A Computed-Error-Input Based Learning Scheme for Multi-Robot Systems

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Abstract In this paper, a learning control problem is formulated for cooperating multiple-robot manipulators with uncertain system parameters. The commonly held object is also assumed to be unknown and the multiple-robots themselfs experience uncertain operating conditions such as link parameters, viscous friction parameters, stictions, actuator bias, and etc.. Under these conditions, the learning controllers designed for learning of uncertain parameters and robot control inputs for multiple-robot systems are shown to drive the multiple-robot manipulators to follow the desired Cartesian trajectory with the desired internal forces to the unknown object.

Keywords Computed-Torque Control, Learning Control, Cooperating Robot System

I. Introduction

Recently, a substantial number of learning control schemes for robotic systems have been reported [1,2,3,4,5,6,7,8,9,10,11]. In general. these learning controllers can be regarded as asymptotic optimal controllers in the sense that the optimality of the robot system is obtained gradually along with the elapse of iteration. That is, the robot trajectory error and the learning input error decrease asymptotically as the learning continues. However, the learning control differs from the classical optimal control theory, since it provides the optimal control input without complete knowledge or precise description of the robot dynamic model. On the other hand, the learning control concept can be extended to construct the learning controller for cooperating multi-robot systems, while the controller design of multi-robot systems is more involved than that of single robot. That is, the existence of closed kinematic chains and the internal forces between the multiple-robot manipulators and the held object makes the learning controller for multiple-robots much more complicated than for single-robot. Nevertheless, it is expected with the learning controller that uncertain operating conditions such as parametric uncertainties in robot systems and held object as well as the unknown external disturbances can be overcome. Moreover, since the learning controller learns the entired profile of trajectory in the given time interval, the tracking performance in the transient phase as well as in the steady state will be improved with the learning controller.

II. Multiple-Robot Systems

Consider $k_i(\geq 6,\ i=1,2,\cdots,r)$ degree of freedom r rigid robots holding a common rigid object. If we let $n=\sum_{i=1}^{r} k_i$, the dynamics of the cooperating r rigid robots can be described as

$$A(q)\ddot{q}+B(q,\dot{q})\dot{q}+c(q,\dot{q})+d+J^T f=\tau, \qquad (1)$$

where $q\in R^n$ is the augmented generalized vector of joint position and

$$A(q)=diagonal(A_i(q_i))$$
$$B(q,\dot{q})=diagonal(B_i(q_i,\dot{q}_i))$$
$$c(q,\dot{q})=[\ c_1^T(q_1,\dot{q}_1),\cdots,c_r^T(q_r,\dot{q}_r)]^T$$
$$d(t)=[\ d_1^T(t),\cdots,d_r^T(t)]^T$$
$$\tau(t)=[\ \tau_1^T(t),\cdots,\tau_r^T(t)]^T$$
$$J(q)=diagonal(J_i(q_i))$$
$$f(t)=[\ f_1^T,\cdots,f_r^T]^T \qquad \text{for} \quad i=1,2,\cdots,r.$$

$J^T \in R^{n\times 6r}$, $A(q)$, $B(q, \dot{q}) \in R^{n\times n}$ and $c(q, \dot{q})$, d, $\tau \in R^n$ are respectively the augmented robot Jacobian matrix, the augmented inertia matrix, the centripetal plus Coriolis force matrix, the gravity and viscous friction vector, the external disturbance vector and the augmented control input vector. $f \in R^{6r}$ is the interaction force vector. The dynamics of held object is given by

$$A_2(p_o)\ddot{p}_o + B_2(p_o, \dot{p}_o)\dot{p}_o + c_2 = f_o, \qquad (2)$$

where

$$A_2(p_o) = diagonal(m_o I_{3\times 3}, J_o)$$
$$B_2(p_o, \dot{p}_o) = diagonal(0_{3\times 3}, E(\dot{p}_o))$$
$$c_2 = [\ -m_o g^T, 0^T_{1\times 3}]^T$$
$$f_o = Gf$$
$$\dot{p}_o = [\ \dot{p}^T_c, w^T_c]^T.$$

$p_o \in R^6$ is the position and orientation of the center of mass of object with respect to the world frame. $A_2(p_o)$, $B_2(p_o, \dot{p}_o) \in R^{6\times 6}$, c_2, $f_o \in R^6$ are the mass-inertia matrix, the centripetal plus Coriolis force matrix, the gravity vector and the resultant force vector on the object, respectively. m_o, $J_o \in R^{3\times 3}$ and $E(\dot{p}_o) \in R^{3\times 3}$ are the mass, the inertial matrix of the object and the skew-symmetric matrix satisfying $E(\dot{p}_o)w_c = w_c \times J_o w_c$. $\dot{p}_c \in R^3$ and $w_c \in R^3$ represent the linear and angular velocity of the center of mass. The grasping matrix $G \in R^{6\times 6r}$ transforms the end-effectors' interacting force to the resultant force of the object and is defined as

$$G = \begin{bmatrix} I_{3\times 3} & 0_{3\times 3} & I_{3\times 3} & 0_{3\times 3} \cdots I_{3\times 3} & 0_{3\times 3} \\ P_1 & I_{3\times 3} & P_2 & I_{3\times 3} \cdots P_r & I_{3\times 3} \end{bmatrix},$$

where $P_i \in R^{3\times 3}$ denotes the skew-symmetric matrix such that $P_i f_{e_i} = f_{e_i} \times p_{ce_i} (i = 1, 2, \cdots, r)$. $f_{e_i} \in R^3$ and $p_{ce_i} \in R^3$ represent the interacting force vector of the ith end-effector with the object and the distance vector between the center of mass of the object and the ith end-effector's contacting point with the object, respectively. The dynamic equations of multiple-robot system and the object are merged into a compact form by using the kinematic relationship of the cooperating robot system:

$$\dot{p}_e = J\dot{q} = G^T \dot{p}_o \qquad (3)$$
$$\ddot{q} = J^{\#}[\ G^T \ddot{p}_o + \dot{G}^T \dot{p}_o - \dot{J}\dot{q}] + v_n, \qquad (4)$$

where $\dot{p}_e \in R^{6r}$ is the linear and angular velocity of multiple-robots' end-effectors with respect to the world frame. $J^{\#} = J^T(JJ^T)^{-1} \in R^{n\times 6r}$ is the generalized pseudo-inverse of $J \in R^{6r\times n}$, where the Jacobian J is assumed to be full-rank. $v_n \in R^n$ is a vector in the null-space of the augmented Jacobian J representing any multiple- robot motion which does not result in the motion of the held object. Using the kinematic equations (3) and (4), the multiple-robot system (1) ca be written as

$$A_1 \ddot{p}_o + B_1 \dot{p}_o + c_1 + d_1 + Gf = \tau_0, \qquad (5)$$

where we let $v_n = 0$ and

$$A_1 = GJ^{\#T}AJ^{\#}G^T$$
$$B_1 = GJ^{\#T}AJ^{\#}\dot{G}^T - GJ^{\#T}(AJ^{\#}\dot{J} - B)J^{\#}G^T$$
$$c_1 = GJ^{\#T}c$$
$$d_1 = GJ^{\#T}d$$
$$\tau_0 = GJ^{\#T}\tau.$$

Then, the learning control problem for the cooperating robot system is to generate the learning control input $\tau_i(t)$ for $[\ 0, t_f]$ to drive the robot system to the desired configuration trajectories (the position, orientation and interacting fоrce). The followings on the multiple-robot system and the object are assumed to solve the learning control problem.

A1) The inertia matrices $A_i (i = 1, 2)$ are positive definite.

A2) $\dot{A}_i - 2B_i$ $(i = 1, 2)$ are skew-symmetric.

A3) The following holds for the multiple-robot system (1) and the object dynamics (2)

$$A_1 \ddot{p}_o + B_1 \dot{p}_o + c_1 + d_1 = Y_1(p_o, \dot{p}_o, \ddot{p}_o)\theta_1$$
$$A_2 \ddot{p}_o + B_2 \dot{p}_o + c_2 = Y_2(p_o, \dot{p}_o, \ddot{p}_o)\theta_2,$$

where $Y_i(p_o, \dot{p}_o, \ddot{p}_o) \in R^{6\times l_i}$ and $\theta_i \in R^{l_i} (i = 1, 2)$ are the regression matrices and unknown parameter vectors, respectively.

A4) The desired input τ_d consists of τ_{1_d} and τ_{2_d}.

$$\tau_d = \tau_{1_d} + \tau_{2_d},$$

where

$$\tau_{1_d} = A_{1_d} \ddot{p}_{o_d} + B_{1_d} \dot{p}_{o_d} + c_{1_d} + d_{1_d}$$
$$\tau_{2_d} = A_{2_d} \ddot{p}_{o_d} + B_{2_d} \dot{p}_{o_d} + c_{2_d}.$$

A5) The augmented Jacobian matrix J is full rank.

III. The Multiple-Robot Learning System

The control input for the multiple-robot learning system (1) is defined as at the j-th iteration

$$\tau^j = J^{j^T}(G^{j^*}(\tau^j_1 + \tau^j_2) + f_d), \qquad (6)$$

where $G^{\#}=G^T(GG^T)^{-1}\in R^{6r\times 6}$ is the generalized pseudo-inverse of $G\in R^{6\times 6r}$ and

$$f^j=f_d+G^{j^*}\tau_2^j \ . \qquad (7)$$

$f_d\in R^{6r}$ is in the null-space of G(, i.e., $Gf_d=0$) and $\tau_i^j=\tau_{ie}^j+\tau_{ib}^j+\tau_{il}^j$ for $i=1,2$. $\{\tau_{ie}^j, \tau_{ib}^j, \tau_{il}^j\}$ $(i=1,2)$ represent the computed error input, the position feedback input and the learning input, respectively, for the multiple-robot system and the held object.

$$\tau_{1e}^j=\widehat{A_{1e}^j}\ddot{p}_{od}+a\widehat{A_1^j}\dot{e}_p^j+\widehat{B_{1e}^j}\dot{p}_{od}+a\widehat{B_1^j}e_p^j + \widehat{c_{1e}^j}+\widehat{d_{1e}^j}$$

$$\tau_{2e}^j=\widehat{A_{2e}^j}\ddot{p}_{od}+a\widehat{A_2^j}\dot{e}_p^j+\widehat{B_{2e}^j}\dot{p}_{od}+a\widehat{B_1^j}e_p^j+\widehat{c_{2e}^j}$$

$$\tau_{ib}^j=\beta(L_i+Y_{ie}^jS_i^{-1}Y_{ie}^{j^T})z^j \quad \text{for } i=1,2,$$

where $e_p^j=p_{o_d}-p_o^j$ and $z=\dot{e}_p^j+ae_p^j$. β is a positive gain and L_i and S_i are symmetric and positive definite gain matrices. $\widehat{(\cdot)}$ is an estimated system such that

$\widehat{A_{ie}^j}=\widehat{A_i^j}-\widehat{A_{id}^j}$, $\widehat{B_{ie}^j}=\widehat{B_i^j}-\widehat{B_{id}^j}$,

$\widehat{c_{ie}^j}=\widehat{c_i^j}-\widehat{c_{id}^j}$ $(i=1,2)$ and $\widehat{d_{1e}^j}=\widehat{d_1^j}-\widehat{d_{1d}^j}$,

where the $(\cdot)_d$ denotes the desired trajectories being used in the system description $(\cdot)$. The regression matrix $Y_{ie}^j\in R^{6\times l_i}$ $(i=1,2)$ is derived from the equation $Y_{ie}^j\widetilde{\theta_i^j}=\tau_{ie}^{j^*}-\tau_{ie}^j$, where $\tau_{ie}^{j^*}$ is the computed error input τ_{ie}^j with replacement of the estimated parameter $\widehat{\theta_i^j}$ with the system parameter vector θ_i. Substituting (6) and (7) into (2) and (5), respectively, yields for $i=1,2,3$,

$$A_i^j\dot{z}^j+B_i^jz^j+\beta(L_i+Y_{ie}^jS_i^{-1}Y_{ie}^{j^T})z^j = Y_e^j\widetilde{\theta^j}+\widetilde{\tau_l^j}, \qquad (8)$$

where $\widetilde{\tau_l^j}=\tau_d-\tau_l^j$ and $\widetilde{\theta^j}=\theta-\widehat{\theta^j}\in R^l$. The matrices and vectors $(\cdot)_3$ in error sytem (8) are formed by adding or augmenting the corresponding matrices or vectors $(\cdot)_1$ and $(\cdot)_2$.

The learning rules for the uncertain error dynamic system (8) are given by

$$\tau_{il}^{j^*}=\tau_{il}^j+\beta L_iz^{j-1} \qquad (9)$$

$$\hat{\theta}_i^{j^*}=\hat{\theta}_i^j+\beta S_i^{-1}Y_{ie}^{j-1^T}z^{j-1} \text{ for } i=1,2, \qquad (10)$$

where β is a positive learning gain and L_i and S_i are symmetric and positive definite gain matrices. The initial conditions for the first period of time are $z^j(0)=0$, $\tau_{il}^1=\hat{\tau}_{id}^1$ and $\hat{\theta}_i^1=\theta_{i0}$ for $t\in[0,t_f]$, where θ_{i0}'s are the nominal parameter vectors. The projection applied to the learning signals is defined as follows:

$$x_i^j=\begin{cases}\bar{x}_i^j, & \text{if } x_i^{j^*}>\bar{x}_i^j\\ x_i^{j^*}, & \text{if } \underline{x}_i^j\le x_i^{j^*}\le\bar{x}_i^j\\ \underline{x}_i^j, & otherwise.\end{cases}$$

x_i^j represents the i-th element of τ_{il}^j or $\widehat{\theta_i^j}$, where τ_{il}^j $(1\le i\le 6)$ and $\widehat{\theta_i^j}$ $(1\le i\le l)$.

The learning system is now shown to converge in the following statements.

Theorem 1: Suppose that the learning system consists of the equations of system (1) and (2), the control inputs (6) and (7) and the learning rules (9) and (10). Then, the learning system converges globally asymptotically as follows:

i) $\lim_{j\to\infty}e^j=0$ and $\lim_{j\to\infty}\dot{e}^j=0$

ii) $\lim_{j\to\infty}\dot{z}^j=0$

iii) $\lim_{j\to\infty}\widetilde{\tau_l^j}=0$

iV) $\lim_{j\to\infty}e_f^j=0$ for almost all $t\in[0,t_f]$,

where e_f^j represent the internal force error $e_f^j=f_d-f_n^j$.

Proof: Let an index functional $J^j(t)$ be

$$J^j(t)=\frac{1}{2\beta}\int_0^t(\tilde{\tau}_l^{j^T}L^{-1}\tilde{\tau}_l^j+\tilde{\theta}^{j^T}S\tilde{\theta}^j)d\eta,$$

where $\tilde{\tau}_l=[\tilde{\tau}_{1l}^T, \tilde{\tau}_{2l}^T]^T$,

$\tilde{\theta}=[\tilde{\theta}_1^T, \tilde{\theta}_2^T]^T$.

Further, let

$$J^{j^*}(t)=\frac{1}{2\beta}\int_0^t(\tilde{\tau}_l^{j^{*T}}L^{-1}\tilde{\tau}_l^{j^*}+\tilde{\theta}^{j^{*T}}S\tilde{\theta}^{j^*})d\eta,$$

where $J^{j^*}(t)$ is computed as in $J^j(t)$ by using the learning signals before projection. Then, it is obvious that $J^j(t)\le J^{j^*}(t)$.

Let $\nabla J^j(t)=J^j(t)-J^{j-1}(t)$. Then, from the error system (8), we obtain

$\nabla J^j(t)\le -z^{j^T}A_3^jz^j\le 0$,

implying $\lim_{j\to\infty}z^j=0$.

To prove ii), we first show the following holds for each $t \in [\ 0, t_f]$ by using the error system (8):

$$| \dot{z}^j - z^{j-1}| \leq O(z^j, z^{j-1}),$$

where $\lim_{j \to \infty} O(z^j, z^{j-1}) = 0$.

Then, since $J^j(t) \leq J^{j-1}(t)$ and $J^j(t)$ is equi-continuous, it can be shown $\lim_{j \to \infty} z^j = 0$ uniformly for each $t \in [\ 0, T]$ implying that of $O(z^j, z^{j-1})$. With this, the inequality implies that $\lim_{j \to \infty} \dot{z}^j = 0$ in the compact set $[\ 0, t_f]$. This, in turn, implies that $\lim_{j \to \infty} \tilde{\tau}^j_l = 0$ in the error system (8). Further, from the equation (2), the end-effectors' force of multiple-robots is given by

$$f^j = G^{j^*}[\ A^j_2 \ddot{p}^j_o + B^j_2 \dot{p}^j_o + c^j_2] \ + f^j_n \ ,$$

where $G^{\#} = G^T(GG^T)^{-1} \in R^{6r \times 6}$ is the generalized pseudo-inverse of $G \in R^{6 \times 6r}$ and $f_n \in R^{6r}$ is the force vector in the null-space of G. With this and (6) the internal force error is described as

$$e^j_f = G^{j^*}[\ A^j_2 \dot{z}^j + B^j_2 z^j + \beta(L_2 + Y^j_{2e} S^{-1}_2 Y^{j^T}_{2e}) z^j - Y_{2e}{}^j \ \tilde{\theta}_2{}^j - \ \tilde{\tau}_{2l}{}^j] \quad .$$

Since the right side of the equation converges to zero pointwisely for each $t \in [\ 0, t_f]$, $\lim_{j \to \infty} e^j_f = 0$

Q.E.D.

Now, let the learning rules (9) and (10) be replaced with the following learning rules for $i = 1, 2$

$$\tau^{j^*}_{il} = \tau^{j-1}_{il} + \beta L_i z^j \qquad (11)$$

$$\hat{\theta}_i{}^{j^*} = \ \hat{\theta}_i{}^{j-1} + \beta S^{-1}_i Y^{j^T}_{ie} z^j \quad . \qquad (12)$$

The control inputs for these learning rules are given by

$$\tau^j_i = \tau^j_{ib} + \tau^j_{ie} + \tau^{j-1}_{il} \quad (i = 1, 2) . \qquad (13)$$

These new learning rules cnverge as follows.

Theorem 2: The learning system with the learning rules (11) and (12) and the control input (13) for the cooperating robot system converges as follows:

i) $\lim_{j \to \infty} e^j = 0$ and $\lim_{j \to \infty} \dot{e}^j = 0$

ii) $\lim_{j \to \infty} \dot{z}^j = 0$

iii) $\lim_{j \to \infty} \tilde{\tau}^j_l = 0$

iV) $\lim_{j \to \infty} e^j_f = 0$ for almost all $t \in [\ 0, t_f]$,

where the internal force error is defined in *Theorem 1*.

The proofs of *Theorem 2* is similar to *Theorem 1* .

V. Conclusion

We present in this paper a learning controller for multiple-robot systems which coorporate to handle a common unknown object. The learning algorithm includes the parameter learning rules to learn the uncertain parameters of robot system and object as well as the desired control inputs. It is shown that all the learning signals in the learning system are bounded and the robot motion converges to the desired one as the learning continues.

References

[1] C.G. Atkeson and J. McIntyre, ''Robot Trajectory Learning Through Practice'', *IEEE Conf. Robotics and Automation*, pp.1737-1742, 1986.

[2] S.Arimoto, S.Kawamura, and F.Miyasaki, ''Bettering operation of robots by learning,'' *J.Robotic Syst.*,pp123-140, 1984.

[3] P.Bondi, G.Cassaline, and L.Gambardella, ''On the iterative learning control theory for robotic manipulators,'' *IEEE J.Robotics Automat.*, vol.4, no.1, Feb. 1988.

[4] D. Jeon and M. Tomizuka, ''Learning Hybrid Force and Position Control of Robot Manipulators'', *IEEE J. Robotics and Automation*, Vol.9, No.4, pp.423-431, August 1993.

[5] T. Kuc and Jin.S. Lee, ''An Adaptive Learning Control of Robot Manipulators'', *IEEE Conf. Dec. Contr.*, U.K., Dec. 1991.

[6] A. de Luca and F. Mataloni, ''Learning Control of Redundant Manipulators'', *IEEE Conf. on Robotics and Automation*, pp.1442-1450, Sacramento, CA, Apr. 1991.

[7] W. Messner, R. Horowitz, W. Kao, and M. Boals, ''A New Adaptive Learning Rule'', *IEEE Conf. Robotics and Automation*, 1990.

[8] W.T.Miller III, F.H.Glanz, and L.G.Kraft III, ''Application of a general learning algorithm to the control of robotic manipulators,'' *Int. J. Robotics Res.*, vol.6, no.2, Summer 1987.

[9] H.Miyamoto, M.Kawato, T.Setoyama, and R.Suzuki, ''Feedback-error-learning neural network for trajectory control of a robot manipulator,'' *IEEE Trans. Neural Networks*, vol.1, pp.251-265, 1988.

[10] N. Sadegh, R. Horowitz, W. -W. Kao, and M. Tomizuka, ''A Unified Approach to the Design of Adaptive and Repetitive Controllers for Robotic Manipulators'', *ASME J. of Dynamic Systems, Measurement, and Control*, Vol.112, pp. 618-629, Dec. 1990.

[11] D. Wang, Y.C. Soh, and C.C. Cheah, ''Robust Learning Control for Constrained Robots'', *IEEE Conf. Dec. and Contr.* pp.608-613, Dec. 1992.