tracking Controller for Mobile Robot

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Abstracts Autonomous Mobile Robot(AMR) is a field of study which is under active research along with rapid development of the engineering technology. The main reasons for the high interest in AMR are because of its ability to change work space freely and its capability to replace human being for difficult and dangerous jobs. Also the fact that AMR provides a variety of research fields, such as path planning, navigation algorithm, sensor fusion, image processing, and controller design is part of the reason for its popularity. But relatively few researches are concerned with controller. So in this paper, a control strategy of mobile robot with nonholonomic constraint for tracking ordered discontinuous motion is proposed. The proposed control strategy has been designed as a state feedback shape to allow the AMR to obtain continuous velocity and track the path which is composed of discontinuous motions. In order to design such controller, 3 states have been reduced to 2 states through coordinate projection. These ideas are tested for validity through simulation and simulation result is compared with experiments result.

Keywords Mobile Robot, Autonomous Mobile Robot, AMR, Motion Tracking, Path Tracking

1. INTRODUCTION

Recently, much research has been carried out in Autonomous Mobile Robot(AMR). Path planning, mechanical design, vision interface, navigation, etc. Among these specifications, controller design is very important because it is basic element of an AMR. But relatively few researches are carried on it. In this paper, first, we design a feedback controller, based on Canudas de Wit and Sørdalen[1][3] of which shape is piecewise smooth feedback controller and it can make the AMR follow a ordered discontinuous motions with continuous tangent velocity changing. Second, we compare its result by simulation with result by experiment. Theoretically, there are no problems in well designed controller. But, in real situation, AMR did not move as expected because of motor's limitation and so on.

2. CONTROLLER DESIGN

2.1 Exponential Convergence To a Motion of AMR with Two Driving Wheels

Figure 1 shows an AMR with two driving wheels, where v is tangent velocity and ω is angular velocity of the AMR. Cr is distance between driving wheel and center of wheel axis. v_1 and v_2 are left and right wheel tangent velocity respectively.

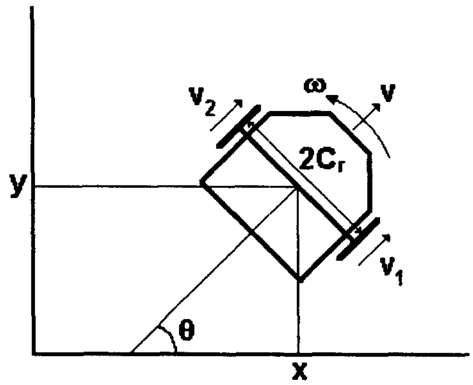

fig. 1. An AMR and variables

The kinematics of the AMR with two driving wheels are given as

$$\begin{aligned} \dot{x} &= \frac{(v_1+v_2)}{2}\cos\theta = v\cos\theta \\ \dot{y} &= \frac{(v_1+v_2)}{2}\sin\theta = v\sin\theta \\ \dot{\theta} &= \frac{(v_1-v_2)}{2C_r} = \omega \end{aligned} \tag{1}$$

where the state of the system (1) is the position of the wheel axis center and the orientation of the AMR: $q=[x,\ y,\ \theta]^T$. The definition of variables are shown by figure 1.

The control input v and ω are related to wheel velocities.

$$u = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2C_r} & -\frac{1}{2C_r} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{2}$$

A nominal state equation with input u is given as

$$\dot{q} = Aq + Bu = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} u = Gu \quad (3)$$

where A matrix is 0-matrix.

Let $q_r = [x_r, y_r, \theta_r]^T$ be a reference point in the configuration space. The closed-loop system of (3) with reference q_r converges for any q to q_r. (figure 2)

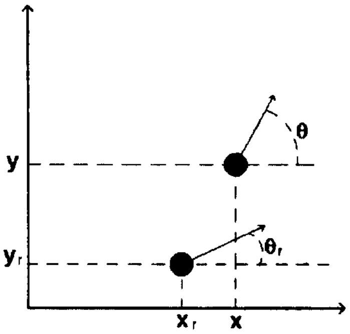

fig. 2. q and $\dot{q}_r$ in coordinate

Defining error vector q_e as (4).

$$q_e = T(\theta_r)(q - q_r), \quad T(\theta_r) = \begin{bmatrix} \cos\theta_r & \sin\theta_r & 0 \\ -\sin\theta_r & \cos\theta_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

and it can be depicted as figure 3.

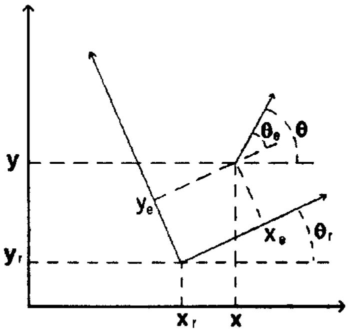

fig. 3 q_e in coordinate

The time derivative of q_e is given from (3) and (4).

$$\dot{q}_e = T(\theta_r)\dot{q} = T(\theta_r)Gu = \tilde{G}u \quad (5)$$

It can be known that the convergence of q to q_r is equivalent to the convergence of q_e to $[0, 0, 2\pi n]^T$, where n is integer.

A set C is defined as following

$$C = \{(x_e, y_e) | x_e^2 + (y_e - r)^2 = r^2\} \quad (6)$$

The set C passes origin and (x_e, y_e). Also its center is located on the y-axis. Let θ_d be an angle of tangent line of C at (x_e, y_e). It is expressed as

$$\theta_d(x_e, y_e) = \begin{cases} 2\mathrm{atan2}(x_e, y_e) & ; (x_e, y_e) \neq (0, 0) \\ 0 & ; (x_e, y_e) = (0, 0) \end{cases} \quad (7)$$

where $\theta_d \in (-\pi, \pi]$

Let a be a length between q_r and q_e and let α be a orientation error.

$$a(x_e, y_e) = \sqrt{x_e^2 + y_e^2} \quad (8)$$

$$\alpha(x_e, y_e, \theta_e) = e - 2\pi n, \quad e = \theta_e - \theta_d \quad (9)$$

where n is a value of integer which makes α belong to $(-\pi, \pi]$. A depiction of these definition is shown in figure 4. (In [1], a is defined as arc-length. But it has problem when the q_e is placed on x-axis.)

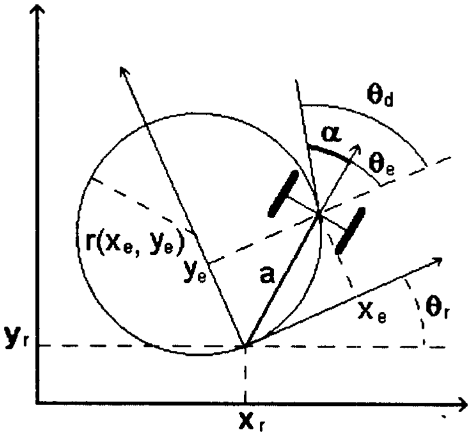

fig. 4. Illustrated α and a

When a and α converge to 0 then $q_e = (x_e, y_e, \theta_e)$ converges to (0, 0, 0).

Mapping function M, which maps q_e to $z = [a, \alpha]^T$, can be found. It make state space $q_e \in R^3$ to two dimensional space $z \in R * (-\pi, \pi]$.

$$z = M(q_e) \quad (10)$$

First derivative of (10) is given as

$$\dot{z} = \frac{\partial M(q_e)}{\partial q_e} \dot{q}_e \quad (11)$$

and from (5) and (11)

$$\dot{z} = \frac{\partial M(q_e)}{\partial q_e} \tilde{G}u = B(q_e)u, \quad B(q_e) = \begin{bmatrix} b_1 & 0 \\ b_2 & 1 \end{bmatrix} \quad (12)$$

where

$$b_1 = (x_e^2 + y_e^2)^{-\frac{1}{2}} (x_e \cos\theta_e + y_e \sin\theta_e) \quad (13)$$

$$b_2 = \frac{2}{x_e^2 + y_e^2} (y_e \cos\theta_e - x_e \sin\theta_e) \quad (14)$$

Lemma 1. The properties of b_1 and b_2.

① $-1 \le b_1(a, \alpha) \le 1$

② $b_1(a, \alpha)$ is continuous

③ $\lim_{\alpha \to 0} b_1(a, \alpha) = \sin(\frac{3}{2}\theta_d)$

④ $|b_2(a, \alpha)a| \le 2$

proof is on the APPENDIX.

Input linearized form of (12) is given as

$$\dot{z} = \begin{bmatrix} b_1 & 0 \\ b_2 & 1 \end{bmatrix} u = \begin{bmatrix} b_1 & 0 \\ b_2 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{b_1} & 0 \\ -\frac{b_2}{b_1} & 1 \end{bmatrix} \begin{bmatrix} b_1 v \\ b_2 v + \omega \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 v \\ b_2 v + \omega \end{bmatrix} \quad (15)$$

Taking closed-loop input as state feedback,

$$b_1 v = -\gamma a \quad (16)$$

$$b_2 v + \omega = -\delta \alpha \quad (17)$$

where γ and δ are state feedback gain.($\gamma>0$, $\delta>0$)

From (16) and (17), real input v and ω are given by

$$v = -\frac{\gamma}{b_1} a \quad (18)$$

$$\omega = -b_2 v - \delta \alpha \quad (19)$$

where b_1 is not 0.

The solutions of (12) by using (18) and (19) are represented by

$$a(t) = a(0)e^{-\gamma t} \quad (20)$$
$$\alpha(t) = \alpha(0)e^{-\delta t}$$

From (20), it can be known that the AMR exponentially converge to reference motion point with bounded v and ω.(see lemma 1 with [1])

2.2. Tracking Discontinuous Ordered Motions

We showed a set point regulation controller for an AMR in 2.1. But in the AMR navigation, to converge the AMR to one point is not an object of control. The problem is to make the AMR track a path. Let well selected sequential motions on the path by some path planning be a set F.[2] An element f_i is a motion on the path and given by

$$f_m = [x_m, y_m, \theta_m]^T \quad (21)$$

where x_m and y_m are position of motion and θ_m is orientation of motion. Each motion has its desired velocity v_m and maximum deviation ε_m. The set V and set ε is defined as follows:

$$V = [v_1, v_2, \ldots, v_e] \quad (22)$$

where $v_1 = v_e = 0$

$$\varepsilon = [\varepsilon_2, \varepsilon_3, \ldots, \varepsilon_e] \quad (23)$$

where $\varepsilon_m > 0$

v_m is desired tangent velocity of the AMR on motion f_m and ε_m is a timing decision factor that the AMR can change its object motion from f_m to f_{m+1}. We change the variable m as (24).

$$m = \begin{cases} 2 & \text{when } t = 0 \\ m+1 & \text{when } |a| \le \varepsilon_m \end{cases} \quad (24)$$

From (8) the distance between each f_m can be known and its set Dis is given by

$$Dis = [d_1, d_2, \ldots, d_{e-1}]$$
$$d_m = \sqrt{(x_{m+1}-x_m)^2 + (y_{m+1}-y_m)^2} \quad (25)$$

To make variation of tangent velocity of the AMR continuously and to make tangent velocity on motion f_m be v_m, (18) is modified as follows

$$v(\tau) = v_{m-1} + 3\Delta v\left(\frac{\tau}{\tau_f}\right)^2 - 2\Delta v\left(\frac{\tau}{\tau_f}\right)^3 \quad (26)$$

where

$$\tau_f = -\frac{2d_{m-1}}{v_{m-1}+v_m}, \quad \Delta v = v_m - v_{m-1} \quad (27)$$

τ is a time variable which is set to 0 when m changes. τ_f is estimated time for arriving f_m. With this velocity input, the AMR can track sequential motions which consist a path.(see [3] for detail information) When the motion is not properly selected then the control input ω will be extremely big. So it is important to select proper motion set.

3. COMPUTER SIMULATION AND EXPERIMENT

In this chapter result of computer simulation and experiment is compared.

3.1 Simulation Result of 2.1

Figure 5 is result of computer simulation with following variable setting.

$x_r = 0$, $y_r = 0$, $\theta_r = 0$,

$x_e = -0.866$, $y_e = 0.5$, $\theta_e = \frac{\pi}{2}$, $\gamma = 1.35$, $\delta = 4.60$

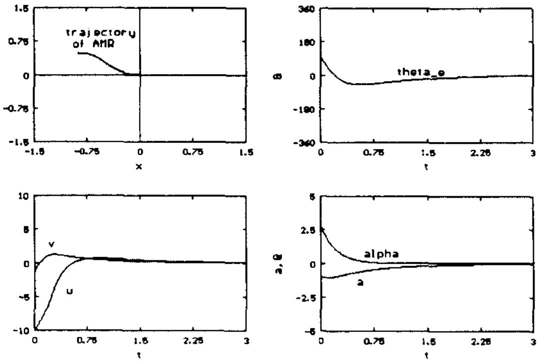

fig. 5. Simulation result of 3.1

In figure 5, upper-left is trajectory of the AMR, down-left is angular and tangent velocity of the AMR, upper-right is orientation angle of the AMR and down-right is a and α.

3.2 Simulation result of 2.2

Figure 6 and figure 7 are result of computer simulation with following variable setting.

$$P = \{(0.9,\ 0.9,\ 0.5),\ (1,\ 1,\ \frac{\pi}{2}),\ (1,\ 2,\ \frac{\pi}{2}),\ (2,\ 3,\ 0),\ (4,\ 3,\ 0),\ (5, 2, -\frac{\pi}{2}),\ (5,\ 1,\ -\frac{\pi}{2})\}$$

$$V = \{0,\ 0,\ 0.5,\ 0.5,\ 0.5,\ 0.5,\ 0)$$

$$\varepsilon = \{0.001,\ 0.01,\ 0.01,\ 0.01,\ 0.01,\ 0.01\}$$

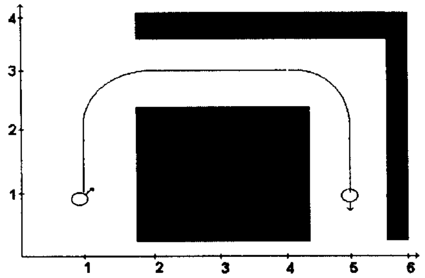

fig. 6. The trajectory of the AMR

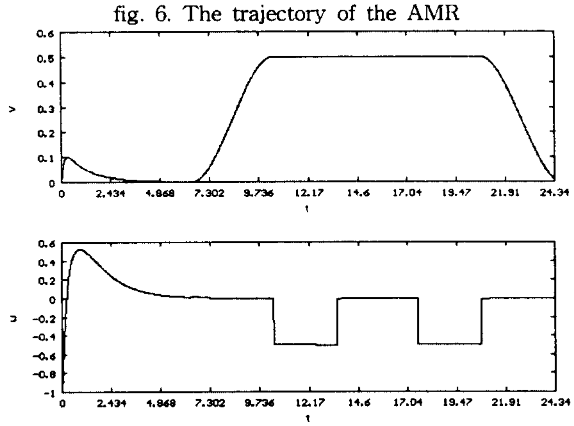

fig 7. Time graph of tangent velocity v and angular velocity ω (by simulation)

Above of figure 7 is a graph of linear velocity and below of figure 7 is a graph of angular velocity of the AMR. It can be easily known that the graph is almost same as the graph shown in [3].

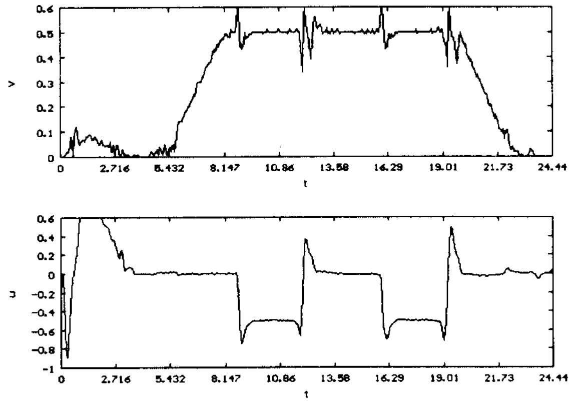

fig. 8. Time graph of tangent velocity v and angural velocity ω (by experiment)

3.3 Experiment Result of 2.2

The experiment is performed with self-made AMR in same setting of variables as 3.2 and C_r is 0.22m. The motors are DME60BB8H30 (JAPAN SERVO) and PID controller IC LM629 is adopted for motor speed controller. From (2), it can be known that the fixed tangent velocity and abrupt variation of angural velocity cause extreme change of tangent wheel velocity. In figure 8, small difference with figure 7 is found. When target motion is changed to next motion, control input ω changes abruptly and it causes extreme change of v_1 and v_2. But various limitations make motor not change its speed as step function and it made the difference. Nevertheless, AMR tracked the motions satisfactorily.

4. CONCLUSION

Discontinuous ordered set motion tracking controller is designed and tested by computer simulation and experiment. The variable arc length a in [1] is replaced by distance between origin and q_e. The results of computer simulation and experiment are proposed for validity. By result of experiment, the problem of controller is shown.

It can be a good topic for research that the controller includes dynamics of motor driving and AMR to make smoother angural velocity change when transition is occurred in motion.

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APPENDIX

Proof of Lemma

From (13) and (14)

$$b_1 = \sin(\frac{\theta_d}{2} + \theta_e) \tag{28}$$

$$b_2 = \frac{2}{\sqrt{x_e^2 + y_e^2}} \cos(\frac{\theta_d}{2} + \theta_e) \tag{29}$$

① From (28)

$$\sin(-\frac{\pi}{2}) \le b_1(a,\ \alpha) \le \sin(\frac{\pi}{2})$$

② θ_d and θ_e is piecewise continuous. So (28) is piecewise continuos.

③ From (9) and (28)

$$b_1 = \sin(\frac{3}{2}\theta_d + \alpha)$$

④ From (8) and (29)

$$b_2(a,\ \alpha)\, a = 2\cos(\frac{\theta_d}{2} + \theta_e)$$