Design of Minimal-order Observer for Linear Dynamical Systems with Unknown inputs

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Abstract-In the last several years, considerable attention has been focused on the problem of designing observers for linear systems with unknown inputs. Since UO(unknown inputs observer) has the derivative of the outputs, it is very sensitive to measurement noises. Therefore this note propose an algebraic approach to UO design to alleviate the prescribed problems. Since the proposed method has simple form to estimate state and unknown input and robustness to sensor noise, we believe that it is very attractive in practice.

1. The Problem

When a linear system is subjected to the disturbance, one needs to estimate the state of the system accurately. The method of estimating the state of the system is called an observer. The state observer is a technique used to estimate the state of a system from its input and output data. The state observer is a mathematical model that represents the system's behavior and is used to predict the system's future state. The state observer is an important tool for control system design, because it allows us to estimate the state of the system even when the system's input and output data are not completely known.

2. The Solution

The solution to the problem of estimating the state of a system is to use a state observer. The state observer is a mathematical model that represents the system's behavior and is used to predict the system's future state. The state observer is an important tool for control system design, because it allows us to estimate the state of the system even when the system's input and output data are not completely known.

\[ x(t) = Ax(t) + Bu(t) + Du(t) \]  
\[ y(t) = Cx(t) \]

When the system is subjected to disturbance, the state observer is used to estimate the state of the system accurately. The state observer is a technique used to estimate the state of a system from its input and output data. The state observer is a mathematical model that represents the system's behavior and is used to predict the system's future state. The state observer is an important tool for control system design, because it allows us to estimate the state of the system even when the system's input and output data are not completely known.

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관측기의 상태변수 $x(t)$에 의해 구체화한 식(3)과 같은 시스템의 수치적 Assurance 섹션의 상태에 \( \hat{x}(t) \)는 다음과 같다.

\[
\hat{x}(t) = \left[ \begin{array}{c}
\dot{x}_1(t) \\
\vdots \\
\dot{x}_n(t)
\end{array} \right], \quad \dot{x}_i(t) = \frac{1}{2m} \left( F_i + H_i \right) \quad (i = 1, 2, \ldots, m)
\]  
(11)

\[
\hat{x}(t) = -K \cdot \hat{x}(t) \quad (K = \text{asso})
\]  
(12)

이에, 미지 입력 $d(t)$를 추정하기 위하여 다음과 같이 정의한다.

\[
\begin{align*}
\hat{x}(t) &= \begin{bmatrix} k_1 & k_2 \\ 0 & 1
\end{bmatrix} \begin{bmatrix} \hat{x}_1(t) \\
\hat{x}_2(t)
\end{bmatrix} \\
\hat{x}(t) &= \begin{bmatrix} k_1 & k_2 \\ 0 & 1
\end{bmatrix} \begin{bmatrix} \hat{x}_1(t) \\
\hat{x}_2(t)
\end{bmatrix}
\end{align*}
\]
(13)

식(13)에 의하여 식(11)은 다음과 두 식으로 표현한다.

\[
\begin{align*}
\hat{x}_1(t) &= k_1 \cdot \hat{x}_1(t) + k_2 \cdot x(t) \\
\hat{x}_2(t) &= k_2 \cdot \hat{x}_1(t) + k_1 \cdot x(t)
\end{align*}
\]
(14)

\[
\begin{align*}
\hat{x}_1(t) &= k_1 \cdot \hat{x}_1(t) + k_2 \cdot x(t) \\
\hat{x}_2(t) &= k_2 \cdot \hat{x}_1(t) + k_1 \cdot x(t)
\end{align*}
\]
(15)

식(3,1)에 의하여 식(10)은 다음과 같이 정의할 수 있다. 따라서, 유도된 관측기 방정식에 의해 다음 식을 구할 수 있다. 

\[
\begin{align*}
\hat{e}(t) &= e(t) + G_1 \cdot x(t) + G_2 \cdot \dot{x}(t) + G_3 \cdot u(t)
\end{align*}
\]
(16)

\[
\begin{align*}
\hat{e}(t) &= e(t) + G_1 \cdot x(t) + G_2 \cdot \dot{x}(t) + G_3 \cdot u(t)
\end{align*}
\]
(17)

이에, 식(3,3)의 양변을 적분하면 다음과 같이 정의한다. 

\[
\begin{align*}
\int \hat{e}(t) \, dt &= \int e(t) \, dt + \int G_1 \cdot x(t) \, dt + \int G_2 \cdot \dot{x}(t) \, dt + \int G_3 \cdot u(t) \, dt
\end{align*}
\]
(18)

이에, 식(18)의 양변을 적분하면 다음과 같이 정의한다.

\[
\begin{align*}
\hat{e}(t) &= \int e(t) \, dt + \int G_1 \cdot x(t) \, dt + \int G_2 \cdot \dot{x}(t) \, dt + \int G_3 \cdot u(t) \, dt
\end{align*}
\]
(19)

\[
\begin{align*}
\hat{e}(t) &= \int e(t) \, dt + \int G_1 \cdot x(t) \, dt + \int G_2 \cdot \dot{x}(t) \, dt + \int G_3 \cdot u(t) \, dt
\end{align*}
\]
(20)

\[
\begin{align*}
\hat{e}(t) &= \int e(t) \, dt + \int G_1 \cdot x(t) \, dt + \int G_2 \cdot \dot{x}(t) \, dt + \int G_3 \cdot u(t) \, dt
\end{align*}
\]
(21)

\[
\begin{align*}
\hat{e}(t) &= \int e(t) \, dt + \int G_1 \cdot x(t) \, dt + \int G_2 \cdot \dot{x}(t) \, dt + \int G_3 \cdot u(t) \, dt
\end{align*}
\]
(22)

\[
\begin{align*}
\hat{e}(t) &= \int e(t) \, dt + \int G_1 \cdot x(t) \, dt + \int G_2 \cdot \dot{x}(t) \, dt + \int G_3 \cdot u(t) \, dt
\end{align*}
\]
(23)

\[
\begin{align*}
\hat{e}(t) &= \int e(t) \, dt + \int G_1 \cdot x(t) \, dt + \int G_2 \cdot \dot{x}(t) \, dt + \int G_3 \cdot u(t) \, dt
\end{align*}
\]
(24)

\[
\begin{align*}
\hat{e}(t) &= \int e(t) \, dt + \int G_1 \cdot x(t) \, dt + \int G_2 \cdot \dot{x}(t) \, dt + \int G_3 \cdot u(t) \, dt
\end{align*}
\]
(25)

\[
\begin{align*}
\hat{e}(t) &= \int e(t) \, dt + \int G_1 \cdot x(t) \, dt + \int G_2 \cdot \dot{x}(t) \, dt + \int G_3 \cdot u(t) \, dt
\end{align*}
\]
(26)
5. 결론

본 연구에서는 미지 입력을 포함한 선형 시스템을 좌표 변환 방법에 의해 새로운 시스템을 구성하여 적정 최소 차수 관측기를 설계하였고, 변환된 시스템에서 미지 입력 추정식을 유도한 뒤, BPFs를 이용하여 이에 대한 다수적 접근 방법을 제안하였다. 제안된 다수적 접근 방법은 관측기 설계식에 미분함을 포함하지 않으므로 관측기의 해를 계산하는데 용이하며 미지 입력 추정식에 포함된 출력의 미분함을 계기하여 올바른 값으로 외판에 매우 강한항을 가할 수 있다.

문고문헌