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Weibull clutter 에 대한 최대사후확률 일정오경보수신기

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Maximum a posteriori CFAR for weibull clutter.

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Abstract

A CFAR algorithm for weibull clutter is discussed. The Maximum a posteriori(MAP) estimator for two parameters(skewness and scale) of the weibull clutter is proposed, assuming the probability density function of skewness parameter is known. And proposed MAP estimator is compared with the Maximum likelihood(ML) esimator.Using this MAP estimator, we can design CFAR detector which is shown to have smaller CFAR loss than ML CFAR detector by the statistical simulation method.

Notations

P_{fa} : false alarm probability
CFAR : constant false alarm rate
OS : Order statistics
OW : optimum weibull in the sense of maximum likelihood
pdf : probability density function
ML : Maximum likelihood
MAP : Maximum a posteriori
i.i.d : independent identically distributed.
SNR : Signal to Noise Ratio
P_d : probability of detection

1. Introduction

The Weibull clutter model is known to represent sea and ground clutter at low grazing angles, especially when high resolution radar is used. The Weibull pdf is two parameter pdf and the Rayleigh pdf is a special case. The Weibull clutter model(clutter power pdf) is given by

$$p_s(x,c,\rho)=\frac{c}{\rho^{\frac{c}{2}}}x^{\frac{c}{2}-1}\exp(-(\frac{x}{\rho})^{\frac{c}{2}}) \qquad (1.1)$$

where ρ is the power scale parameter and c is the skewness parameter.The above model is the power pdf out of an envelop detector. If square law-detector is used, The pdf given by (1.1) is the pdf of input to CFAR detector. If linear detector is used, the Weibull clutter pdf is given by

$$p_L(x,c,b)=\frac{c}{b}\left(\frac{x}{b}\right)^{c-1}\exp(-(\frac{x}{b})^c) \qquad (1.2)$$

where $b = \sqrt{\rho}$

CFAR detectors for Weibull clutter have been suggested in the past.[1],[5],In [1] OW detector was suggested. It estimates the scale parameter in ML sense, assuming the skewness parameter is known and adjusts the threshold from the estimate of the scale parameter. In [5] the ML estimator of both ρ and c are proposed and CFAR property of the detector was proved. In [1] the skewness parameter was assumed to be completely known and in [5] no previous knowledge of the skewness parameter is assumed.

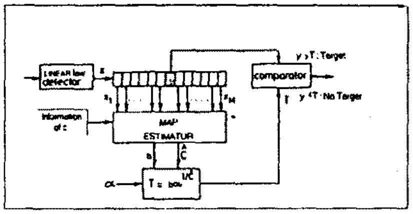

Figure 1 MAP CFAR DETECTOR BLOCK DIAGRAM

In this paper we consider the case the partial information of the skewness parameter is known. Firstly we analyze the sensitivity of P_{fa} with respect to c. OS CFAR detector is analyzed. From this analysis we can see that good estimate of skewness parameter is crucial for the performance of detector concerned. We will also show that the sensitivity of OS is independent of clutter statistics. And then we assume the skewness parameter is gaussian with mean μ_c and standard deviation σ_c. And we propose MAP estimator for two parameters of weibull pdf. And we show that MAP CFAR detector has CFAR property for both b and c as $\sigma_c \rightarrow \infty$ or $\sigma_c \rightarrow 0$.The block diagram of the proposed MAP CFAR detector is given in figure 1.

2. Sensitivity analysis of P_{fa} of OS CFAR

The sensitivity of a quantity α to changes in a quantity β is defined to be

$$S^{\alpha}_{\beta}=\frac{d\alpha}{d\beta}\frac{\beta}{\alpha} \qquad (2.1)$$

The sensitivity is a measure of the relative change in α due to relative change in β.For example, let's consider sensitivity of x^n. $S^{x^n}_x$ is equal to n, which is constant. If the sensitivity of some function is -7.5 at x_0, its variation at x_0 can be locally approximated $x^{-7.5}$.We want to consider the change of P_{fa} according to the variation of c for two CFAR algorithms in a Weibull background. One is OS CFAR, and the other is OW CFAR. It will be shown that the sensitivity of P_{fa} is independent of clutter skewness parameter (i.e clutter statistics).

It is known that P_{fa} of OS CFAR in a Weibull background is [4]

$$P_{fa}=\prod_{i=1}^{K}(1+\frac{\alpha^{\frac{c}{2}}}{M+1-i})^{-1} \qquad (2.2a)$$

where α is threshold multiplier controlling P_{fa},M is the number of reference cells, K is the order of test cell. Define $\alpha^{\frac{c}{2}} \triangleq q$.Then eq (2.2a) becomes

$$P_{fa}= \prod_{i=1}^{K}\left(1+\frac{q}{M+1-i}\right)^{-1} \quad (2.2b)$$

The $S_c^{P_{fa}}$ can be calculated using (2.2).

$$S_c^{P_{fa}}= -q\ln q \sum_{i=1}^{K}\frac{1}{M+1-i}\left(1+\frac{q}{M+1-i}\right)^{-1} \quad (2.3)$$

If M and K are given, the q can be calculated for preassigned value P_{fa} using (2.2b).And using this value, we can calculate the $S_c^{P_{fa}}$. Since q is dependent only on P_{fa}, $S_c^{P_{fa}}$ is also dependent only on P_{fa}. i.e $S_c^{P_{fa}}$ is independent of c, the weibull skewness parameter. Therefore we can plot $S_c^{P_{fa}}$ versus P_{fa}. Figure 2 is the plot of q versus P_{fa}. And figure 3 is the plot of $S_c^{P_{fa}}$.

In figure 3 we can see that (1) if c is overestimated , P_{fa} will increase (2) This increase is large for small value of P_{fa}. For example $S_c^{P_{fa}}$ = -21.51 for $P_{fa} = 10^{-5}$ .This means $P_{fa} \simeq c^{-21.51}$ at $P_{fa} = 10^{-5}$. If the true value of c is actually 85 % of estimated value. P_{fa} increases approximately thirty three times. In other words, P_{fa} becomes 3.3×10^{-4}. This is rather large performance degradation. The threshold multiplier is related to average decision threshold (ADT) [6]. Its formal definition is ADT = $E(\alpha Z)/\mu$, where α is threshold multiplier which is function of estimation method and preassigned P_{fa} , the random variable Z is the result of the estimation method used in CFAR system, μ is the mean clutter power level. E denotes expectation operation. Actually the ADT is the required SNR for $P_d=0.5$ for low P_{fa}. The smaller the threshold multiplier becomes, the smaller becomes the CFAR loss.Therefore if we overestimate c and we are to maintain the preassigned P_{fa} ,the CFAR detector will increase q ,results in increase of α.This will increase CFAR loss. Therefore accurate estimation of skewness parameter is crucial for performance of CFAR detector.

3. MAP estimator for the Weibull clutter

3.1 Motivation

The skewness parameter of the Weibull clutter is varying as grazing angle(ϕ) varies.

The table 1 (figure 1 of [2]) shows the variation of sea clutter skewness parameter for several grazing angle.

grazing angle(ϕ degree)	skewness parameter(c)
1	1.16
5	1.65
30	1.78

table 1 skewness parameter variation

Test condition : (1) sea state 3 (2) K_u band (3) Horizontal polarization (4)0.1 μS pulse width

The skewness parameter can not be known exactly due to conditions such as weather.But we can know the approximate value of the skewness parameter as the grazing angle varies. In [1] the ML(maximum likelihood) estimator for the mean power of Weibull clutter was proposed. The assumption that the scale parameter can be known accurately is not realistic. In [5],R.Ravid et al derived ML estimator for both scale and skewness parameter assuming that no information of the scale parameter is available. This ML estimator is proved to be CFAR, but this ML estimator doesn't use any prior information of scale parameter. In this paper, we assume that partial information of scale parameter is known and propose MAP estimator for the Weibull clutter. Using MAP estimator we can design CFAR detector. It can be shown that the OW estimator proposed in [1] and ML estimator proposed in [5] is special case of proposed estimator.

3.2 Derivation of the MAP estimator for weibull clutter.

Firstly we assume as following.

Assumption

1.Linear law detector is used.

2.The weibull samples are i.i.d

3.The pdf of the skewness parameter is given by $f_c(c)$

The conditional pdf $f_{X|c}(X|c)$ is given by

$$\begin{aligned} f_{X|c}(X|c) &= \prod_{i=1}^{M} p_L(x_i, c, b) \\ &= \left(\frac{c}{b^c}\right)^M \prod_{i=1}^{M} x_i^{c-1}\exp\left(-\frac{x_i^c}{b^c}\right) \end{aligned} \quad (3.4)$$

Let's define likelihood function as

$$\begin{aligned} L(X,c,b) &= \ln(f_c(c)f_{X|c}(X|c)) \\ &= \ln\left(f_c(c)\prod_{i=1}^{M} p_L(x_i,c,b)\right) \\ &= \ln\left(f_c(c)\left(\frac{c}{b^c}\right)^M \prod_{i=1}^{M} x_i^{c-1}\exp\left(-\frac{x_i^c}{b^c}\right)\right) \end{aligned} \quad (3.5)$$

Differentiate (3.5) with respect to b and equating zero, we obtain

$$\frac{\partial L}{\partial b}=0 \ : \ \hat{b}=\left[\frac{1}{M}\sum_{i=1}^{M} x_i^{\hat{c}}\right]^{\frac{1}{\hat{c}}} \quad (3.6)$$

Differentiate (3.5) with respect to c and equating zero, we obtain

$$\frac{\partial L}{\partial c} = \frac{1}{f_c(c)}\frac{\partial f_c(c)}{\partial c} + \frac{M}{c} + \sum_{i=1}^{M}\ln x_i - \left[\sum_{i=1}^{M}\left(\frac{x_i}{b}\right)^c \ln x_i\right]=0$$

(3.7)

If we assume the pdf of skewness parameter is gaussian pdf with mean μ_c and standard deviation σ_c,eq (3.7) becomes

$$\frac{\partial L}{\partial c} = -\frac{(\hat{c}-\mu_c)}{\sigma_c^2} + \frac{M}{\hat{c}} + \sum_{i=1}^{M}\ln x_i - \left[\sum_{i=1}^{M}\left(\frac{x_i}{\hat{b}}\right)^{\hat{c}}\ln x_i\right]=0 \quad (3.8)$$

Substituting $\hat{b}$ in (3.8) using (3.6), eq (3.8) becomes,

$$\frac{\partial L}{\partial c} = -\frac{(\hat{c}-\mu_c)}{\sigma_c^2} + \frac{M}{\hat{c}} + \sum_{i=1}^{M}\ln x_i - M\frac{\left[\sum_{i=1}^{M} x_i^{\hat{c}}\ln x_i\right]}{\sum_{i=1}^{M} x_i^{\hat{c}}}=0 \quad (3.9)$$

The MAP estimate, $\hat{c}$ for the skewness parameter of the Weibull clutter can be obtained solving eq (3.9).Then $\hat{b}$ can be obtained using eq (3.6). If no prior information is available, we can think of c as uniformly distributed. Then $\frac{\partial f_c(c)}{\partial c}=0$ .Therefore eq (3.9) becomes

$$\frac{M}{\hat{c}} + \sum_{i=1}^{M}\ln x_i - M\frac{\left[\sum_{i=1}^{M} x_i^{\hat{c}}\ln x_i\right]}{\sum_{i=1}^{M} x_i^{\hat{c}}}=0 \quad (3.10)$$

Eq (3.6) and eq (3.9) are used to design two parameter ML CFAR [5].

As σ_c goes to zero, $\hat{c}\rightarrow\mu_c$ .Then eq (3.6) becomes

$$\hat{b}=\left[\frac{1}{M}\sum_{i=1}^{M} x_i^{\mu_c}\right]^{\frac{1}{\mu_c}} \quad (3.11)$$

Eq (3.11) can be used to design parametric one parameter OW CFAR [1].

Remark : If we assume the more plausible a posteriori probability $f_c(c)$, the more accurate estimate of the skewness parameter we can obtain.

4.Simulation result

4.1 Performance Comparison between ML and MAP estimator.

Since the closed form solution of P_{fa} for MAP detector is hard to find, we resort to statistical simulation method. The P_{fa} of the MAP detector is given by

$$P_{fa}= P(y>T)=P(y>\hat{b}\alpha^{\frac{1}{\hat{c}}}) \quad (4.1)$$

where y is the sample in test cell and $\hat{b}$ and $\hat{c}$ is estimated parameters using M weibull samples. If we simulate an event with probability p with m trials ,the accuracy of simulation is approximately given by

$$\frac{std(\hat{p})}{p} \approx \frac{1}{\sqrt{mp}} \qquad (4.2)$$

If 5 % error is to be guaranteed, m should be

$$m = \frac{400}{p} \qquad (4.3)$$

If we simulate p = 10^{-5}, m = $4*10^{7}$.The estimated P_{fa} is obtained by

$$\widehat{P_{fa}} = \frac{1}{m}\sum_{j=1}^{m} P(y > T_j) = \frac{1}{m}\sum_{j=1}^{m} \exp\left(-\left(\frac{T_j}{b}\right)^c\right) \qquad (4.4)$$

where $T_j = \hat{b}_j \alpha^{\frac{1}{\hat{c}}}$.

b and c : The b and c used in generating weibull sample.

Algorithm used in obtaining $\widehat{P_{fa}}$.

for j = 1 to m, do

step 1. given α, generate M weibull samples using predetermined b and c.

step 2. calculate $\hat{b}_j, \hat{c}_j$ therefore T_j according to specified CFAR method.

step 3. $P_{fa}(j) = \exp\left(-\left(\frac{T_j}{b}\right)^c\right)$

end for loop.

step 4 $\widehat{P_{fa}} = \frac{1}{m}\sum_{j=1}^{m} P_{fa}(j)$

We calculated the P_{fa} of ML CFAR ($\sigma_c = \infty$ case) MAP CFAR ($\sigma_c = 0.5$ $\sigma_c = 0.2$ $\sigma_c = 0.1$ case),Parametric ML CFAR ($\sigma_c = 0$ case).In figure 4, the original weibull samples has parameter b=3 , c=1 and the number of trial (m) = 10000.We can see that as σ_c decreases, the threshold multiplier decreases. So does the CFAR loss. And we can conclude that the proposed MAP detector performs better than ML detector for uniform weibull clutter.

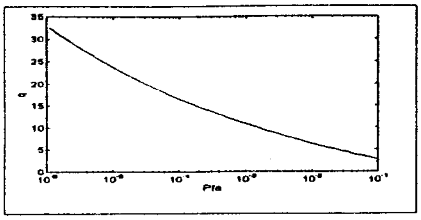

Figure 2. q value as a function of P_{fa} (OS CFAR)

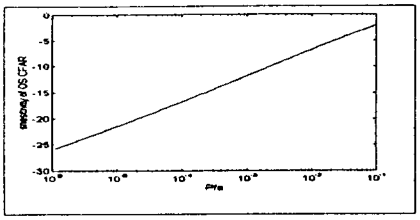

Figure 3. Sensitivity of OS CFAR as a function of P_{fa}

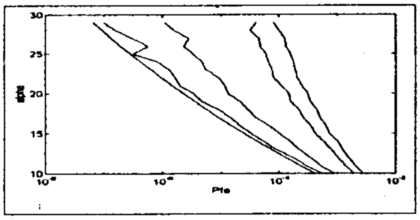

figure 4. Performance comparison between ML CFAR and MAP CFAR (from top to bottom ML CFAR, MAP CFAR $\sigma_c = 0.5, \sigma_c = 0.2, \sigma_c = 0.1$ ideal case)

6. Conclusion

Firstly, we considered the sensitivity analysis of OS CFAR. From this analysis we concluded that the accurate estimation of skewness parameter is crucial for the performance of CFAR detector.We proposed the MAP CFAR algorithm ,which have smaller CFAR loss than ML algorithm.The proposed algorithm is flexible since experimental result (the pdf of skewness parameter) can be embedded into the algorithm. And this algorithm was shown to be the general case of parametric ML detector(the case that full information of skewness is known) and nonparametric ML detector(the case that no information of skewness is known).

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