Duopoly R&D Competition with Flexible Spillovers

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ABSTRACT

A duopoly model of R&D competition is presented to investigate whether an equilibrium R&D level with flexible spillovers is insufficient (or excessive) from the viewpoint of social welfare. The model focus on flexible spillovers which include much portion of externality occuring in R&D activity. Flexible spillovers refer to the spillovers that vary with industry equilibrium level of R&D. Innovating firms have incentives to cooperate in R&D in the presence of large spillovers. For any symmetric R&D profile, socially desirable equilibrium output is larger than equilibrium output produced in duopoly. Cooperative equilibrium R&D investment is observed to be socially insufficient in terms of welfare criterion irrespective of the magnitude of spillovers. While noncooperative R&D yields socially excessive expenditure on research project for a certain range of spillovers.

1. INTRODUCTION

It is widely recognized that "technological leadership" in the rivalrous environment of an industry is the key to the competitive edge. Each firm competitively invests on the R&D program to secure the advantage from innovation. Moreover R&D and production competition take place simultaneously in the market. Accordingly it is necessary to note that R&D competition and production competition should be considered in terms of sequential matters.

In reality firms find it difficult to completely protect the knowledge obtained in the process of R&D, which is referred to appropriability problem regarding research outcomes. Knowledge from innovation may leak out in the course of competition between firms. Many empirical studies demonstrate that innovating firm benefits from others' R&D through spillovers.1) By an analysis on cooperative R&D, Katz [8] states that an innovating firm cannot demand payment from rival competitor who benefits from its R&D through spillovers. This may constrict the idividual firm's incentives to conduct R&D. Furthermore the larger the extent of spillovers, the smaller the amount of research expenditures becomes.2) However that may be, it is knotty for individual firm to

1) For example, pointing out that the previous empirical works distinguished the sources of spillovers ambiguously, Bernstein and Nadiri [1] investigated the effect of interindustry R&D spillovers detached from overall effect of spillovers.

2) Levin and Reiss [10], however, point out that a high level of technological spillover, though reduces the average cost further, does not necessarily reduce the amount of R&D conducted in case that an increase in the extent of process (product) spillovers lead to an increase in product (process) R&D.

precisely take the externality caused by spillovers into endogenous consideration.

Meanwhile, as in every phase of economic decision making, uncertainty causes significant impacts on the behavior of innovating firms. Firms are facing various uncertainties from many sources. So it isn't gauranteed in the presence of uncertainty that R&D investment yields deterministic profit corresponding to the efforts. In the production competition for "market leadership", the views that uncertainty affects the innovating firm's market behavior also become more complicated.

Purpose of this paper is to investigate, whether an equilibrium R&D level in a duopoly with flexible spillovers is insufficient (or excessive) from the viewpoint of social welfare, and under what conditions firms can have incentives to cooperate in research project. In order to answer these questions cooperative and noncooperative R&D schemes are analyzed. Welfare criterion is, of course, established to evaluate the insufficiency (or excessiveness) of firm's R&D behavior.

The configuration of the paper is as follows. In spillovers and uncertainty of Chapter 2, ahead of establishing the model two underlying concepts in the analysis are briefly presented. Defining the flexible spillovers, basic model of analysis is developed in Chapter 3. Chpater 4 demonstrates the analysis on decision making in R&D for two types of R&D behavior - the noncooperative and the cooperative R&D. Economic interpretations about the results obtained in welfare analysis are presented in Chapter 5. The implication and importance of this study are given in Chapter 6 to conclude the paper. Also possible extensions of the model and further research directions are exhibited as an concluding remark.

2. SPECIFICATION OF THE MODEL

In order to reflect the synchronous nature of R&D and production competition, the game is divided into two stages. At the first stage of the game, named R&D stage, firms should determine their R&D investment levels to maximize overall net profit corresponding to the given setting of the game. At the game's second stage, i.e. production stage, firms are supposed to compete with each other in production of which market is assumed to be the *Cournot* duopoly. Linear inverse demand $p(Q)$ can be defined as

(1) $$p(Q) \equiv a - bQ,$$

where $Q = q_i + q_j$ $(i \neq j)$ denotes the total quantity demanded in the market.

In most R&D activities research outcomes are considered to be dependent across firms. Accordingly R&D profile of industry X is the vector which can be constructed by two elements. One is firm i's own expenditures x_i on research project and the other is that of rival firm x_j. Then the effective research expenditure profile is expressed as $X = (x_i, x_j)$. It is assumed that there is a stochastic relationship between research inputs and its performance. In other words, firm's research expenditures yield probabilistic results. Regarding the R&D success probability function $g(x_i; x_j)$, probabilistic results of R&D investment, the following assumption is introduced.

***Assumption* 1:** *(a) Firm's R&D success probability function $g(x_i;x_j)$ is twice contnuously differentiable, (b) $g(x_i,x_j)=0$ for symmetric R&D profile $X=(x_i,x_j)=(0,0)$, (c) $\partial g(x_i;x_j)/\partial x_i>0$ and concave.*

Concavity of $g(x_i,x_j)$ means that the marginal increase of research expenditure marginally decreases the rate of increase in R&D success probability. Taking the concativity of $g(x_i;x_j)$ and the properties of probability function into consideration,

$$\lim_{x_i\to\infty}\frac{\partial g(x_i;x_j)}{\partial x_i}=0 \tag{2}$$

is implied for any symmetric X. The property expressed in (2) serves to guarantee existence of equilibria where R&D investment decisions are probabilistically bounded from above.

Flexible spillovers, which are inherent in the firm's competition throughout production stage and R&D stage, are defined as follows.

***Definition* 1:** *Spillovers in a duopoly are the fraction that is the marginal change in rival's R&D success probability over that of firm i's due to an increase in its own research expenditures on R&D. Formally, they are expressed as*

$$\frac{\partial g_j}{\partial x_i}=\rho(X)\frac{\partial g_i}{\partial x_i}\ ,^{3)} \tag{3}$$

where $g_i=g(x_i;x_j)$ denotes the conditional probability of firm i's R&D success given x_j.

The following Assumption is posited about the flexible spillovers $\rho(X)$.

***Assumption* 2:** *(a) Spillovers $\rho(X)$ have the value from open interval $(-1,\ 1)$ and (b) $\rho(O)=0$, where O in parenthesis denotes zero vector.*

$\rho(X)$ has largely two effects on direction to which spillovers affect firm's R&D behavior - competition effect and technological leakage effect. Statement (a) in Assumption 2 is justified by the fact that spillovers are combined with the competition effect and the technological leakage effect. The former has negative effect on externality and the latter positive in R&D. If an increase in x_i has negative influence on rival firm's research ability and this competition effect is stronger than the technological leakage effect, then spillovers $\rho(X)$ lie in the left half of open interval $(-1,\ 1)$ from $\partial g(x_j;x_i)/\partial x_i<0$ and Assumption 1. Assumption of open interval implies that

3) Note that $\rho(X)$ is the map from R^2 to R.

there are no complete spillovers between firms. For statement (b), it happens that if firms do not invest in R&D there can be no spillovers regarding R&D within the industry.

In this game setting, expenditures on the research project yield process innovation. Namely, firm i expends x_i for the purpose of cost reduction. Therefore, the outcome that firm i obtains from innovation is a unit cost reduction at productin stage. Table 1 demonstrates the scope of cost-reduction when individual firm succeeds or fails in R&D.

[Table 1] Cost-Reduction from R&D Investments

	Cost-Reduction	Cases
Maximum	M^s	$g_i=1$
Minimum	M^f	$g_i=0$

where g_i is given by (3). $M^s > M^f \geq 0$ is properly assumed. When R&D stage is over, the expected cost-reduction obtained by innovating firm is expressed as $M_i = M^s g_i + M^f[1-g_i]$. Firm's production cost c_i is the unit cost minus cost reduction as a result of R&D activity.

(4) $$c_i(x_i;x_j) = c - M_i$$

with $0 < c < a$ and $c_i \geq 0$.

Meanwhile, innovating firm pays x_i for the research project in the process of R&D. Firm i has the following overall net profit throughout two stage of the game.

(5) $$\pi_i(q, x_i, x_j) = [a - bQ]\, q_i - c_i(x_i;x_j) q_i - x_i \,,$$

where $q = (q_i, q_j)$ is the industry-wide output profile. The first term in (5) is total revenue from market and remaining two terms are the cost on production and on R&D respectively.

At production stage Nash-Cournot equilibrium in (5) is computed as (superscript N stands for noncooperative Nash)

(6) $$q_i^N(X) = \frac{1}{3b}[a - 2c_i(x_i;x_j) + c_j(x_j;x_i)]$$.[4)]

Substituting (6) for q_i in (5) yields the following:

(7) $$\begin{aligned} \pi_i(q^N(X);x_i,x_j) &= \frac{1}{9b}[a - 2c_i + c_j]^2 - x_i \\ &= \frac{1}{9b}[a - c + 2M_i - M_j]^2 - x_i \end{aligned}$$

4) $\partial\pi_i/\partial q_i = a - 2bq_i - bq_j - c_i = 0$, $i = 1,2$ and $i \neq j$ is the first order condition for $q_i^N(X)$ where c_i is given by (4). Solving the equation above with respect to q_i gives (6).

with $q^N(X)=(q_i^N(X), q_j^N(X))$. The equation (7) is nothing other than overall net profit of R&D stage.

3. DECISION MAKING IN R&D STAGE

In examining whether the scheme of firm's decision making in R&D stage makes any difference, two types of R&D behavior of innovating firm must be ditinguished. One is noncooperative R&D and the other is cooperative R&D. In noncooperative R&D firms make decision on research expenditures to maximize their own overall net profit of the first stage. On the other hand, firms jointly decide their R&D spendings in maximizing the combined profits in cooperative R&D.

4. 1. Noncooperative R&D

In this case, as production stage, innovating firm maintains the position of fierce competitor with each other at R&D stage. An SPNE for the first stage is determined by solving

$$\max_{x_i} \ \pi_i(q^N(X); x_i, x_j) \quad i=1,2 \text{ and } j \neq i, \tag{8}$$

where $\pi_i(q^N(X); x_i, x_j)$ is given by (7).

The first order necessary conditions for an SPNE are

$$\frac{\partial \pi_i(q^N(X); x_i, x_j)}{\partial x_i} = \frac{2A_i}{9b}[\ 2-\rho(X)]\ \frac{\partial g_i}{\partial x_i} - 1 = 0,\ \ i=1,2, \tag{9}$$

where $A_i=(M^s-M^f)(a-2c_i+c_j)$.[5]

Since both firms are assumed to be identical ex ante, only symmetric solution is considered, that is, $X=X^N$ for $i=1,2$. The noncooperative equilibrium investments $X^N=(x_i^N, x_j^N)$ is characterized by

$$\frac{2A^N}{9b}[\ 2-\rho(X^N)]\ \frac{\partial g(x_i^N, x_j^N)}{\partial x_i} = 1, \tag{10}$$

where $A^N=(M^s-M^f)[\ a-c_i(x_i^N; x_j^N)]$.[6]

Using X^N which satisfies (10) and rearranging yield

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5) Note that $\frac{\partial c_i}{\partial x_i} = -(M^s-M^f)\frac{\partial g_i}{\partial x_i}$ and that $\frac{\partial c_j}{\partial x_i} = -(M^s-M^f)\rho(X)\frac{\partial g_i}{\partial x_i}$.

6) $x_i=x_j$ is implied from symmetry, from which $c_i(x_i; x_j)=c_j(x_j; x_i)$ holds. Accordingly $c_i(x_i; x_j)=c-(M^s-M^f)g(x_i; x_j)-M^f$ and $-2c_i+c_j=-c_i(x_i; x_j)$ follow at any symmetric R&D profile $X=(x_i, x_j)$.

$$
(11) \qquad \begin{aligned} q_i^N(X^N) &= \frac{1}{3b}[\ a-c_i(x_i^N;x_j^N)] \\ &= \frac{1}{3b}[\ a-c+M_i(x_i^N;x_j^N)] \quad . \end{aligned}
$$

(11) is the equilibrium output in noncooperative R&D, where $M_i(x_i;x_j)=(M^s-M^f)g(x_i;x_j)+M^f$ at any symmetric R&D profile $X=(x_i,x_j)$.

4. 2. Cooperative R&D

Under this setting firms determine their research expenditures cooperatively in order to maximize the combiend profits. Accordingly, the equilibrium R&D levels are computed by solving

$$
(12) \qquad \max_{x_i} \sum_{i=1}^{2} \pi_i(q^N(X);x_i,x_j),
$$

where $\pi_i(q^N(X);x_i,x_j)$ is given by (7).

Let $D \equiv \sum_{i=1}^{2} \pi_i(q^N(X);x_i,x_j)$, then the first order necessary conditions for an SPNE are as follows.

$$
(13) \qquad \frac{\partial D}{\partial x_i} = \frac{2}{9b}[\ A_i(2-\rho(X))+A_j(2\rho(X)-1)]\ \frac{\partial g_i}{\partial x_i}-1=0
$$

for i=1,2 and $j \neq i$, where A_i and A_j are given by (9) and (16) respectively. Again, assuming the symmetry in (13), namely, $X=X^C=(x_i^C,x_j^C)$ for $i=1,2$, one finally obtains

$$
(14) \qquad \frac{2A^C}{9b}[\ 1+\rho(X^C)]\ \frac{\partial g(x_i^C;x_j^C)}{\partial x_i}=1,
$$

where $A^C=(M^s-M^f)[\ a-c_i(x_i^C;x_j^C)]$ (C represents cooperation). Substituting X^C for X in (6) results in the equilibrium output in cooperative R&D which is written as

$$
(15) \qquad \begin{aligned} q_i^N(X^C) &= \frac{1}{3b}[\ a-c_i(x_i^C;x_j^C)] \\ &= \frac{1}{3b}[\ a-c+M_i(x_i^C;x_j^C)] \quad , \end{aligned}
$$

where $M_i(x_i;x_j)$ is given by (11).

4. 3. Incentives to Cooperate in R&D

As mentioned before, in the presence of spillovers innovating firms face a tradeoff between incentives to undertake R&D and the costless benefits from rival competitor's innovation. In those contexts cooperative R&D can be interpreted as an attempt to internalization of the extenality which takes place. In order to investigate what makes innovating firm to cooperate in research project, cooperation in determining R&D investments is analyzed. Thus, the main question in this section is regarding the incentives to cooperate in R&D. The sign of $\partial \pi_j(q^N(X);x_j,x_i)/\partial x_i$ is necessary to be scrutinized in examining this question. Taking derivative for $\pi_j(q^N(X);x_j,x_i)$, which is the rival firm's overall net profit in R&D stage, with respect to x_i, yields

$$(16) \qquad \frac{\partial \pi_j(q^N(X);x_j,x_i)}{\partial x_i} = \frac{2A_j}{9b}[\ 2\rho(X)-1]\ \frac{\partial g_i}{\partial x_i},\ j \neq i,$$

where $A_j=(M^s-M^f)(a-2c_j+c_i)$. An increase in $\pi_j(q^N(X);x_j,x_i)$ due to an increase in x_i means the internalized effect of innovating firm's research expenditures on cooperative R&D. If its sign is positive, then inovating firm finds it attractive to participate in cooperative R&D. For any symmetric R&D profile $X=(x_i,x_j)$, the right-hand side of (16) leads to the following:

$$(17) \qquad \frac{2A}{9b}[\ 2\rho(X)-1]\ \frac{\partial g(x_i;x_j)}{\partial x_i},$$

where $A=(M^s-M^f)[\ a-c_i(x_i;x_j)]$. As assumed in Chapter 3, it is satisfied that $2(M^s-M^f)/9b>0$. In addtion to this coupling Assumption 2 and equation (4), one can finally find that $\partial \pi_j(q^N;x_j,x_i)/\partial x_i>0$ holds if and only if $2\rho(X)-1>0$. Proposition 1 is given by summarizing these.[7)]

***Proposition* 1**: *Given the assumptions posited before, there exist incentives to cooperate in R&D if and only if $2\rho(X)-1>0$ for any symmetric equilibrium.*

In terms of Definition 1, spillovers are the combination of two effects - competition effect and technological leakage effect. Accordingly $\rho(X)$ varies with the conditions for the combination of those two effects. Technological nature and competitive environment of an industry is considered to determine that conditions.

If the competition between firms is so intense that an increase in firm i's R&D spendings reduces the competitor's research ability and knowledge leaked out is so small, then competition effect prevails over technological leakage effect such that $\rho(X)$ can have small positive value less than $1/2$ or even negative value. In that case there are no incentives for the firm to conduct R&D cooperatively. On the other hand, in case that $\rho(X)$ increases with an increase in $X=(x_i,x_j)$,

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7) This Proposition is similar to the results obtained by Kamien, et al. [7]. However, it is noted that R&D spillovers defined in this model vary with the equilibrium research investment unlike their constant spillovers. So the magnitude of spillovers differs according to the condition for an SPNE. Moreover it can vary by virtue of the combination of competition effect and technological leakage effect in spillovers.

comparatively insufficient research expenditures can satisfy the condition that $2\rho(X)-1>0$ holds. This case also demonstrates that spillovers are dependent on the conditions for the combination of competition effect and technological leakage effect.

4. SOCIAL WELFARE ANALYSIS

The welfare analysis on R&D investment is needed to investigate whether a certain type of behavior is desirable from a social point of view. For any R&D profile $X=(x_i, x_j)$ and any output profile $q=(q_i, q_j)$, welfare function $W(q, X)$ is defined as the sum of the consumer's and the producer's surplus:

$$W(q, X) \equiv \int_0^Q p(z)dz - \sum_{i=1}^{2}[\ c_i(x_i; x_j)q_i + x_i] \quad i \neq j, \tag{18}$$

where $Q=q_i+q_j$ $(i \neq j)$. Given any R&D profile $X=(x_i, x_j)$, let $q_i^S(X)$ be the socially desirable output obtained by maximizing (18) with respect to q_i. Clearly $q_i^s(X)$ is characterized by

$$a-bQ^S(X)=c_i(x_i; x_j) \quad i=1,2, \tag{19}$$

where $Q^S(X)=q_i^S(X)+q_j^S(X)$ $(i \neq j)$, which is nothing other than the marginal cost principle of firm i. Lemma 1 is obtained by comparing $q_i^S(X)$ and $q_i^N(X)$.

Lemma 1: *Firm's socially desirable output $q_i^S(X)$ is larger than the output produced noncooperatively in production stage, $q_i^N(X)$, for any symmetric R&D profile $X=(x_i, x_j)$.*

***Proof*.**

Assuming symmetry for any output profile $q=(q_i, q_j)$ allows us to let $Q(X)=2q_i(X)$. This leads to $q_i^S(X)=[\ a-c_i(x_i; x_j)]\ /2b$ from (19). In maximizing $\pi_i(q, x_i, x_j)$ in (5) with repect to $q=(q_i, q_j)$, $a-bQ^N(X)-bq_i^N(X)=c_i(x_i; x_j)$ for $i=1,2$ and $i \neq j$ is given. $q_i^N(X)=[\ a-c_i(x_i; x_j)]\ /3b$ is obtained by using symmetry in $q=(q_i, q_j)$ again. Comparing the output produced in each case gives

$$\begin{aligned} q_i^S(X)-q_i^N(X) &= \frac{1}{6b}[\ a-c_i(x_i; x_j)] \\ &= \frac{1}{6b}[\ a-c+M_i(x_i; x_j)] \end{aligned} \tag{20}$$

which is positive from equation (4).

Q.E.D.

Substituting $q^S(X)$ for q in (18) yields the welfare function of R&D stage defined by

$$W^S(X) \equiv W[\, q^S(X), X] \, . \tag{21}$$

The sign of $\partial W^S(X)/\partial x_i$ is needed to compare R&D behaviors of innovating firm, X^N and X^C respectively, with socially desirable R&D profile $X^S = (x_i^S, x_j^S)$ which satisfies the conditions for maximization of $W^S(X)$ with respect to $X = (x_i, x_j)$. Resorting to (19) for i and j, one obtains

$$\frac{\partial W^S(X)}{\partial x_i} = -\frac{\partial c_i(x_i; x_j)}{\partial x_i} q_i^S(X) - \frac{\partial c_j(x_j; x_i)}{\partial x_i} q_j^S(X) - 1. \text{ [8]} \tag{22}$$

Theorem 1 is implied by evaluating the sign of $\partial W^S(X)/\partial x_i$ at X^N in terms of standard marginal principle of economics. The positive(negative) value of $\partial W^S(X)/\partial x_i$ implies that marginal increase(decrease) in x_i at X^N increases the social welfare given by (21). This is interpreted as follows; if the sign of $\partial W^S(X)/\partial x_i$ is positive(negative), noncooperative R&D behavior is socially insufficient (excessive) from the welfare point of view.

***Theorem* 1**: *The noncooperative equilibrium R&D investment is socially excessive compared with the socially desirable R&D investment for a certain level of spillovers.*

***Proof*.**
Let the second term in (22) be expressed as

$$[\, q_j^N(X) - q_j^S(X)] \frac{\partial c_j(x_j; x_i)}{\partial x_i} - \frac{\partial c_j(x_j; x_i)}{\partial x_i} q_j^N(X). \tag{23}$$

Invoking (20) for j and rearranging for symmetric X^N, the equation (23) is reduced to

$$\frac{1}{2b} \rho(X^N)(M^s - M^f)[\, a - c_i(x_i^N; x_j^N)] \frac{\partial g(x_i^N, x_j^N)}{\partial x_i} \, . \tag{24}$$

On the other hand, at $X = X^N$, $-q_i^S(X)[\, \partial c_i(x_i; x_j)/\partial x_i] \, -1$ in (22) is rewritten as

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8) Taking derivative with respect to x_i for (21) gives

$$\frac{\partial W^S[\, q^S(X), X]}{\partial x_i} = \left(\frac{\partial q_i^S(X)}{\partial x_i} + \frac{\partial q_j^S(X)}{\partial x_i} \right) [\, a - bQ^S(X)] \, -1$$
$$- \frac{\partial c_i}{\partial x_i} q_i^S(X) - \frac{\partial c_j}{\partial x_i} q_j^S(X) - c_i \frac{\partial q_i^S(X)}{\partial x_i} - c_j \frac{\partial q_j^S(X)}{\partial x_i} \, .$$

The first term and the last two terms in the left-hand side of the equation above are eliminated from (19) for i and j respectively.

(25) $$[\ q_i^N(X^N)-q_i^S(X^N)]\ \frac{\partial c_i(x_i^N;x_j^N)}{\partial x_i}-\frac{\partial c_i(x_i^N;x_j^N)}{\partial x_i}q_i^N(X^N)-1 \qquad i=1,2.$$

The first order conditions (9) at symmetric X^N lead to

(26) $$-\frac{\partial c_i(x_i^N;x_j^N)}{\partial x_i}q_i^N(X^N)-1=\frac{A^N}{9b}[\ 2\rho(X^N)-1]\ \frac{\partial g(x_i^N;x_j^N)}{\partial x_i},$$

where $A^N=(M^s-M^f)[\ a-c_i(x_i^N;x_j^N)]$. In addtion to (24), putting (20) and (26) into (25) for X^N finally turn the equation (22) into the following:

(27) $$\frac{A^N}{18b}[\ 13\rho(X^N)+1)]\ \frac{\partial g(x_i^N;x_j^N)}{\partial x_i},$$

which is negative if and only if $\rho(X)$ is smaller than $-1/13$.

Q.E.D.

Again in terms of standard marginal principle of economics, evaluating the sign of $\partial W^S(X)/\partial x_i$ at X^C yields Theorem 2.

***Theorem** 2: The cooperative equilibrium R&D investment is socially insufficient compared with the socially desirable R&D investment for any level of spillovers.*

***Proof**.*

As in the verification of Theorem 1, for symmetric X^C, the equation (22) changes into

(28) $$\frac{A^C}{18b}[\ 5\rho(X)+5]\ \frac{\partial g(x_i^C;x_j^C)}{\partial x_i},$$

where $A^C=(M^s-M^f)[\ a-c_i(x_i^C;x_j^C)]$ .9) $\partial W^S(X)/\partial x_i$ always has positive value from Assumption 1, 2 and the equation (4) at $X=X^C=(x_i^C,x_j^C)$.

Q.E.D.

Theorem 2 implies that in the presence of spillovers innovating firms are confronted with the divergence between private incentives to conduct R&D and socially desirable research incentives irrespective of their R&D behavior. Theorem 1, while, suggests that if competition between firms is so hard that $\rho(X)$ is smaller than $-1/13$ there happens to be socially excessive investment in R&D.

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9) The equation (23) is the same as (24) in the proof of Theorem 1 for cooperative R&D profile $X^C=(x_i^C,x_j^C)$. The left-hand side of (26), however, turns into $\frac{A^C}{9b}[\ 1-2\rho(X^C)]\ \frac{\partial g(x_i^C,x_j^C)}{\partial x_i}$.

5. CONCLUSION

A duopoly model of R&D competition is presented to investigate whether an equilibrium research expenditures in two schemes of cooperative and noncooperative R&D with flexible spillovers are insufficient (or excessive) from the viewpoint of social welfare. Welfare criterion established in the model follows that of D'Aspremont and Jacquemin [7], which defines social welfare as consumer's surplus and producer's surplus.

Technological leakage effect has the socially beneficial impact of forcing firms to share their research outcome and hence raises incentives to conduct R&D with lower competition effect. Innovating firm have incentives to cooperate in R&D in the presence of large spillovers. Equilibrium research expenditures in cooperative R&D are socially insufficient compared with the socially desirable R&D investment for any level of spillovers. Equilibrium R&D investment in noncooperative case, however, are socially excessive in terms of welfare criterion for a certain level of spillovers. For any symmetric R&D profile, socially desirable output is larger than the output produced noncooperatively in production stage.

Compared with the study using deterministic functional form of cost reduction [Kamien, et al. 7], probabilistic nature is incorporated in the model to reflect uncertainty underlying in R&D process. Since research outcome are assumed to be dependent across firms, this paper introduces conditional probability of R&D success for firm i given competitor's research expenditures. This is the point that brings the study closer to reality. The model focuses on flexible spillovers including most portion of externality occuring in R&D activity. Flexible spillovers make the model different from other literatures on R&D with spillovers. This paper can be more advanced by allowing spillovers to vary with industry equilibrium level of R&D and by incorporating the direction to which spillovers affect firm's R&D behavior - the sources of spillovers.

Although a simple model of duopoly is demonstrated, extended application of the model to an oligopoly are expected to carry meaningful implications. Detailed insight into the externality that operates in R&D process is possibly gained by more broad analysis on flexible spillovers. For instance, study on spillovers, which take competition effect and technological leakage effect separately into consideration, will be more contributive to the work on R&D competition embracing externality.

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