

**Hyperbolicity Breaking Model and Drift-Flux Model for the Prediction of Flow Regime Transition after Inverted Annular Flow**

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**Abstract**

The concept of hyperbolicity breaking is applied to predict the flow regime transition from inverted annular flow (IAF) to agitated inverted annular flow (AIAF). The resultant correlation has the similar form to Takenaka's empirical one. To validate the proposed model, it is applied to predict Takenaka's experimental results using R-113 refrigerant with four different tube diameters of 3, 5, 7 and 10 mm. The proposed model gives accurate predictions for the tube diameters of 7 and 10 mm. As the tube diameter decreases, the differences between the predictions and the experimental results increase slightly. The flow regime transition from AIAF to dispersed flow (DF) is described by the drift flux model.

**1. Introduction**

In some transients or accident conditions of a nuclear power plant, the heated wall can no longer sustain liquid contact, and the safety of the boiling system is threatened by the high wall temperature. Such a situation is called the critical heat flux (CHF) or dryout condition. The heat transfer characteristic after dryout in the flow channel is of great importance in nuclear reactor systems or in other cryogenic systems.

The boiling regime beyond the dryout location can occur with various different flow patterns depending on the given flow quality and mass flux at the dryout location. If the dryout occurs at subcooled or low quality region, it is believed that the flow regime at dryout is the inverted annular flow (IAF) in which a liquid core flows at the center of the channel surrounded by a vapor film. The liquid core in inverted annular flow is disturbed with vapor velocity increase and becomes the inverted slug flow (ISF) or the agitated inverted annular flow (AIAF) (Takenaka *et al.* 1990). The inverted annular flow develops into the inverted slug flow (ISF) at lower mass flux and it changes into agitated inverted annular flow (AIAF) at higher mass flux. At further downstream there exists only dispersed droplet flow.

Several works on the flow regime transition in the post dryout region have been performed. Groeneveld (1975) suggested that the flow regime would be dispersed from annular flow when the void fraction at dryout was larger than 80% and the flow regime would be inverted annular flow when the void fraction was less than 50%. Based on their experimental observations, Ishii *et al.* (1986) and Ishii (1992) proposed the concepts that flow regimes just beyond dryout might be viewed as the inverse forms of pre-CHF regimes.

To describe the transition from IAF to AIAF in a vertical tube, Takenaka *et al.* (1991) derived an empirical correlation from the similarity of the regime transition with the onset of slugging in a horizontal channel suggested by Wallis and Dobson (1973).

$$U_g - U_f = 250\sqrt{\alpha D}. \quad (1)$$

The transition from agitated inverted annular flow to dispersed flow was determined assuming that in dispersed flow  $\alpha \geq 0.7$ . They utilized the drift flux model to describe this boundary condition.

Since the above mentioned Takenaka et al.'s regime transition criterion from IAF to AIAF is purely empirical, its application for wide range can be limited. Therefore, it is meaningful to generate the flow regime transition criteria in the post-dryout region from appropriate theoretical bases.

## 2. Flow Regime Transition after Inverted Annular Flow

### 2.1 Hyperbolicity Breaking for the Prediction of Transition Criterion from IAF to AIAF

The momentum equations of one-dimensional two-fluid model of inverted annular flow in a vertical tube can be written as follows:

. vapor momentum

$$\alpha_g \rho_g \frac{\partial U_g}{\partial t} + \alpha_g \rho_g U_g \frac{\partial U_g}{\partial z} + \alpha_g \frac{\partial P}{\partial z} - \Delta P_{gi} \frac{\partial \alpha_g}{\partial z} = \Gamma(U_i - U_g) - 4 \frac{\sqrt{\alpha_f}}{D} \tau_i - \frac{4}{D} \tau_w - \alpha_g \rho_g g. \quad (2)$$

. liquid momentum

$$\alpha_f \rho_f \frac{\partial U_f}{\partial t} + \alpha_f \rho_f U_f \frac{\partial U_f}{\partial z} + \alpha_f \frac{\partial P}{\partial z} - \Delta P_{fi} \frac{\partial \alpha_f}{\partial z} = -\Gamma(U_i - U_f) + 4 \frac{\sqrt{\alpha_f}}{D} \tau_i - \alpha_f \rho_f g. \quad (3)$$

In case of inverted annular flow,  $\Delta P_{ki}$  terms are considered to be negligibly small for both cases. In addition, the pressure difference between two phases is defined as follows:

$$P_f - P_g = \Delta P.$$

Rearranging the momentum equations previously described and the mass conservation equations about  $\underline{X} = (P_g, \alpha_g, U_g, U_f)^T$ , the following form of equation array can be introduced:

$$\underline{A} \frac{\partial \underline{X}}{\partial t} + \underline{B} \frac{\partial \underline{X}}{\partial z} = \underline{C}. \quad (4)$$

Transforming the coordinate into  $\xi = z + \lambda t$  and neglecting the sonic velocity terms, the gradient of the void fraction can be expressed as

$$\frac{\partial \alpha_g}{\partial \xi} = \frac{N(\alpha_g)}{\Delta}, \quad (5)$$

If  $\Delta \neq 0$ , the points in the phase space are regular. If  $\Delta = 0$ , they become singular points and the characteristics of the system become unstable. In a hyperbolic system, the propagation velocities of information,  $\lambda$ , remain all distinct and real. But when  $\lambda$  becomes imaginary, hyperbolicity breaking occurs. The singular point with hyperbolicity breaking becomes a bifurcating point where a sudden flow transition occurs (Lee and No 1994).

From the solution of the characteristic equation,  $\Delta(\lambda) = 0$ , we can obtain the information about the stability of the flow system,  $\lambda = p \pm \sqrt{p^2 - q}$ . The neutral stability condition, obtained from  $p^2 - q = 0$ , can be reduced into

$$(U_g - U_f)^2 = -\frac{\partial \Delta P}{\partial \alpha_g} \left( \frac{\alpha_g \rho_f + \alpha_f \rho_g}{\rho_g \rho_f} \right). \quad (6)$$

The transition from inverted annular flow to agitated inverted annular flow is considered to occur if the neutral stability is broken at the peak of the most dominant wave at interface. Now, if we properly evaluate the term,  $-\left(\frac{\partial \Delta P}{\partial \alpha_g}\right)$ , the criterion of flow regime transition from inverted annular flow to agitated inverted annular flow can be determined from Eq.(6).

The dominant pressure difference between the two phases can come only from the surface tension force at surface. Assuming the wave form at the interface of liquid and vapor as sinuous function, we can describe the pressure difference between phases as follows:

$$\Delta P = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \text{ at } \xi = 0,$$

$$\delta(\xi) = h \cdot \cos k\xi + a.$$

Now, the pressure difference between the two phases can be evaluated from the following equation:

$$\Delta P = \frac{\sigma}{a + h} + \sigma h k^2. \quad (7)$$

For the determination of the flow regime transition criterion, two empirical constants are determined. One is the void fraction at the peak of the dominant wave. And the other is the wavelength of the most dominant wave at the liquid-vapor interface. It can be thought that the void fraction at the peak of the wave is not far from the average void fraction in a sinuous form of wave. And it is well known that a capillary wave having a certain wave length begins to grow and develops into solitary wave. It is assumed that a capillary wave having the most dangerous wavelength is responsible for the surface breakup of IAF. Therefore, it is assumed that the wavelength of the most dominant wave is proportional to the wavelength of the fast growing wave at the marginal stability of Helmholtz instability:

$$\lambda_{cr} = \frac{2\pi b}{\sqrt{\Delta \rho g / \sigma}}. \quad (8)$$

If we assume again that the average vapor film thickness is nearly constant near the transition point, the neutral stability condition can be reduced into the following form of equation:

$$\frac{j_g^*}{\alpha_g} - \sqrt{N_\rho} \frac{j_f^*}{\alpha_f} = N_{ip} \sqrt{\alpha_g + N_\rho \alpha_f}. \quad (9)$$

Because  $N_\rho \ll 0$ , and through the order analysis of  $N_{ip}$ , the transition criterion from IAF to AIAF (or ISF) can be arranged as follows:

$$j_g^* = \frac{1}{2b} \alpha_f^{-1/4} \alpha_g^{3/2} + \frac{\alpha_g}{\alpha_f} \sqrt{N_\rho} j_f^*. \quad (10)$$

## 2.2 Transition from Agitated Inverted Annular Flow to Dispersed Flow

From the results of Song (1995) it is assumed that the existence of dispersed flow can be described by the following equation:

$$\alpha_g = 0.45 + 2.37 \left( \frac{d_d}{D} \right). \quad (11)$$

Now, the only information needed is the diameter of droplet resulting from the breakup of AIAF or ISF.

The transition from agitated inverted annular flow or inverted slug flow into dispersed flow is described by use of the drift flux model in the dispersed flow region. Using the drift flux parameters suggested by Ishii (1977), the transition criterion from AIAF to DF can be described by the following drift flux equation:

$$j_g = \frac{C_0 \alpha_g}{1 - C_0 \alpha_g} j_f + \frac{\alpha_g}{1 - C_0 \alpha_g} U_{GJ}. \quad (12)$$

## 3. Validation of Proposed Model

To determine the transition criterion from IAF to AIAF using the hyperbolicity breaking model, one should know  $\alpha_g$  at the peak of the interfacial wave and the proportionality constant  $b$  describing the wavelength of the most dominant wave. Since it is assumed that the wave form is sinuous, the void fraction at the wave peak will be nearly the average void fraction. The constant  $b$  remains as a sensitivity parameter to be determined.

The only reliable data set showing the regime transition in inverted flow is Takenaka et al.'s R-113 refrigerant data (Takenaka *et al.* 1989; Takenaka *et al.* 1991). The correlation suggested by Takenaka et al., Eq.(1), is reduced into a nondimensional form for the comparison with the results predicted with the hyperbolicity breaking model:

$$j_g^* = 250 \sqrt{\frac{\rho_g}{g \Delta \rho} \alpha_g^{3/2}} + \frac{\alpha_g}{\alpha_f} \sqrt{N_\rho} j_f^*. \quad (13)$$

To validate the proposed model, the results obtained from Eq.(10), which has the similar form to Eq.(13), are compared with experimental data and those from Takenaka's correlation. The comparison results are shown in Figs.2 through 4 for various diameters. A constant value of  $b = 1/7\sqrt{2}$  is used in the analyses. All the data predicted by hyperbolicity breaking model are plotted on a domain of  $j_g^*$  in Fig.5. The proposed model predicts well the experimental results for the diameter,  $D = 7 \text{ mm}$  and  $D = 10 \text{ mm}$ . However, the deviation from experimental results increases as the diameter decreases. It is presumed that the assumption that the void fraction at the wave peak is not far from the average becomes invalid for small diameter tubes.

In Fig.6, Eq.(12) is compared with the experimental data of Takenaka et al.(1989). The following equation is recommended for the slug breakup diameter in R-113 flow:

$$d_d = 5.99 \times 10^{-3} \left( \frac{\sigma}{g \Delta \rho} \right)^{1/2} \left( \frac{\rho_f + \rho_g}{\rho_g} \right). \quad (14)$$

The transition from AIAF (or ISF) to DF can be predicted well if the droplet sizes in dispersed flow at the transition are evaluated appropriately.

#### 4. Summary

Studies have been done on the flow regime transition in an inverted flow which is very important for the understanding of the flow characteristics and the heat transfer mechanism in a post-dryout flow. The experimental data on the flow regime transition from IAF to AIAF or ISF are analyzed theoretically. For the application of hyperbolicity breaking model, it is wanted to know the information of transition void fraction and the most dominant wavelength on liquid core surface.

It is validated that the present model can be successfully applied to describe the regime transition. The capillary wave, having  $b = \frac{1}{7\sqrt{2}}$ , is selected as the fast growing wave. This criterion gives good prediction results when it is applied to Takenaka et al.'s R-113 data for rather large diameter tubes. However, when it is applied to small diameter tubes, the difference between the predicted and the experimented increases.

It is also validated that the criterion of regime transition from AIAF (or ISF) to dispersed flow(DF), which is derived from the basis of the drift-flux model, is suitable to describe the transition when the information of droplet diameter at the transition is given.

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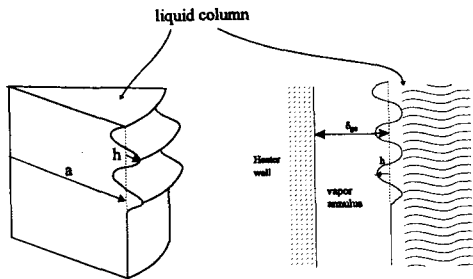


Fig.1 Wave form at the Surface of Liquid Core in IAF

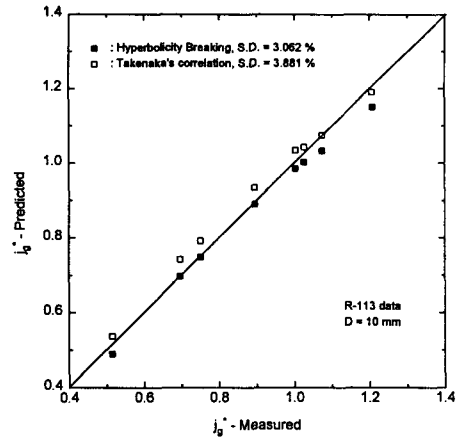


Fig.2 Transition from IAF to AIAF predicted by Hyperbolicity Breaking Model and Takenaka Model (D=10 mm)

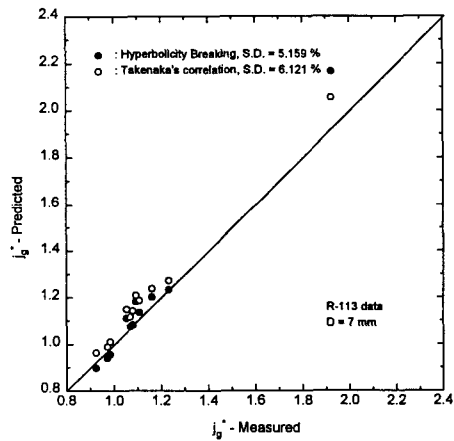


Fig.3 Transition from IAF to AIAF predicted by Hyperbolicity Breaking and Takenaka Model (D=7 mm)

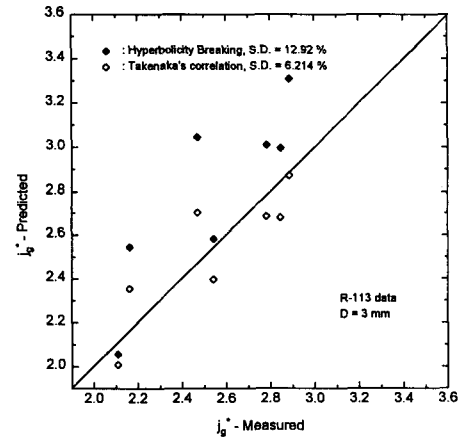


Fig.4 Transition from IAF to AIAF predicted by Hyperbolicity Breaking Model and Takenaka Model (D=3 mm)

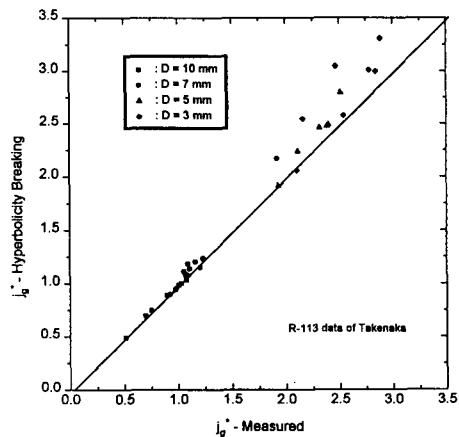


Fig.5 Transition from IAF to AIAF predicted by Hyperbolicity Breaking for various tube diameters

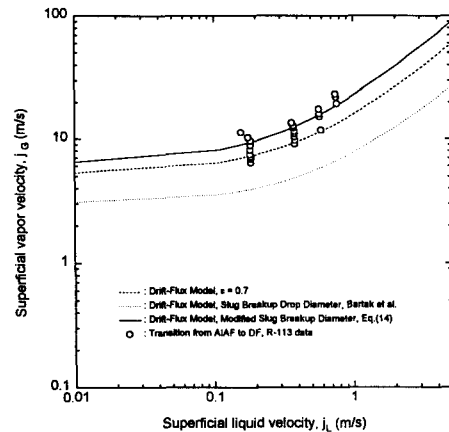


Fig.6 Transition from AIAF to DF predicted by Drift Flux Model (D=10 mm)