Optimal Capital/Labor Ratio in R&D Sector As a Policy Variable

유평일 · 장영현

Abstract

The purpose of this paper is to investigate the role of government in innovation. That is, how much should government invest in R&D activity for social welfare enhancement. An optimal control problem is proposed to answer the above question. Because of difficulties in solving the problem, simulation utilized to find an approximate solution.

All results obtained from the simulation are very similar. The investment in R&D activity increases and reaches its maximum and then decreases continuously. If the importance of technology grows the investment in R&D activity should be expanded.

I. Introduction

Innovation has long been recognized as providing benefits to society far beyond those that accrue to any particular participant in the private sector, and as being important contributor to economic growth[Stoneman (1983), Dasgupta (1987)]. However, in the absence of government intervention, private-sector participants consider only their particular benefits when choosing the appropriate level of commitment to the innovation process. This result in "market failure."

In case of R&D, especially of basic research, market failure is often a result of features intrinsic to the production of information. Hence, government has intervened to eliminate market failure in the provision of R&D output. It is found that technological change has contributed greatly to output growth since the industrial revolution[Gomulka (1990)]. For this reason it is an important issue what is the optimal role of government for economic growth through R&D activity. The goal of this paper is to answer the following question : To maximize social welfare, how much should government provide R&D output?

In general, social welfare is represented by aggregate amount of consumption. In order to answer the question, a dynamic economic model is proposed where government decides the size of research sector represented by both the number R&D personnel and the capital stock of R&D sector.

The output of R&D activity is knowledge. In general, technology is defined to be the human knowledge applied in production. Through the decision of R&D activity, government decides the supply of technology. This model tries to give some lights on government's role in innovation, that is, knowledge provider.

The paper is organized as follows. In section II theoretical backgrounds are presented. Section III defines a basic economic model. In section IV an optimal control problem of government will be proposed and its solution will be explored by simulation. Section V summarizes the results and discusses policy implications.

II. Theoretical Backgrounds

1. Technology in Neoclassical Economics

In neoclassical economics, technology was regarded as an exogenous factor, that is, technological change was a given factor irrelevant to the inside mechanism of the economy. Hence, technology was a non-controllable factor in neoclassical framework. In neoclassical economics technology was represented in the typical production function as

$$Q = A(t)F(K,L) = A(t)K^{\alpha}L^{\beta} \tag{1}$$

where $A(t)$ is exogenous technological change.

The change of parameter $A(t)$ stands for exogenous technological change and assumed to be determined exogenously. Hence, in neoclassical economy technology policy -- government's role in knowledge creation -- cannot be disputed.

Technological change is represented in neoclassical economics as a residual. By taking the logarithm and differentiating above equation with respect to time(total differentiation), we get

$$\frac{\dot{Q}}{Q} = \mathring{A} + \alpha \mathring{K} + \beta \mathring{L} \tag{2}$$

where the superscript dot means time derivative.

2. Endogenous Technological Change

Through the studies of economic growth it was found that the contribution of technology in production was not negligible but was great[Arrow (1962)]. Hence, the concern about the inside mechanism of technological change was extended. Economists have embraced technology as an internal economic factor.

This work distinguishes an endogenous outcome of an economic system not the result of forces that impinge from the outside. For this reason, theoretical work does not invoke exogenous technological change to explain why income per capita has increased by an order of magnitude since the industrial revolution. The empirical works do not settle for measuring a growth accounting residual that grows at different rates across countries. It tried instead to uncover private and public sector choices that cause the rate of growth to vary across the countries. As in neoclassical growth theory, the focus in endogenous growth is on the behavior of the economy as a whole. As a result, this work is complementary to, but different from, the study of research and development or productivity at the level of the industry or firm.

One of the most important factor in endogenous growth is the accumulation of knowledge. This plays a crucial role in economic growth. It deserves comment about the process of R&D activity.

Teitel[1994] regressed output of R&D activity represented by patent on R&D input. R&D input factors are R&D expenditures, the number of scientists and engineers. The regression model in logarithmic form is

$$\log R = r_0 + r_1 \log S + r_2 \log E + \varepsilon \tag{3}$$

where R is the number of patents granted to residents of the country, S is the total number of scientists and engineers available in a country, E is the expenditures in research and experimental development for the year.

For all countries, the results of study indicate that a 1% increase either in R&D expenditures, or in the stock of scientists and engineers, leads to a 1% increase in the number of patents granted to residents. For low income countries, both elasticities are less than one. For high income countries, however, both elasticities are greater than one and the elasticity of stock of scientists and engineers is greater than that of R&D expenditures(1.35 and 1.06, respectively).

III. Basic Model for Analysis

1. Building of Economic Model

The economy consists of two sectors. one is production sector and the other research sector. The former produces industrial goods used either for investment or consumption and the latter knowledge for production sector. The most fundamental assumption is that perfect competition dominates economy. That is, marginal cost is equal to marginal product. Production sector utilizes knowledge stock for output production. The main role of government in this model is to supply knowledge stock for production sector. For this government subsidizes research sector by means of tax. We use indices p and r to denote the production sector and research sector, respectively. Let L and K denote the total population and total capital stock. It is also assumed that labor is homogenous and always fully employed at any point of time. The structure of model is presented in the follow sections.

(1) Production Sector

In order to examine the economic role of knowledge in production, a kind of Cobb Douglas production function is proposed. It is assumed that production is carried out by combining knowledge, capital, and labor force. The production function can be specified as follows[Zhang(1993), Kazuyuki(1989)]

$$Q = AZ^{\alpha} K_p^{\beta} L_p^{1-\beta} \tag{4}$$

where A is a constant, Z is R&D stock, β is output elasticity of capital, K_p and L_p are capital and labor of production sector, respectively. The parameter A indicates the exogenous technological change assumed to be constant in this model.

(2) Capital and Labor Markets

All labor is allocated to either research sector or production sector. It is assumed that labor is homogenous and can be freely transferred from one sector to another. Let r and w be interest rate and wage rate, respectively. Since perfect competition dominates the market, wage rate and interest rate are equal to the marginal product of labor and capital, respectively.

$$w = MP_{L_p} = \frac{(1-\beta)}{L_p}Q,\quad r = MP_{K_p} = \frac{\beta}{K_p}Q \tag{5}$$

(3) Saving and Capital Formation

The net income of the population, *Y(t)*, is equal to the earnings from labor and capital. Hence, it is given by

$$Y(t) = rK + wL \tag{6}$$

Denote by $s(0 < s < 1)$ the constant saving rate of the population out of all income. It is assumed that all savings are invested in capital, the capital accumulation is formulated by

$$\dot{K} = sY - \delta K \tag{7}$$

in which δ is the given constant depreciation rate of capital. Price has no particular meaning in this model because there is only one kind of product.

(4) Knowledge Creation(Output of R&D)

The characteristic of research output is that its output is both accumulated and depreciated through time. Assume that knowledge creation is the function of R&D capital and R&D personnel devoted to research activity. Then, knowledge creation function can be specified as follows[Teitel].

$$\dot{Z} = K_r^i L_r^j - \rho Z \tag{8}$$

in which ρ is the given constant depreciation rate of knowledge stock, i and j are elasticities of knowledge creation of capital and labor, respectively. Equation (8) means that knowledge creation is proportional to the capital and labor of research sector and its output is depreciated through time. The depreciation rate can be measured patent renewal data.

(5) The budget of Research Sector and policy variables

For welfare enhancement, government subsidizes research sector with tax. The total tax is determined by total wage and capital cost of research sector. Hence, the total tax will be

$$Tax = rK_r + wL_r \tag{9}$$

It is time to design a way for government to determine K_r and L_r. The number of scientists and the capital stock of research sector are determined by the following ways,

$$L_r = aL,\ K_r = bK,\ 0 < a,b < 1 \tag{10}$$

Since the total tax is equal to the budget of research sector, the tax rate is determined endogenously. There are two variables having the dynamic properties, that is, K and Z. Dynamics of the system can be expressed in terms of K and Z as

$$\begin{aligned} \dot{K} &= sQ\left[\frac{1-\beta}{1-a} + \frac{\beta}{1-b}\right] - \delta K \\ \dot{Z} &= [bK]^i[aL]^j - \rho Z. \end{aligned} \tag{11}$$

Equation (11) describes the time-dependent paths of capital and knowledge stock.

2. Existence of Equilibrium

In equilibrium, state variables do not change any more. However, this does not mean that all variables are constants. This means all variables are constants in relative meaning. When the economy is in equilibrium, a balanced growth path can be defined. The balanced growth path can be specified by setting in equation (11) $\dot{K} = 0$ and $\dot{Z} = 0$, which yields

$$sQ\left[\frac{1-\beta}{1-a} + \frac{\beta}{1-b}\right] = \delta K \tag{12}$$

$$[bK]^i[aL]^j = \rho Z \tag{13}$$

By solving the above equilibrium equations, we can find out the equilibrium values of capital and knowledge in a steady state. From equation (12) and (13), we have

$$K = BZ^{\alpha/(1-\beta)} \tag{14}$$

where $B = (1-a)L\left[\frac{\delta}{sA(1-b)^{\beta}[(1-\beta)/(1-a)+\beta/(1-b)]}\right]^{1/(\beta-1)}$

Substituting equation (14) into the equation (13) yields

$$Z = [(bB)^i(aL)^j]^{\frac{(\beta-1)}{\alpha i+\beta-1}} \tag{15}$$

Equation (14) describes the relation between capital and knowledge stock in a steady state. The values of capital and knowledge stock are dependent on policy variables, production technology, depreciation rate, and total labor.

It is worth while to note that the equilibrium of the model is not static but dynamic. This means that the equarion (14) and (15) move as time goes by and the equilibrium will also change continuously by this shift.

IV. Model Solution with Optimal Control Method

In this section, the role of government as a policy maker will be investigated. Because of the dynamic property of the system, the government's problem can be transformed into a dynamic optimization problem. This sort of problem is called optimal control problem.

In this problem, decision variables of government are the ratio of capital and labor which are employed in research sector. Because of the dynamic property of the model, we can set up an optimal control problem of government as follows. As mentioned before, the objective function of government is to maximize aggregate consumption. The aggregate amount of consumption is

$$C = Y - sY - tax$$
$$= Q\left[\frac{(1-\beta)(1-s-a)}{1-a} + \frac{\beta(1-s-b)}{1-b}\right] \qquad (16)$$

Government's problem is a kind of optimal control. There are two kinds of state variables, that is, K (capital stock) and Z (knowledge stock). If social welfare is dependent on the aggregate consumption, government's problem can be formulated as follows

$$Max \int U(C)dt = \int Q\left[\frac{(1-\beta)(1-s-a)}{1-a} + \frac{\beta(1-s-b)}{1-b}\right]dt$$
$$s.t \quad \dot{K} = sQ\left[\frac{1-\beta}{1-a} + \frac{\beta}{1-b}\right] - \delta K \qquad (17)$$
$$\dot{Z} = [bK]^i [aL]^j - \rho Z$$
$$Z(0) = Z_0,\ K(0) = K_0.$$

Since government determines the ratio of capital and labor in research sector, the control variables are 'a' and 'b'. 'a' is the ratio of labor and 'b' is the ratio of capital. The state variables are knowledge stock and capital stock.

1. Phase Diagram Analysis

Because of the difficulties in solving optimal control problem qualitative analysis called phase diagram analysis is utilized. Using this analytical tool, we can trace the solution of optimal control problem. This analysis begins with sketching the steady state paths of the state variables. Then, we can search the movement's direction of state variables in each location. By sketching the movement of state variables, we can find out the property of equilibrium too.

To construct phase diagram, our first business is to draw two demarcation curves. Individually, each of these curves serves to delineate the subset of points in the space where the variable in question can be stationary.

Setting $\dot{K} = 0$ and $\dot{Z} = 0$ we get,

$$K = BZ^{\alpha/(1-\beta)} \tag{18}$$

$$Z = \frac{1}{\rho}[bK]^{i}[aL]^{j} \tag{19}$$

Points off the demarcation curves are very much involved in dynamic motion. The direction of the movement depends on the signs of the derivative K and Z at a particular point in the KZ space; the speed of movement depends on the magnitudes of those derivatives. From equation (11), we find by differentiation that

$$\begin{aligned} d\dot{Z}/dt &= -\rho < 0 \text{ and} \\ d\dot{K}/dt &= c \cdot MP_k > 0 \end{aligned} \tag{20}$$

where c is some constant greater than zero.

Following the directional restrictions imposed by the arrowheads and sketching bars, we can draw a family of streamlines, or trajectories, to portray the dynamics of the system from any conceivable initial point. Following the above equations, phase diagram can be sketched and its equilibrium is very stable. This implies that there is driving force making all points out of equilibrium go to the equilibrium.

2. Numerical Solution of Optimal Control problem

In this section solution which are satisfying the objective of government will be explored. The government's problem is described in equation (17). To begin with, it is necessary to define Hamiltonian function used for optimal control problem.

The Hamiltonian function is given by

$$\begin{aligned} H = Q\left[\frac{(1-\beta)(1-s-a)}{1-a} + \frac{\beta(1-s-b)}{1-b}\right] + \lambda_1\left[sQ\left[\frac{1-\beta}{1-a} + \frac{\beta}{1-b}\right] - \delta K\right] \\ + \lambda_2\left[[bK]^{i}[aL]^{j} - \rho Z\right] \end{aligned} \tag{21}$$

in which 'a' and 'b' are costate variables of capital and knowledge stock. The costate variable is akin to a Lagrange multiplier and measures the shadow price of an associated state variable.

It is almost impossible to find out smart solution of optimal control problem unless the functional form is very simple. Hence, solution under the assumption of $a = b$ will be unearthed. This assumption means that the ratio of capital and labor is same. Though this assumption is strict it can be observed that increase in labor is accompanied by increase in capital in real world.

Many empirical analyses show that knowledge creation is proportional to R&D personnel and capital[Teitel,(1994)]. For simplicity of the problem the assumption that $i = j = 1$ is introduced. In general, i is not equal to j. This assumption makes it possible to find a solution. However, this assumption does not cut the meaning of the solution seriously. Under these assumptions the Hamiltonian function can be rewritten as follows.

$$H = Q\left[\frac{(1-s-a)}{1-a}\right] + \lambda_1\left[\frac{sQ}{1-a} - \delta K\right] + \lambda_2\left[a^2KL - \rho Z\right]. \tag{22}$$

The optimal solution must satisfy the necessary condition which can be written in equation (23).

$$a = \frac{C_1}{\lambda_2}, \text{ where } C_1 = \frac{AZ^\alpha}{2K^{1-\beta}L^\beta} \tag{23}$$

Equation (23) is the relation between parameters in the middle of the dynamic process. Hence, nothing can be inferred from the equation. We must know about the λ_1 and λ_2 to find an exact solution. However, it is hard to find λ_1 and λ_2 in functional form from the results. However, we can know better by simulation. In next section, each parameter is assigned as initial value and the path of each parameter is traced.

3. Solution by Simulation

(1) Simulation Procedures

The solution obtained from optimal control problem was a naive solution. It is hardly possible to find implications from the results. Hence it is somewhat useful to search a solution by simulation. The procedure of simulation is as follows

1. Define an initial value of each parameter.
2. From the equation of motion, calculate the derivative of each parameter.
3. Calculate the new value of each parameter by approximation.
4. Return to initial stage.

2) The Outcomes of Simulation and its Interpretation

All results obtained from simulation show that the optimal value of 'a' follows path shown Fig.1

The ratio of capital and labor occupied by research sector should follow the Figure 1. The shape of the optimal path is not much influenced by the initial value of 'a', its shapes are very alike. This implies that the solution of optimal control problem is like Figure 1. The contour of a^* (optimal size of research sector) shows that at first the value a^* increases and reaches its maximum and then decreases as time goes by. The path of a^* can be categorized into two stages.

In the first stage, a^* increases and in the second stage, it decreases. The solution has economic implications as follows

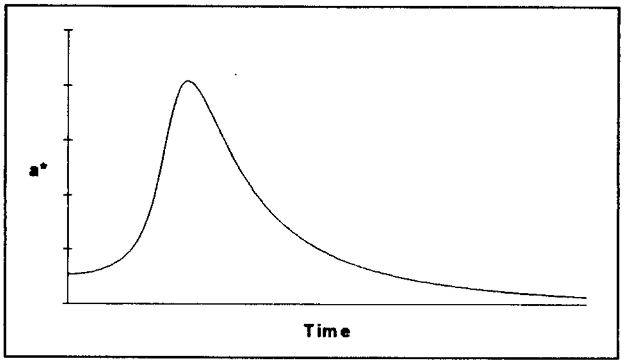

Figure 1. The optimal size of research sector

In the first stage, the shadow price of knowledge stock is greater than that of capital stock. In other words, the value of knowledge stock is greater than that of capital stock. Hence, the investment in R&D activity should be increased. In this stage, output growth mainly results from the expansion of knowledge stock. In the second stage, the shadow price of capital stock is greater than that of knowledge stock. Hence, the investment in research activity should be reduced. In this stage, the aggregate capital stock is greater than the knowledge stock. Because government's decision is dependent on the contribution to output, the size of research sector will be decreased as time goes by.

It is worthwhile to comment about the results. As shown in Figure 1, the optimal value of a^* goes to zero as time goes by. Time horizon is infinite in the framework of the model. However, infinite time horizon has little meaning in reality. Because of the framework in the model, growth of capital stock is greater than that of knowledge stock. This difference in growth rate makes the optimal value of a^* goes to zero as time goes to infinity.

Figure 2 and Figure 3 make sure of the existence of solution again. Given some initial value of knowledge stock, the optimal value of 'a' according to initial 'a' follows the curve like Figure 2.

Figure 3 makes sure of the existence of solution because the shapes of curve are very alike regardless of the initial value of 'a'. Overall pattern of a^* is same regardless of initial knowledge stock as well as capital stock. As shown in Figure 3, the shapes of curve vary according to initial value of 'a' but its pattern is same: at first, value of 'a' increases until reaches its maximum then decreases as time goes by.

The size of research sector goes zero as time goes by. But the time horizon is infinite, so it has little meaning.

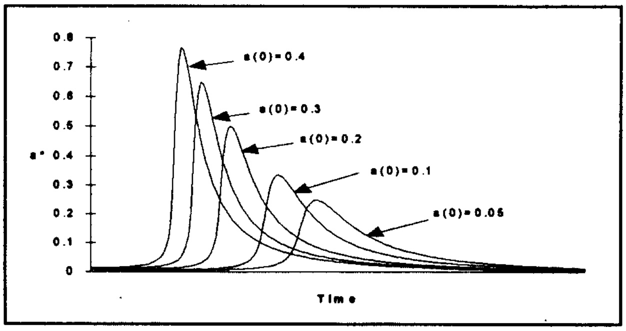

Figure 2. The optimal value of 'a' when $Z(0) = 100$

Given the initial value of 'a', the optimal value of 'a' is portrayed as Figure 3. Overall shapes of curve vary according to initial value of 'a' but its pattern is very similar.

The approximate shapes of results is very alike regardless of initial value of 'a'. Figure 3 shows that as the initial value of knowledge stock increases the maximum value of a^* and its time of maximum decreases. This results can be interpreted that the more knowledge stock has at initial stage the less value research output has. Hence, the time of maximum 'a' is earlier and the size of research sector should be small compared to other cases.

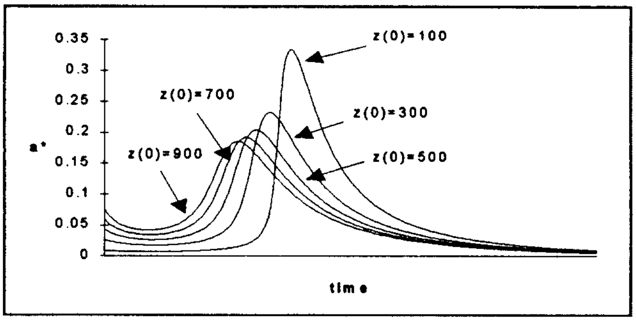

Figure 3. The optimal value of 'a' when $a(0) = 0.1$

This results have practical implications. Because developing countries have less knowledge stock in initial stage, they need more investment in research activity. Because of shortage of knowledge stock, they also need some time to accumulate knowledge stock. Hence, the time of maximum is later than that of developed countries.

Given an initial value of 'a' the relation between maximum value of a^* and its time is illustrated in Figure 4. The relation shows that as the initial stock of knowledge increase the time which a^* reaches its maximum value will be earlier. It is also recognized that as the initial knowledge stock changes the maximum value of a^* will vary. However, there is specific time when maximum value of a^* has minimum.

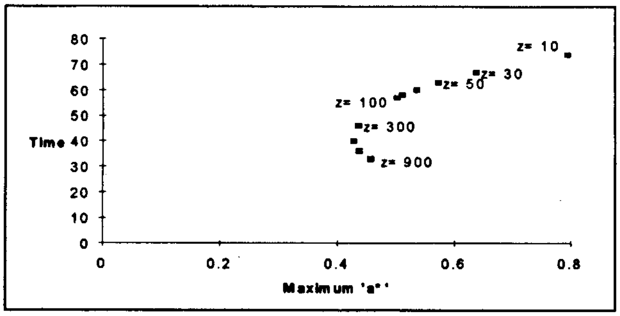

Figure 4. The relation between maximum a^* and its time

(3) Sensitivity Analysis

In previous section, optimal solution and its shift along the initial value has been analyzed according to the initial value. However, it is very useful to examine change of solution according to parameters. The effects of change in output elasticity of knowledge stock and output elasticity of capital stock will be explored. When $a(0) = 0.1$ and $Z(0) = 100$ the optimal value of 'a' is portrayed according to output elasticity of knowledge stock in Figure 5 and.

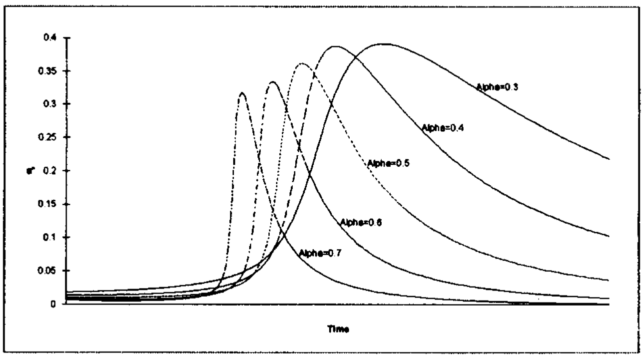

Figure 5. The change effect of alpha on a^*

As depicted in Figure 5, as the value of 'alpha' increases the maximum value decreases and its time becomes earlier. Since 'alpha' is the output elasticity of knowledge stock, this augment means the value of knowledge stock increased. Hence, the investment in research activity should be expanded.

The effect of increase in parameter''alpha' is same as the increase in the initial knowledge stock. This result is very intuitive. If the efficiency of the knowledge stock increase less investment will do enough.

How about the increase of the beta? Intuitively, its effect will reduce investment in research activity. This result is depicted in Figure 6.

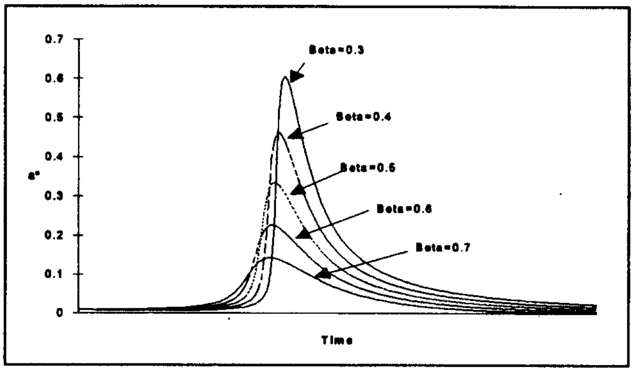

Figure 6. The change effect of beta on a^*

As depicted in Figure 6, the maximum value increases and its time becomes later as the value of beta decrease. Since the beta is the output elasticity of capital stock, this augment means the increase the value of capital stock. Hence, the investment in research activity should be reduced. This result is coincide with intuition. This result has practical meaning. If the efficiency of capital stock is high, the investment in research activity must be reduced.

V. Conclusion

Because of the difficulties of finding exact solution, simulation was applied. Outputs of simulation are very alike. The optimal research policy of government is as follows.

The investment on research activity is decreased slightly at first but soon increased until reaches its maximum value and then decreased as time goes by. This result has practical meaning. The importance of technology has been growing. So the investment in R&D activity should be expanded utill its importance is small.

In general, technology level of developing countries is lower than those of developed countries. Hence, developing countries need more investment in research activity to fulfill government's duty. Because of shortage knowledge stock, they need more time to accumulate knowledge stock. Hence the time of maximum is later than that of developed countries.

According to initial value the path of R&D investment is somewhat changed but its overall shape is not much changed. It was found that the more initial knowledge stock has the less investment need in R&D activity. This result agrees to intuition.

The relation between maximum value of R&D intensity and its time of maximum shows interesting thing. The more initial knowledge stock has the earlier its time of maximum and maximum value of R&D intensity is minimized at some initial knowledge stock.

The effect of change in output elasticity of knowledge stock and capital stock was examined. The increase of output elasticity of knowledge stock reduced the time of maximum of R&D investment and also reduced the amount of R&D investment. The increase of output elasticity of physical capital has inverse effect on R&D investment. This result coincides with intuition.

From the result mentioned above some comment can be offered. When the value of knowledge stock is greater than that of physical capital stock, government should expand investment in research activity. According to the initial state the optimal policy is slightly varied but its overall pattern is the same.

As the contribution of technology to output growth increases the investment in R&D activity is growing now. This effect is the same as to the increase of elasticity of knowledge stock in production. The importance of technology in economic growth will be expanded more and more. Hence, policy implications can be proposed as follows

Government should expand R&D investment to enhance social welfare if the contribution of technology is growing. The amount of investment is dependent on economic environment such as production technology, total labor, depreciation rate, and etc.

Above all results indicate the existence of solution and its stability as well as practical meaning. However, there are many drawbacks. First of all, this model do not consider private R&D activity. The incorporation of private R&D activity into this model is a very difficult job. However, the portion of private R&D activity is larger or equal to that of government's. This job is left to further research. Many aspects of private activity is ignored or much simplified. Results obtained from the model are a half because of ignorance of private sector. However, its practical meaning is not cut from this ignorance.

References

[1] Akira Goto and Kazuyuki Suzuki, "R&D Capital, Rate of Return on R&D Investment and Spillover of R&D in Japanese Manufacturing Industries" *The Review of Economics and Statistics* 1989 Nov, 555-564

[2] Alpha C. Chiang, *Elements of Dynamic Optimization* McGraw-Hill, Inc 1992

[3] Barro, "Government Spending in a Simple Model of Economic Growth" *J. Political Economy*, 1990, 103-125

[4] Eltis, W. A., The rate of technical progress, *The Economic Journal* 81(823), 1971, 501-524

[5] Gerhard Glomm and B. Ravikumar, "Public Investment in Infrastructure in a Simple Growth Model" *J. of Economic Dynamics and Control* 18, 1994, 1173-1187

[6] Gomulka Stanislaw, *The Theory of Technological Change and Economic Growth* Routledge, 1990

[7] Gregory T. Papanikos, "Macroeconomic Impacts of Endogenous Technical Progress" *Technological Forecasting and Social Change* 46, 1994, 17-27

[8] Griliches, "Issues in Assessing the Contribution of R&D to Productivity growth" *Bell J. of Economics* 10, 1979, 92-116

[9] James D. Adams, "Fundamental Stocks of Knowledge and Productivity Growth" *J. of Political Economy* Vol.98, No.4, 1990, 673-702

[10] Paul M. Romer, "Endogenous Technical Change" *J. of Political Economy* Vol.98 No.5, 1990, S71-S102

[11] Paul M. Romer, "The Origins of Endogenous Growth" *J. of Economic Perspectives* Vol.8 No.1, 1994, 3-22

[12] Robert G. King and Sergio Rebelo, "Public policy and Economic Growth" *J. of Political Economy* Vol.98 No.5 1990, s127-s150

[13] Sergio Rebelo, "Long-Run Policy Analysis and Long-Run Growth" *J. of Political Economy* Vol.99, 1991, 500-521

[14] Steven A.Y. Lin, "Government Spending and Economic Growth" *Applied Economics* 26, 1994, 83-94

[15] Stoneman, Paul *The Economic Analysis of Technological Change* Oxford University Press, 1983

[16] Wei-Bin Zhang, "Government's Research Policy and Economic Growth: Capital, Knowledge and Economic Structure" *Research Policy* 22, 1993, 327-336