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A Detection Scheme of Input Estimation Filter

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Abstracts In this paper, a new detection scheme, the detectable maneuver set(DMS) scheme, is proposed by incorporating the trade-off property between target maneuver magnitude and detection time delay. With this new detection scheme, small maneuvers can be effectively detected without enlarging window size. Simulation results show that the proposed DMS scheme gives better tracking performance

Keywords Input Estimation, Target Maneuver Detection, Detectable Maneuver Set

1 Introduction

An input estimation filter consists of two parts [1] : one is to detect the presence of target maneuver and the other is to estimate the target maneuver magnitude. To increase the detection probability, detection should be delayed. On the other hand, it is also required to detect the target maneuver as quickly as possible to improve the estimation performance. Thus the maneuver detection problem has a trade-off property.

Therefore it is desirable to minimize detection delay. It is known that the detection delay is a function of the system model and noise model, maneuver model and maneuver magnitude, probability of false alarm, and window length [2]. It has not been succeeded to find the optimal detection time delay by considering all those parameters. In many applications, however, a suboptimal finite sample size detection procedure is preferable for the simplicity of calculation [3]-[5]. For a given finite window length, detection time delay will be inversely proportional to the maneuver magnitude. In the sequel, it is desirable to include the inverse proportionality, or the trade-off property, into the detection scheme of target tracking problem.

In this paper, a new detection scheme of input estimation filter is proposed to incorporate the detectable maneuver set. The detectable maneuver set has its elements as the maneuvers whose magnitudes are greater than a minimum detectable maneuver magnitude which is determined from the worse-case noise level. If an estimate of the maneuver magnitude belongs to the detectable maneuver set, then target is said to have maneuvered. This new detection scheme can work for low maneuvering cases as well as for high maneuvering cases without enlarging the window size. The derived scheme is applied to a 1-dimensional tracking problem. Simulation results show that the proposed detection scheme has better performance than conventional detection schemes.

2 Problem Formulation

The target model can be expressed as

$$X(k+1) = FX(k) + BU(k) + G_d w(k) \tag{1}$$

where the state vector $X(k)$ is composed of the position, velocity and acceleration of the target at time k with respect to a given coordinate system, the target maneuver $U(k)$ is expressed as an abrupt change in acceleration at time k_m as $U(k) = U\delta(k-k_m)$, and the system noise $w(k)$ is zero mean and its covariance is given as

$$E\{w(k)^T w(j)\} = \begin{cases} Q(k) & , \quad k = j \\ 0 & , \quad k \neq j \end{cases} \tag{2}$$

The measurement equation is given by

$$y(k) = CX(k) + \nu(k) \tag{3}$$

where the measurement noise is also zero mean and its covariance is given as

$$E\{\nu(k)^T \nu(j)\} = \begin{cases} R(k) & , \quad k = j \\ 0 & , \quad k \neq j \end{cases} \tag{4}$$

Now the Kalman filter for target tracking is expressed as

$$\hat{X}(k) = F\hat{X}(k-1) + K(k)z(k) \tag{5}$$

where the innovation sequence $z(k)$ is defined by [6]

$$z(k) \triangleq y(k) - CF\hat{X}(k-1) \tag{6}$$

and $K(k)$ is the Kalman gain matrix.

The innovation sequence $z(k)$ can be expressed in terms of the target maneuver $U(k)$ and noise processes $w(k)$ and $\nu(k)$ in the frequency domain from the above equations as

$$Z(z) = L(z)[G(z)U(z) + G_w(z)W(z) + N(z)] \tag{7}$$

where $W(z)$ and $N(z)$ are the transformed noise processes and $G(z) = C(zI-F)^{-1}B, G_w(z) = C(zI-F)^{-1}G_d, L(z) = [I + CF(zI-F)^{-1}K(z)]^{-1}$.

The noise processes are assumed to be bounded as

$$\sup_{\omega_D} |W(e^{j\omega_D T})| = \bar{w}, \quad \sup_{\omega_D} |N(e^{j\omega_D T})| = \bar{\nu}. \tag{8}$$

This assumption is not true because it is one of the basic assumptions of Kalman filter that the noises are Gaussian. However, if the bounds are set as β-times greater than the standard deviation values, Eq. (8) is reliable with the probability

$$\Pr[-\beta < N(0,1) < \beta] = 2F(\beta) - 1 \qquad (9)$$

where $F(\cdot)$ is the standard normal distribution function. For example, Eq. (8) is reliable with the probability 99.73% when $\beta = 3$.

For tracking requirements, the innovation system, Eq. (7), should be stable for the given input sequences $U(z), W(z)$ and $N(z)$. Hence the following statements should be satisfied

i) $L(z)$ is stable,

ii) $L(z)G(z)$ is stable, and

iii) $L(z)G_w(z)$ is stable.

3 Innovations Approach to Maneuver Detection

In this section, the detectable maneuver set, which is a set of maneuvers whose magnitudes are greater than the minimum detectable maneuver magnitude, will be defined. And a new detection scheme will be derived using the detectable maneuver set.

First, a measure of the innovation sequence is defined in a finite window $[k-N, k-1]$ as

$$J(k;N) = \|Z(z)\|_k^N \qquad (10)$$

where the norm is calculated in the time domain as

$$\|Z(z)\|_k^N \triangleq \left[\frac{1}{N}\sum_{i=1}^{N} z^T(k-i)z(k-i)\right]^{1/2} \qquad (11)$$

The norm operation $\|\cdot\|_k^N$ is based on an RMS measure of the innovations within the window. The norm can also be calculated in the frequency domain using Parseval's theorem[7]. A major requirement on the maneuver detection problem is to reduce or prevent false alarms. In the absence of target maneuvers, the upper bound of $J^o(k;N)$, $J_U^o(k;N)$, is obtained by setting $U(z) = 0$ as

$$J_U^o(k;N) = \sup_{w,\nu} \|L(z)[G_w(z)W(z) + N(z)]\|_k^N. \qquad (12)$$

The superscript o means that there is no maneuver.

The probability of false alarm can be calculated as

$$\text{PFA} = \Pr[J^o(k;N) > J_U^o(k;N)]. \qquad (13)$$

For 1-dimensional case, PFA satisfies the following inequality

$$\text{PFA} < \frac{\Pr\left[\chi_N^2 > \frac{\sigma_2^2\beta^2 N}{4\sigma_1^2}\left(1+\sqrt{\frac{Q}{R}}\right)^2\right]}{\Pr\left[F_{N,N} < \frac{R}{Q}\right]} \qquad (14)$$

where

$$\begin{aligned}\sigma_1 &= \max[\bar{\sigma}\{L(z)G_w(z)\},\ \bar{\sigma}\{L(z)\}]\\ \sigma_2 &= \min[\underline{\sigma}\{L(z)G_w(z)\},\ \underline{\sigma}\{L(z)\}]\\ \bar{w} &= \beta\sqrt{Q},\ \ \bar{\nu} = \beta\sqrt{R}.\end{aligned}$$

Now, the detectable maneuver set can be defined.

Definition 1 *The detectable maneuver set at time k, $\Omega_d(k;U(k),N)$, is defined as*

$$\Omega_d(k;U(k),N) = \{U(k) | \inf_{U,w,\nu} J(k;N) > J_U^o(k;N)\} \qquad (15)$$

where $J_U^o(k;N)$ is defined in Eq. (12).

The inequality relation in Eq. (15) is considered as a detection scheme. This inequality, named as the threshold selector, is suggested in [5]. The inequality condition means that if the measure of the innovation sequence at time k is greater than that of the worst-case noise level in the absence of maneuvers, then the target has maneuvered within the window $[k-N, k-1]$.

Now a new detection scheme can be derived from the above definition.

Theorem 1 *If an estimate of maneuver magnitude is greater than $U_{\min}$ given by*

$$U_{\min}(k;N,k_m) = \frac{2J_U^o(k;N)}{\Delta(k;N,k_m)} \qquad (16)$$

where

$J_U^o(k;N) \triangleq \|L(z)G_w(z)\|_k^N \bar{w} + \|L(z)\|_k^N \bar{\nu}$

$\Delta(k;N,k_m) \triangleq \left[\frac{1}{N}\sum_{i=1}^{k-k_m} |\mathcal{Z}^{-1}\{L(z)G(z)\}(i)|^2\right]^{1/2}$

$N \triangleq$ no. of steps within the window

$\bar{w}, \bar{\nu} \triangleq$ bounds on RMS magnitude of process and sensor noises

$k_m \triangleq$ maneuver time

then it can be said that the target has maneuvered within the the window $[k-N, k-1]$ with the PFA of Eq. (13).

The above theorem re-defines the detectable maneuver set for given noises and detection window size as

$$\Omega_d(k) = \{U | U(k) = U\delta(k-k_m), |U| > U_{\min}\}. \qquad (17)$$

By using theorem 1, the detection scheme of input estimation filter can be modified. For example, the Bogler's algorithm can be modified as:

Step.1 Set the window size at time k as $[k-N, k-1]$.

Step.2 Compute an estimate of maneuver magnitude $\hat{U}$ and an estimate of the maneuver time $\hat{k}_m$ using the least-squares method.

Step.3 Check whether $\hat{U}$ belongs to the detectable maneuver set $\Omega_d(k;N,\hat{k}_m)$; If $\hat{U} \in \Omega_d$, then update the state estimate and error covariance using $\hat{U}$. Otherwise, go to step 4.

Step.4 Change $k \to k+1$ and go to step 1.

The proposed DMS(Detectable Maneuver Set) scheme can detect small maneuvers as well without increasing window size or PFA. In the next section, it will be shown that the proposed detection scheme is valid by applying to a 1-dimensional tracking problem.

4 Application to a 1D Tracking Filter

The proposed DMS scheme is applied to a 1-dimensional target tracking problem to show the superiority in tracking performance over conventional detection scheme. Target is assumed to move with a constant acceleration. The required matrices to express the target model are

$$F = \begin{pmatrix} 1 & T & \frac{1}{2}T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad G_d = \begin{pmatrix} \frac{1}{2}T^2 \\ T \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}. \tag{18}$$

It is assumed that the noise covariance values $Q(k)$ and $R(k)$ are constants. And we further assume that the tracking Kalman filter has arrived at a steady state for which the steady state covariance matrix can be calculated from the discrete-time algebraic Riccati equation

$$\bar{P} = F\bar{P}F^T + G_d Q G_d - F\bar{P}C^T R^{-1} C\bar{P}F^T. \tag{19}$$

From the solution of ARE, the steady state Kalman gain is calculated as

$$K_{ss} = \bar{P}C^T[C\bar{P}C^T + R]^{-1}. \tag{20}$$

Fig. 1 shows the minimum detectable maneuver magnitude $U_{\min}$ with respect to the detection delay $k - k_m$ when window size $N = 10$. It is assumed that the sampling time $T = 1$ sec and noise levels $Q = 1$, $R = 100$. The bounds of the system and sensor noises are taken as 3.5 times greater than the corresponding standard deviations. The detection delay time, t_d, can be expressed as

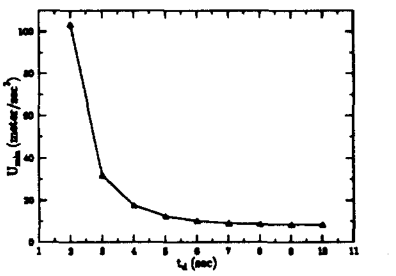

Fig. 1: $U_{\min}$ vs t_d when $N = 10$

$$t_d = (k - k_m)T. \tag{21}$$

Fig. 1 tells if the detection delay becomes larger, the smaller maneuver can be detected. Hence, Eq. (16) is very reasonable for a detection scheme because the calculated relation between $U_{\min}$ and t_d shows the trade-off property.

Simulation study is carried out for a 1-dimensional tracking problem. The proposed detectable maneuver set (DMS) scheme is compared with Bogler's algorithm of threshold $\lambda = 5$ and 15. The initial states for target model and filter are given identically like, $X(0) = \hat{X}(0) = \begin{pmatrix} 0 & 200 & 0 \end{pmatrix}^T$ under the assumption that the filter has arrived at a steady state. Initial error covariance matrix is $P(0) = 10^4 I_3$. The target maneuver scenario is given in Fig. 2. 10 filters are used for both of conventional Bogler and the DMS scheme. Monte Carlo simulations with 20 runs were performed for each case.

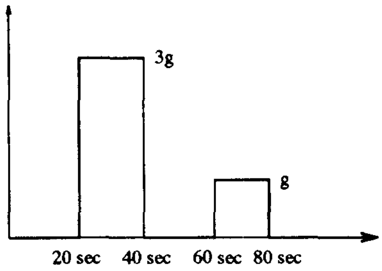

Fig. 2: Target maneuver scenario

Figs. 3 thru 5 show the number of estimated maneuver onset time $\hat{t}_m$ and the number of time when detection occurred. In Bogler's. algorithm, when the $\lambda = 5$, a relatively small value, as in Fig. 3, small maneuver $1g$ can be detected due to the increased detection probability. However, there are many false alarms. On the other hand, when $\lambda = 15$ as in Fig. 4 to decrease false alarms, the high maneuver $3g$ could be detected but the low maneuver $1g$ couldn't be detected. However, when the DMS scheme

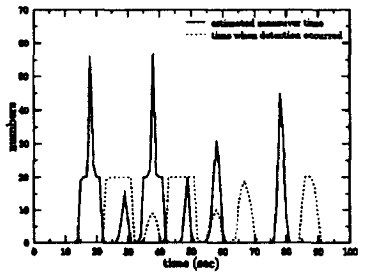

Fig. 3: Detection performance: Bogler with $\lambda = 5$

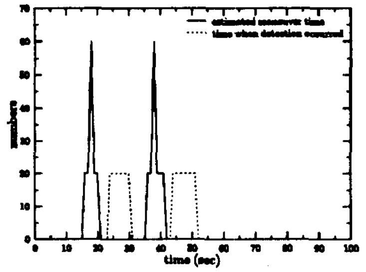

Fig. 4: Detection performance: Bogler with $\lambda = 15$

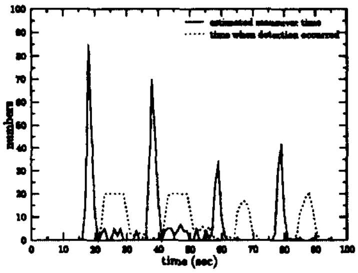

Fig. 5: Detection performance: the DMS scheme

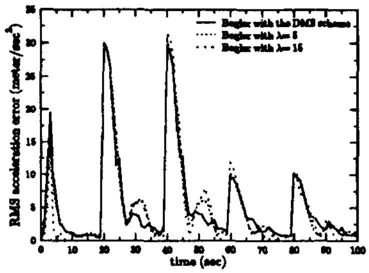

Fig. 8: Acceleration estimation error

is used as in Fig. 5, false alarms are reduced compared to Bogler with $\lambda = 5$ and the small maneuver could be detected without increasing window size.

Besides, there are two more advantages of the proposed detection scheme. One is that the estimated maneuver onset time is more consistent when the DMS scheme is used. This consistent estimation could be expected to improve the estimation performance. The other is that the DMS scheme has smaller detection delay time as. Smaller detection delay could reduce estimation error extremely.

Figs. 6 thru 8 shows the tracking performances of the three detection schemes. The estimation performances

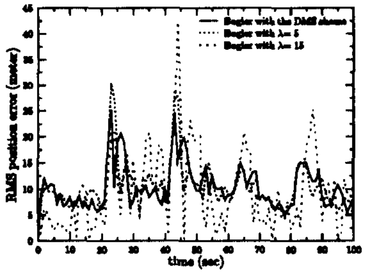

Fig. 6: Position estimation error

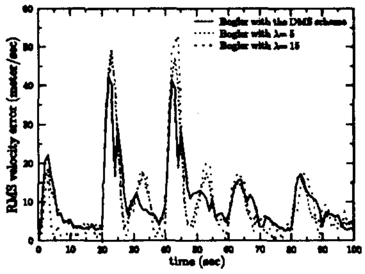

Fig. 7: Velocity estimation error

of position and velocity are much better when the DMS scheme is used. Especially at the maneuver onset times, the peaks of position and velocity errors are reduced due to smaller detection delay. The simulation results verifies that the proposed DMS scheme can produce much improved estimates of input estimation filters.

5 Concluding Remarks

In this paper, a new maneuver detection scheme incorporating target maneuver magnitude is presented. The simulation results showed that the DMS scheme is superior to conventional detection schemes due to the consistent estimation of maneuver onset time and reducing detection delay. Furthermore, the DMS scheme could detect small maneuvers as well as large maneuvers without enlarging the window size.

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