A Traffic and Interference Adaptive DCA Algorithm with Rearrangement in Microcellular Systems

Seong-Lyun Kim and Youngnam Han

System Engineering Section
Mobile Telecommunications Division
Electronics and Telecommunications Research Institute
161 Kajong-Dong, Yusong-Gu, Taejon 305-350, Korea

Abstract - A new dynamic channel assignment (DCA) algorithm with rearrangement for cellular mobile communication systems is suggested. Our DCA algorithm is both traffic and interference adaptive, which is based on the mathematical formulation of the maximum packing under a realistic propagation model. In developing the algorithm, we adopt the Lagrangean relaxation technique that has been successfully used in the area of mathematical programming. Computational experiments of the algorithm reveal quite encouraging results. Although our algorithm primarily focuses on microcellular systems, it can be effectively applied to conventional cellular systems under highly nonuniform traffic distributions and interference conditions.

I. INTRODUCTION

Because of the severe deficiency in radio frequency spectrum, *frequency assignment problem* (FAP) becomes one of the critical issues in FDMA or TDMA based cellular systems. Fundamentally, there are two types of schemes for FAP, *fixed channel assignment* (FCA) and *dynamic channel assignment* (DCA) (see [1]-[3] and the references therein).

As a straightforward remedy to meet huge traffic demand, cellular systems evolve into microcells. Composed of much smaller cells, a microcellular system experiences larger variations in traffic and interference conditions than a conventional cellular system does. This explains why FCA is not adequate for microcellular systems, although it has been successfully applied to conventional systems. To overcome this problem, DCA is more favorable because of its flexibility in channel assignment, where all frequency channels are available in each cell.

For decades, numerous DCA algorithms have been proposed; and these can be classified into two categories, according to their adaptability to either traffic or interference [4]. However, so far, most efforts have been devoted to traffic adaptive DCA algorithms, where a simple propagation model represented by the *compatibility matrix* [2] is used. The compatibility matrix cannot describe the change of interference conditions since it must be based on "worst case" assumptions about mobile locations and propagation situations [5]. The *maximum packing* (MP) [6]-[8] performs best among traffic adaptive DCA algorithms. In MP, a new call is blocked only when there is no possible *rearrangement* (or *reassignment*) that results in the new call being served. By rearrangement, we mean reallocating the frequency channels that are assigned to ongoing calls (i.e., *intracell handoff*). Unfortunately, MP requires great computing capability to reallocate assigned channels into an optimum state, which seriously restricts implementing MP in real systems. Therefore, MP can only provide the theoretical bound on the system performance that is achievable by traffic adaptive DCA algorithms.

The objective of this paper is to propose a traffic and interference adaptive DCA algorithm that will allow channel rearrangement. The proposed algorithm is based on our mathematical formulation of MP under a propagation model that neatly reflects instantaneous interference conditions. In developing the algorithm, we adopt the *Lagrangean relaxation technique* [9] that has been successfully used in the area of *mathematical programming*. To the best of our knowledge, our DCA algorithm is the first one in which the mathematical programming approach has been utilized. Simulation experiments indicate that the proposed algorithm significantly improves the DCA performance with a small number of intracell handoffs. Although our algorithm is primarily for microcellular systems, it can be effectively applied to conventional cellular systems under highly nonuniform traffic distributions and interference conditions. In addition, our formulation of MP may be a good starting point for developing traffic and interference adaptive DCA algorithms.

The organization of this paper is as follows: Section II provides a mathematical formulation of MP. In section III, based on the formulation, we propose a DCA algorithm with rearrangement that is both traffic and interference adaptive. Simulation results of the algorithm are shown in section IV. Finally, section V concludes the paper.

II. MAXIMUM PACKING (MP): MATHEMATICAL FORMULATION

As a preliminary to our formulation of MP, we provide next the propagation model considered throughout this paper.

A. Propagation Model

In the propagation model (see Fig. 1), the coefficient $g_{ij}^p (> 0)$ denotes the link gain of frequency channel p on the path from cell j to cell i (note that g_{ij}^p and g_{ji}^p are not necessarily identical). If the transmitter power of the base station (BS) in cell j for channel p is $T_j^p (> 0)$, then the received power of channel p in cell i is $g_{ij}^p T_j^p$. Suppose that

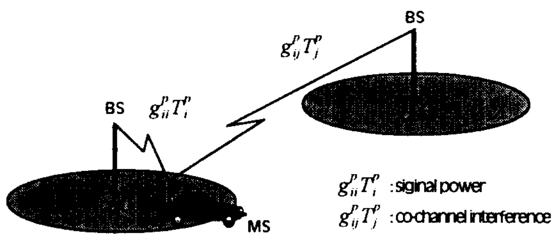

Fig. 1. Propagation model of channel p

the same channel p is reused in cell i with the transmission power $T_i^p(>0)$. Then, for a mobile station (MS) in cell i, $g_{ii}^p T_i^p$ is the received signal power from the desired BS, and $g_{ij}^p T_j^p$ becomes an amount of cochannel interference from cell j.

B. Mathematical Formulation

Let us consider a (micro) cellular system and denote $X=\{1,2,\ldots,n\}$ as the set of cells in the system. A finite number of frequency channels are designated to the system, represented by the set $F=\{1,2,\ldots,f\}$. We assume that Δt is the time interval between consecutive channel assignment epochs of DCA. That is, channel assignment is updated after the interval Δt that is usually smaller than a few seconds, depending on traffic conditions. Let $\{t^l\}$ be the set of discrete points in time, where $t^{l+1}=t^l+\Delta t$ for $l=1,2,\ldots$. Let the n-tuple $M^l=(m_i^l)$ represent the number of ongoing calls in each cell at time t^l (i.e., m_i^l is the number of calls being served in cell i at time t^l). Let f_{ip}^l be a binary variable that is equal to 1 if channel p is allocated to cell i at time t^l and equal to 0, otherwise. Now, let us assume that each BS uses the same transmission power for all channels (this can be easily generalized to *nonhomogeneous power controlled* systems). Then, at time t^l, MP under the propagation model of Fig. 1 can be formulated as the following problem:

(P^l)

maximize $\sum_{i\in X, p\in F} f_{ip}^l$

subject to

$$\frac{g_{ii}^p f_{ip}^l}{\sum_{j\in X, j\neq i} g_{ij}^p f_{jp}^l} \geq \gamma_{ip}, \text{ for all } i\in X \text{ and } p\in F \text{ such that } f_{ip}^l=1, \quad (1)$$

$$m_i^l \leq \sum_{p\in F} f_{ip}^l \leq m_i^l + C_i^l, \text{ for all } i\in X, \text{ where } C_i^l\in\{0,1,\ldots\}, \quad (2)$$

$$f_{ip}^l = 0 \text{ or } 1, \text{ for all } i\in X \text{ and } p\in F. \quad (3)$$

The constraint (1) relies on the model of Fig. 1. It determines which cells are compatible in the sense that the same channels may be used simultaneously: If channel p is allocated to cell i at time t^l, then the carrier-to-(cochannel) interference ratio (CIR) on channel p in cell i should not be less than a *protection ratio* $\gamma_{ip}(>0)$. In real systems, the protection ratio is predetermined such that a call cannot be served if the CIR on the assigned channel is smaller than that. In the constraint (2) of (P^l), C_i^l denotes the number of new calls arrived in cell i during the interval $\Delta t=(t^{l-1},t^l)$. The constraint (2) restricts that the ongoing calls in each cell at time t^l should not be disconnected to serve new calls. It also says that the number of channels allocated to each cell need not be greater than the total number of calls in that cell. The objective function of (P^l) is to maximize the number of served calls at time t^l with a given quality level γ_{ip}. Let $\{\bar{f}_{ip}\}$ be a feasible solution of (P^l). Then, some new calls of a cell, say i^* can be served when $\sum_{p\in F}\bar{f}_{i^*p}^l > m_{i^*}^l$. In ($P^l$), it is not required that each ongoing call at time t^l should use the same channel assigned prior to t^l. That is, the problem (P^l) fully allows rearrangement to serve new calls.

MP under our propagation model can now be restated as follows: At time t^l, if there are new calls during the interval $\Delta t=(t^{l-1},t^l)$, then a control center (e.g., *mobile telephone switching office*) determines which channels should be allocated to each cell by solving (P^l). After allocating channels to each cell, each BS then assigns these channels to respective calls with a simple control scheme. For this purpose, the control center should have the values of g_{ij}^p, m_i^l and C_i^l.

The constraint (1) can be restated as follows:

$$\gamma_{ip}\sum_{j\in X, j\neq i} g_{ij}^p f_{jp}^l - g_{ii}^p f_{ip}^l \leq M_{ip}(1-f_{ip}^l), \text{ for all } i\in X \text{ and } p\in F, \quad (1')$$

where $M_{ip}=\gamma_{ip}\sum_{j\in X, j\neq i} g_{ij}^p$.

In (1'), when $f_{ip}=1$, the right hand side of (1') becomes zero, and (1') can be transformed to (1). If $f_{ip}=0$, then (1') becomes redundant and is satisfied for all combinations of $\{f_{ip}^l\}$.

For the sake of mathematical treatment, (1') can be rewritten as

$$\sum_{j\in X}\bar{g}_{ij}^p f_{jp}^l \leq M_{ip}, \text{ for all } i\in X \text{ and } p\in F, \quad (1'')$$

where $\bar{g}_{ij}^p = \begin{cases} M_{ip}-g_{ii}^p & \text{if } i=j, \\ \gamma_{ip} g_{ij}^p & \text{otherwise.} \end{cases}$

With (1''), (P^l) can be reformulated as the following 0-1 *integer programming problem* that will be used hereafter to denote MP under the model of Fig. 1.

(P_I^l)

maximize $\sum_{i\in X, p\in F} f_{ip}^l$

subject to (1''), (2) and (3).

For a system with a moderate number of cells and channels, (P_l^l) becomes a quite large sized problem. Since (P_l^l) has NP-completeness, it is practically impossible to get optimal solutions of such a problem in real time. In the following section, we propose a heuristic algorithm for (P_l^l) that is a traffic and interference adaptive DCA algorithm allowing rearrangement. This algorithm provides near optimal solutions of (P_l^l) with a small computational burden.

III. A New DCA Algorithm with Rearrangement

First, we sketch general features of our algorithm (see Fig. 2).

Algorithm *DCA with Rearrangement*

Step 1. At time t^l, if new calls arrived during the interval $\Delta t = (t^{l-1}, t^l)$, then the control center searches available frequency channels that satisfy the CIR condition (i.e., the constraint (1") of (P_l^l)) and allocates them appropriately to the cells where the new calls arrived. If there are still new calls blocked by the lack of available channels, then goto *Step 2.*

Step 2. (Rearrange) By using Algorithm Rearrangement, the control center rearranges the channels assigned to the ongoing calls. If this rearrangement does not make room for the new calls blocked in *Step 1*, then these calls cannot be served.

We now describe Algorithm Rearrangement of *Step 2*, which utilizes *Lagrangean relaxation problems* [9] of (P_l^l).

A. Lagrangean Relaxation Problems of (P_l^l)

For a vector of nonnegative Lagrangean multipliers $\lambda = (\lambda_{ip})$, a Lagrangean relaxation problem of (P_l^l) is given by

$(L^l(\lambda))$

maximize $\Sigma_{i\in X}\Sigma_{p\in F}(f_{ip}^l - \lambda_{ip}(\Sigma_{j\in X}\bar{g}_{ij}^p f_{jp}^l - M_{ip}))$

$$= \Sigma_{i\in X}\Sigma_{p\in F} f_{ip}^l - \Sigma_{i\in X}\Sigma_{j\in X}\Sigma_{p\in F}\lambda_{ip}\bar{g}_{ij}^p f_{jp}^l + \Sigma_{i\in X}\Sigma_{p\in F}\lambda_{ip}M_{ip}$$

$$= \Sigma_{i\in X}\Sigma_{p\in F} f_{ip}^l - \Sigma_{i\in X}\Sigma_{j\in X}\Sigma_{p\in F}\lambda_{jp}\bar{g}_{ji}^p f_{ip}^l + \Sigma_{i\in X}\Sigma_{p\in F}\lambda_{ip}M_{ip}$$

$$= \Sigma_{i\in X}(\Sigma_{p\in F}(1 - \Sigma_{j\in X}\lambda_{jp}\bar{g}_{ji}^p) f_{ip}^l + \Sigma_{p\in F}\lambda_{ip}M_{ip})$$

subject to (2) and (3).

The Lagrangean problem $(L^l(\lambda))$ was constructed as follows: The constraint (1") of (P_l^l) was multiplied by $\lambda(\geq 0)$ and then incorporated into the objective function of (P_l^l). It can be easily seen that the optimal objective function value of $(L^l(\lambda))$ provides an upper bound for (P_l^l) [9]. Algorithm Rearrangement employs Lagrangean multiplier vectors in $(L^l(\lambda))$. Hence, the particular method of deriving *good* vectors of Lagrangean multipliers is

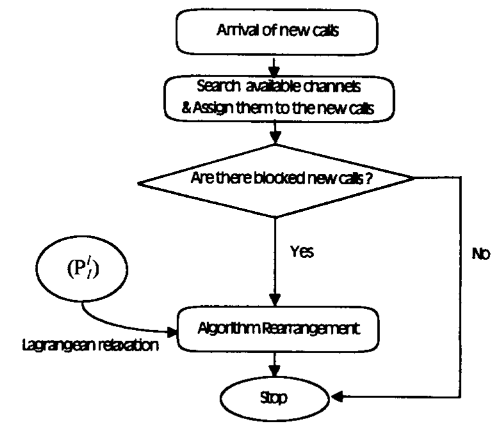

Fig. 2. New DCA algorithm with rearrangement

critical. For this purpose, we suggest using near optimal solutions of the following problem called *Lagrangean Dual* of $(L^l(\lambda))$[9]:

(LD^l)

$$\text{minimize } v(L^l(\lambda))$$

$$\text{subject to } \lambda \geq 0,$$

where $v(L^l(\lambda))$ is the optimal objective function value of $(L^l(\lambda))$.

The *subgradient optimization method* [10] is a widely accepted one to solve problems such as (LD^l). It is an iterative method and, at an iteration k, updates the current vector of Lagrangean multipliers (denoted by $\lambda^k = (\lambda_{ip}^k)$) as follows:

$$\lambda_{ip}^{k+1} = \max\{0,\ \lambda_{ip}^k - s_k u_{ip}^k\}, \text{ for all } i \in X \text{ and } p \in F. \tag{4}$$

In (4), $u^k = (u_{ip}^k)$ is a *subgradient* of $v(L^l(\lambda))$ at λ^k and characterized by

$$u_{ip}^k = M_{ip} - \Sigma_{j\in X}\bar{g}_{ij}^p f_{jp}^{l(k)}, \text{ for all } i \in X \text{ and } p \in F, \tag{5}$$

where $\{f_{ip}^{l(k)}\}$ is an optimal solution of $(L^l(\lambda^k))$. From the unique structure of $(L^l(\lambda^k))$, we can easily obtain $\{f_{ip}^{l(k)}\}$ by solving very simple *linear programming problems* (see APPENDIX). The *step size* $s_k (>0)$ in (4) can be obtained by applying such efficient techniques provided as in [10].

B. Algorithm Rearrangement

Our rearrangement algorithm is also an iterative one in the sense that it requires Lagrangean multiplier update by the subgradient method at each iteration.

Algorithm *Rearrangement*

Step 0. (Initialize) Set $k = 0$. Determine the upper limit of iteration number k^0. Start with a vector of initial Lagrangean multiplies $\lambda^0 = (\lambda_{ip}^0)$.

Step 1. (Aggregate link gain) For given Lagrangean multiplies $\lambda^k = (\lambda_{ip}^k)$, compute $\hat{g}_j^p = \Sigma_{i \in X} \lambda_{ip}^k \bar{g}_{ij}^p$ for all $j \in X$ and $p \in F$.

Step 2. (Rearrange) For each cell i^*, where new calls to be served still exist, execute the followings. If done, then goto *Step 3*.

Step 2.1 Find $p^* \in F$ such that
$\hat{g}_{i^*}^{p^*} - \hat{g}_{(i*)}^{p^*} = \min_{p \in F}\{\hat{g}_{i^*}^{p} - \hat{g}_{(i*)}^{p} | f_{i^*p}^l = 0\}$,
where $\hat{g}_{(i*)}^{p} = \max_{i \in X - \{i^*\}}\{\hat{g}_i^p | f_{ip}^l = 1\}$.

Step 2.2 Set $f_{i^*p^*}^l = 1$. Find $i^{**} \in X - \{i^*\}$ such that currently $f_{i^{**}p^*}^l = 1$, but $f_{i^{**}p^*}^l = 0$ satisfies the constraint (1") of (P_l^l). If such i^{**} exists, then set $f_{i^{**}p^*}^l = 0$ and goto *Step 2.3*.
Otherwise, set $f_{i^*p^*}^l = 0$ and goto *Step 2*.

Step 2.3 Find $p^{**} \in F$ such that currently $f_{i^{**}p^{**}}^l = 0$, but $f_{i^{**}p^{**}}^l = 1$ satisfies the constraint (1") of (P_l^l).
If such p^{**} exists, then set $f_{i^{**}p^{**}}^l = 1$ and goto *Step 2*.
Otherwise, set $f_{i^*p^*}^l = 0$ and $f_{i^{**}p^*}^l = 1$ and goto *Step 2*.

Step 3. (Lagrangean multiplier update) Set the iteration number $k = k+1$. If $k > k^0$ then stop and take the best solution among those obtained in *Step 2*. Otherwise, update current Lagrangean multipliers by using the equation (4), and goto *Step 1*.

In *Step 1* of the algorithm, for each $j \in X$ and $p \in F$, coefficients $\bar{g}_{ij}^p$ are *aggregated* as one value $\hat{g}_j^p$ by a current vector of Lagrangean multipliers λ^k. We will call $\hat{g}_j^p$ the *aggregate link gain* of cell j on channel p. *Step 2* includes the rearrangement process: Channel p^* assigned to an ongoing call in cell i^{**} is replaced by channel p^{**} to assign channel p^* to a new call in cell i^* (see Fig. 3). Therefore, the rearrangement is made in cell i^{**}. In the rearrangement process, an aggregate link gain coupled with a vector of Lagrangean multipliers plays a vital role. Hence, a good vector of Lagrangean multipliers is required for the accuracy of the aggregate link gain. In *Step 3*, current Lagrangean multipliers are updated by the subgradient method.

IV. SIMULATION RESULTS

This section provides simulation results of the proposed algorithm. The simulated cellular system consists of 49 regular hexagonal cells as shown in Fig. 4. Calls arrive in a

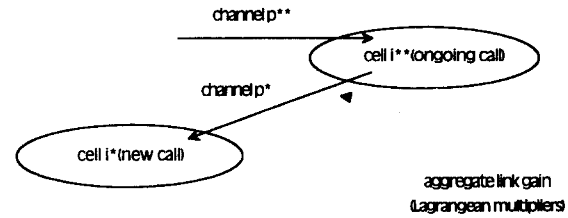

Fig. 3. Rearrangement process in cell i^{**}

cell with a Poisson process, whose duration is exponentially distributed with mean 3 minutes. In Fig. 4, the number in each cell represents mean call arrival rates ranging from 40 to 400 calls/hour. Let the total number of frequency channels available in the system be 70.

In the formulation of (P_l^l), link gain coefficients are generated as

$$g_{ij}^p = (\tfrac{1}{d(i,j)})^{\alpha(p)}, \quad (6)$$

where $d(i,j)$ is the distance between cells i and j (refer to [1] for discussions on the distance between cells in a regular hexagonal cellular system), and $\alpha(p)$, the *propagation loss constant* on channel p is assumed to be a Uniform distribution $U(1.5,3.5)$. We generate the protection ratio γ_{ip} from a Uniform distribution $U(1.2,2.4)$ in *dB*.

The experiment is executed for one hour of simulation time on a Hewlett Packard 9000/827. For computational simplicity, the call arrival and its duration time is discretized to steps of one second. Moreover, we set Δt, the interval between consecutive channel assignment epochs to 1 second. We set the upper limit of the iteration number, k^0 to 5 in Algorithm Rearrangement.

Simulation results are provided in Fig. 5 for the comparison of the performance with two other reference algorithms, an FCA algorithm and the *First Available* DCA algorithm [3]. The FCA algorithm permanently allocates some number of frequency channels to each cell according to the traffic distribution and the CIR constraint (1"). Blocking probabilities, ratio of the total number of blocked calls to the total number of calls arrived during the simulation time, are plotted in Fig. 5 as a function of the percentage load increase for all cells in Fig. 4.

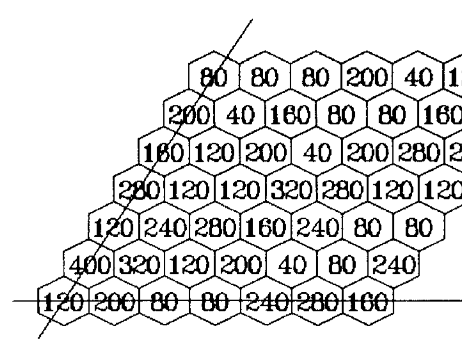

Fig. 4. Nonuniform traffic distribution in a 49-cellular system

The simulation results show that the proposed algorithm consistently offers substantially lower blocking probabilities than those reference algorithms over the whole range of traffic conditions under consideration.

V. Concluding Remarks

This paper attempted to propose a DCA algorithm with rearrangement that is traffic and interference adaptive. The proposed algorithm was based on the mathematical formulation of MP, where actual propagation conditions were taken into consideration. Computational results were very promising, and the mathematical formulation of MP may provide a good starting point for developing traffic and interference adaptive DCA algorithms.

Appendix

The problem $(L^l(\lambda))$ has a special structure, which allows us to decompose the problem into relatively small n-subproblems described as follows:

For a given $i \in X$,

$(L_i^l(\lambda))$

$$\text{maximize} \quad \sum_{p\in F}(1-\sum_{j\in X}\lambda_{jp}\bar{g}_{ji}^{p})f_{ip}^{l} + \sum_{p\in F}\lambda_{ip}M_{ip}$$

$$\text{subject to} \quad m_i^l \le \sum_{p\in F} f_{ip} \le m_i^l + C_i^l,$$

$$f_{ip}^{l} = 0 \text{ or } 1, \text{ for all } p \in F.$$

An optimal solution of $(L^l(\lambda))$ can be obtained by solving independently the subproblems $(L_i^l(\lambda))$ for all $i \in X$. The subproblem $(L_i^l(\lambda))$ is essentially a very simple linear programming problem and easily solved by the following procedure within $m_i^l + C_i^l$ iterations at most.

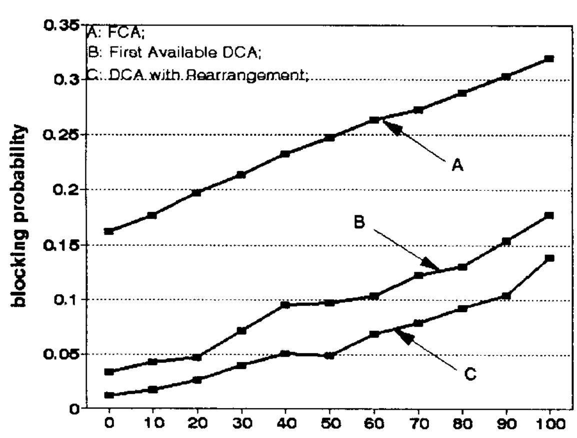

Fig. 5. Blocking performance under nonuniform traffic distributions

Procedure *LP*

Step 0. Set the iteration number $k = 0$ and $f_{ip}^{l} = 0$ for all $p \in F$.

Step 1. Find p^* such that
$1-\sum_{j\in X}\lambda_{jp^*}\bar{g}_{ji}^{p^*} = \max_{p\in F}\{1-\sum_{j\in X}\lambda_{jp}\bar{g}_{ji}^{p} \mid f_{ip}^{l}=0\}$.
Set $f_{ip^*}^{l} = 1$ and $k = k+1$.

Step 2. If $k = m_i^l$, then goto *Step 3*. Otherwise, goto *Step 1*.

Step 3. If $C_i^l = 0$, then stop. Otherwise, goto *Step 4*.

Step 4. Find p^* such that
$1-\sum_{j\in X}\lambda_{jp^*}\bar{g}_{ji}^{p^*} = \max_{p\in F}\{1-\sum_{j\in X}\lambda_{jp}\bar{g}_{ji}^{p} \mid f_{ip}^{l}=0\}$.
If $1-\sum_{j\in X}\lambda_{jp^*}\bar{g}_{ji}^{p^*} \ge 0$, then set $f_{ip^*}^{l} = 1$ and $k = k+1$.
Otherwise, stop. If $k = m_i^l + C_i^l$, then stop.
Otherwise, goto *Step* 4.

References

[1] S. Kim and S.-L. Kim, "A two-phase algorithm for frequency assignment in cellular mobile systems," *IEEE Trans. Veh. Technol.*, vol. VT-43, pp. 542-548, 1994.

[2] A. Gamst, "Some lower bounds for a class of frequency assignment problems," *IEEE Trans. Veh. Technol.*, vol. VT-35, pp. 8-14, 1986.

[3] K. Okada and F. Kubota, "On dynamic channel assignment strategy in cellular radio systems," *IEICE Trans. Fundamentals*, vol. E75-A(12), pp. 1634-1641, 1992.

[4] R. Beck and H. Panzer, "Strategies for handover and dynamic channel allocation in micro-cellular mobile systems," *Proc. IEEE VTC '89*, pp. 178-185, 1989.

[5] J. Zander and H. Eriksson, "Asymptotic bounds on the performance of a class of dynamic channel assignment algorithms," *IEEE J. Select. Areas Commun.*, vol. 11, pp. 926-933, 1993.

[6] D. Everitt and N. W. Macfadyen, "Analysis of multicellular mobile radio telephone systems with loss," *Brit. Telecomm. Technol. J.*, vol. 1 (2), pp. 37-45, 1983.

[7] D. Everitt and D. Manfield, "Performance analysis of cellular mobile communication systems with dynamic channel assignment," *IEEE J. Select. Areas Commun.*, vol. 7, pp. 1172-1180, 1989.

[8] P. Raymond, "Performance of cellular networks," *IEEE Trans. Commun.*, vol. COM-39, pp. 1787-1793, 1987.

[9] M. L. Fisher, "The Lagrangian relaxation method for solving integer programming problem," *Management Science*, vol. 27 (1), pp. 1-18, 1981.

[10] S. Kim and H. Ahn, "Convergence of a generalized subgradient method for nondifferentiable convex optimization," *Mathematical Programming*, vol. 50, pp. 75-80, 1991.