'96 대한전기학회 하계학술대회 논문집 (1996. 7. 22~24)

적응 입출력 선형화 제어 기법을 이용한 매입형 영구 자석 동기 전동기의 토오크 궤적 제어

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Torque Trajectory Control of Interior PM Synchronous Motor Using Adaptive Input-Output Linearization Technique

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Abstract: **A torque trajectory control of the IPM synchronous motor using an adaptive input-output linearization technique is proposed. The input-output linearization is performed using the estimated torque output with the knowledge of machine parameters. The linearized model gives the output torque error under the variation of the flux linkage. To give a good torque tracking in the presence of the flux linkage variation, the flux linkage will be estimated where the adaptation law is derived by the Popov's hyperstability theory and the positivity concept. This estimated value is also used for the generation of the d-axis current command for the maximum torque control. Thus, a good torque tracking and the exact maximum torque-per-current operation will be obtained.**

I. INTRODUCTION

In an interior PM (IPM) synchronous motor, the magnets are buried inside the rotor so that the mechanically robust construction is obtained which can be used for the high speed applications since the magnets are physically contained and protected. Furthermore, in the IPM synchronous motor, the q-axis stator inductance L_q is larger than the d-axis stator inductance L_d. The relation $L_d < L_q$ has direct effect on the electromagnetic torque production. In contrast to the surface PM (SPM) synchronous motor, the electromagnetic torque in the IPM synchronous motor contains two components. One is so called excitation torque due to the interaction of the magnet flux and the q-axis stator current and the other one is the reluctance torque component, which is proportional to the difference $(L_d - L_q)$. Because of this reluctance torque component, higher torque per ampere can be produced in an IPM synchronous motor than in a SPM synchronous motor. In the conventional control method of the IPM synchronous motors for servo drive applications, the stator current vector is aligned with the induced voltage due to the permanent magnet flux. Because the d-axis component of the stator current is always kept zero, this control method is called "$i_d = 0$ control"[2]. Recently, It has been shown that several control methods which control not only the q-axis current but also the d-axis current give the improvement of the drive system such as the power factor, efficiency, and power capability[2]-[3]. These improvement can be obtained by the maximum torque control compared with the conventional $i_d = 0$ control. By the maximum torque control method, the reluctance torque can be used effectively by controlling a stator current vector according to load condition. This requires the proper orientation of the stator current vector since the d-axis current for the maximum torque control is represented as a function of the flux linkage and the magnitude of the q-axis current. When the flux linkage is varied, the exact maximum torque control can not be obtained. However, in the most drive applications, the influence of the flux linkage variation for maximum torque-per-current control has not been considered.

In recent years, state feedback linearizations and input-output decoupling techniques have been applied to the control of the robot manipulators, induction motors, and PM synchronous motors[4]-[5]. The basic idea is to first transform a nonlinear system into a linear one by a nonlinear state feedback, and then use the well-known linear design techniques to complete the controller design[4]. In these schemes, the nonlinear terms can be effectively cancelled out, and the output dynamics can be designed to give an arbitrary trajectory following based on the linear-based model. These techniques, however, require the full knowledge of system parameters with the sufficient accuracy. In order to guarantee the robust response under the parameter variation, the controller parameters must be adaptively changed with the variations of the plant parameters.

In this paper, an adaptive input-output linearization scheme with the flux linkage estimation is presented for the robust torque trajectory control of the IPM synchronous motor. The estimated flux linkage is also used for the proper orientation of the stator current vector. Thus, under the variation of the flux linkage, the robust torque trajectory response and exact maximum torque-per-current control will be simultaneously achieved.

II. MODELING OF IPM SYNCHRONOUS MOTOR

The stator voltage equation of an IPM synchronous motor is described as follows:

$$\begin{aligned} v_{qs} &= R_s i_{qs} + L_q \dot{i}_{qs} + L_d \omega_r i_{ds} + \lambda_m \omega_r \\ v_{ds} &= R_s i_{ds} + L_d \dot{i}_{ds} - L_q \omega_r i_{qs} \end{aligned} \qquad (1)$$

where R_s is the stator resistance, L_q and L_d are the q- and d-axes inductances, respectively, ω_r is the electrical rotor angular velocity, and λ_m is the flux linkage. The electromagnetic torque developed by the machine is expressed as

$$T_e = \frac{3}{2} P[\lambda_m i_{qs} + (L_d - L_q) i_{qs} i_{ds}]. \qquad (2)$$

Fig. 1 shows the relation of the q-axis, the d-axis and the stator current vector. Using the angle β in the Fig. 1, the q- and d-axes components of the stator currents are represented as

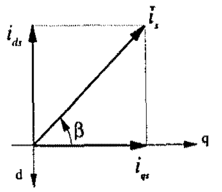

Fig. 1 Relation of the q-axis, d-axis, and stator current vector

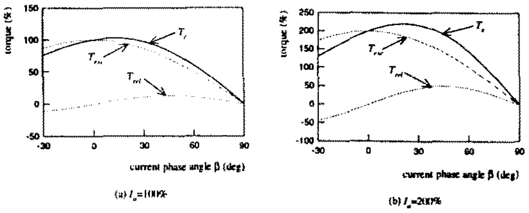
Fig. 2 Effects of the current phase angle to the developed torque

$$i_{qs} = I_a \cos\beta$$
$$i_{ds} = -I_a \sin\beta \tag{3}$$

where $I_a = |\bar{i}_s| = \sqrt{i_{qs}^2 + i_{ds}^2}$ and β is a leading angle of the stator current vector with respect to the q-axis. Using (3), the electromagnetic torque (2) can be rewritten as

$$T_e = \frac{3}{2} P\left(\lambda_m I_a \cos\beta + \frac{1}{2}(L_d - L_q) I_a^2 \sin 2\beta\right). \tag{4}$$

The first term of (4) represents the excitation torque and the second term represents the reluctance torque due to a saliency. As can be shown in (4), the excitation torque is proportional to I_a and the reluctance torque is proportional to I_a^2. Fig. 2 shows the excitation torque T_{exc}, the reluctance torque T_{rel}, and the total torque T_e as a fuction of the current phase angle β when I_a=100% and I_a=200%. The excitation torque is maximum at $\beta = 0°$ and the reluctance torque is maximum at $\beta = 45°$. Therefore, the total torque becomes maximum within the range of $0° < \beta < 45°$ and the effect of the reluctance torque is more significant at high current level. From (4), the relationship between the magnitude I_a and phase angle β for the maximum torque-per-current control is derived as

$$\beta = \sin^{-1}\left(\frac{-\lambda_m + \sqrt{\lambda_m^2 + 8(L_q - L_d)^2 I_a^2}}{4(L_q - L_d) I_a}\right) \tag{5}$$

or equivalently, in terms of the d- and q- axes components of the stator currents, it can be expressed as follows:

$$i_{ds} = \frac{\lambda_m}{2(L_q - L_d)} - \sqrt{\frac{\lambda_m^2}{4(L_q - L_d)^2} + i_{qs}^2}\,. \tag{6}$$

As can be shown in (5), the exact information of the flux linkage is required for the orientation of the stator current vector. When the actual flux linkage varies from the nominal value, the phase angle β has to be adaptively changed for the proper maximum torque-per-current operation. Fig. 3 shows the maximum torque-per-current trajectory with respect to the flux linkage variation. If the exact flux linkage information is given, the phase angle β, and thus, d-axis current is determined at constant value under a constant q-axis current level. However, when the flux linkage is varied form its nominal value, the phase angle β must be changed. Therefore, under the flux linkage variation, the maximum torque per current operation can not be obtained.

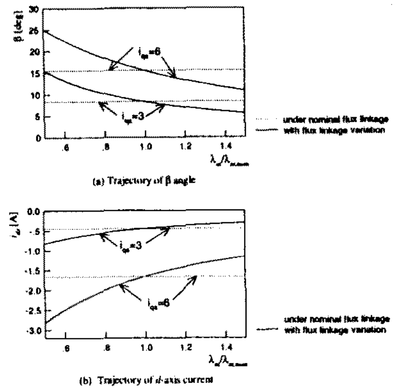
Fig. 3 Maximum torque-per-current trajectory with respect to the flux linkage variation

III. TORQUE CONTROL WITH I/O LINEARIZATION

In this section, a torque trajectory control of the IPM synchronous motor using input-output linearization scheme is presented. A specification of the output torque dynamics is determined based on the linearized model which is obtained by the input-output linearization. Torque output dynamics is influenced by the variation of the flux linkage. To operate the drive system at the maximum torque and to obtain a robust torque response under the flux linkage variation, the flux linkage is estimated and this estimated value is employed for the calculation of the current phase angle β and the torque controller. Using the q- and d- axes stator currents as the states, the state equation of an IPM synchronous motor is described as

$$\dot{x} = f(x) + g_1 v_{qs} + g_2 v_{ds} \tag{7}$$

where $x = [i_{qs} \;\; i_{ds}]^T$, $g_1 = \left(\frac{1}{L_q} \;\; 0\right)^T$, $g_2 = \left(0 \;\; \frac{1}{L_d}\right)^T$

$$f(x) = \begin{pmatrix} -\frac{R_s}{L_q} i_{qs} - \frac{L_d}{L_q}\omega_r i_{ds} - \frac{\lambda_m}{L_q}\omega_r \\ -\frac{R_s}{L_d} i_{ds} + \frac{L_q}{L_d}\omega_r i_{qs} \end{pmatrix}.$$

When the controlled variables are the developed torque and d-axis stator current, the output equation is described as

$$h(x) = [h_1(x) \;\; h_2(x)]^T \tag{8}$$

$$h_1(x) = T_e = \frac{3}{2} P\left\{\lambda_m i_{qs} + (L_d - L_q) i_{qs} i_{ds}\right\} \tag{9}$$

$$h_2(x) = i_{ds} \tag{10}$$

The objective of the control is to maintain the torque and d-axis current to their reference trajectories with a specified behavior. To perform an input-output linearization, the output signal must be given. For the torque trajectory control, the torque feedback signal must be required. The instantaneous torque feedback signal can be estimated from the knowledge of machine parameters and the measurements of currents and rotor position. To control the torque using only the current and position feedback signal, the output torque is estimated with estimated flux linkage as follows:

$$y_1 = \hat{T}_e = \frac{3}{2} P\left\{\hat{\lambda}_m i_{qs} + (L_d - L_q) i_{qs} i_{ds}\right\}$$
$$y_2 = i_{ds} \tag{11}$$

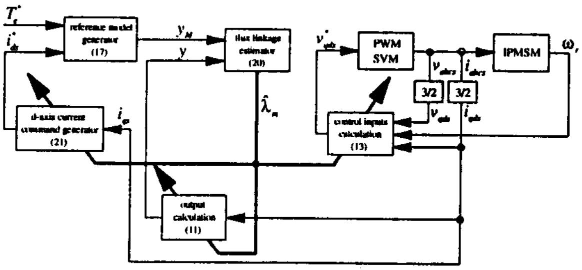

Fig. 4 Overall block diagram of the proposed control scheme

where $\hat{\lambda}_m$ is the estimated flux linkage which will be updated using the estimation algorithm. Using (11), the dynamic equation is described as follows:

$$\begin{aligned} \dot{y}_1 &= L_f h_1 + L_{g1}h_1 \cdot v_{qs} + L_{g2}h_1 \cdot v_{ds} + L_d h_1 \cdot \Delta\lambda_m \\ \dot{y}_2 &= L_f h_2 + L_{g2}h_2 \cdot v_{ds} \end{aligned} \tag{12}$$

where $L_{g1}h_1 = \frac{3}{2}\frac{P}{L_q}\left\{\hat{\lambda}_m + (L_d - L_q)i_{ds}\right\}$

$$L_{g2}h_1 = \frac{3}{2}\frac{P}{L_d}(L_d - L_q)i_{qs}$$

$$L_d h_1 = -\frac{3}{2}P\frac{\omega_r}{L_q}\left\{\hat{\lambda}_m + (L_d - L_q)i_{ds}\right\}$$

$$L_f h_2 = -\frac{R_s}{L_d}i_{ds} + \frac{L_q}{L_d}\omega_r i_{qs}$$

$$L_{g_2}h_2 = \frac{1}{L_d}$$

$$L_f h_1 = \frac{3}{2}P\hat{\lambda}_m\left\{-\frac{R_s}{L_q}i_{qs} - \frac{L_d}{L_q}\omega_r i_{ds} - \frac{\hat{\lambda}_m}{L_q}\omega_r\right\}$$
$$+\frac{3}{2}P(L_d - L_q)\left\{-\frac{R_s}{L_q}i_{qs}i_{ds} - \frac{L_d}{L_q}\omega_r i_{ds}^2 - \frac{\hat{\lambda}_m}{L_q}\omega_r i_{ds} - \frac{R_s}{L_d}i_{qs}i_{ds} + \frac{L_q}{L_d}\omega_r i_{qs}^2\right\}$$

To linearize the nonlinear model in (12), the required control inputs are calculated as

$$\begin{pmatrix} v_{qs} \\ v_{ds} \end{pmatrix} = D(x)^{-1}\begin{pmatrix} -L_f h_1 + u_1 \\ -L_f h_2 + u_2 \end{pmatrix} \tag{13}$$

$$\text{where } D(x) = \begin{pmatrix} L_{g1}h_1 & L_{g2}h_1 \\ 0 & L_{g2}h_2 \end{pmatrix} \tag{14}$$

When the linearizing control inputs (13) are applied, the nonlinear motor model (12) becomes

$$\begin{aligned} \dot{y}_1 &= u_1 + L_d h_1 \cdot \Delta\lambda_m \\ \dot{y}_2 &= u_2 \end{aligned} \tag{15}$$

where u_1 and u_2 are the equivalent control inputs and can be properly assigned to give a specified dynamic performance.

IV. ESTIMATION OF FLUX LINKAGE

To control the output torque dynamics with a specified performance, the flux linkage has to be estimated. As soon as the flux linkage is estimated, the disturbance term in the linearized model (15) is disappeared. To estimate the flux linkage, the equivalent control inputs are chosen as follows:

$$\begin{aligned} u_1 &= -k_T(y_1 - T_e^*) + \dot{T}_e^* \\ u_2 &= -k_{id}(y_2 - i_{ds}^*) + \dot{i}_{ds}^* \end{aligned} \tag{16}$$

where T_e^* and i_{ds}^* are the torque and d-axis current references. This equivalent control input gives the first order torque and d-axis current error dynamics, respectively. To estimate the flux linkage, a reference model is chosen as follows:

$$\begin{aligned} \dot{y}_{1M} &= -k_T(y_{1M} - T_e^*) + \dot{T}_e^* \\ \dot{y}_{2M} &= -k_{id}(y_{2M} - i_{ds}^*) + \dot{i}_{ds}^* \end{aligned} \tag{17}$$

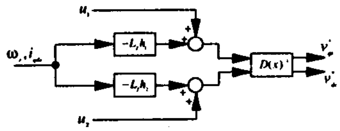

Fig. 5 Calculation of the control input voltages

By subtracting (17) from (15), the error dynamic equation can be obtained as follows:

$$\dot{\tilde{y}} = A_M\tilde{y} - W \tag{18}$$

where $\tilde{y} = y - y_M$, $W = -B\cdot\Delta\lambda_m$

$$A_M = \begin{pmatrix} -k_T & 0 \\ 0 & -k_{id} \end{pmatrix},\ B = \begin{pmatrix} L_d h_1 & 0 \end{pmatrix}^T.$$

Define the adaptation mechanism as follows:

$$\hat{\lambda}_m(\tilde{y},t) = \int_0^t \Psi_1(\tilde{y},t)d\tau + \Psi_2(\tilde{y},t) + \hat{\lambda}_m(0) \tag{19}$$

where Ψ_1 and Ψ_2 are the nonlinear adaptation mechanisms for the estimation of the flux linkage. The design problem to obtain the asymptotic adaptation is as follows:

1. Determine Ψ_1, and Ψ_2 such that $\lim_{t\to\infty} \tilde{y}(t) = 0$.
2. Find the conditions which lead to $\lim_{t\to\infty}\hat{\lambda}_m(\tilde{y},t) = \lambda_m$.

Based on the model reference adaptive control method and the hyperstability theory, the flux linkage is estimated as[6]

$$\hat{\lambda}_m = \left(k_{p\lambda} + \frac{k_{I\lambda}}{s}\right)\cdot(\tilde{y}_1 \cdot L_d h_1) \tag{20}$$

where $\tilde{y} = [\tilde{y}_1 \quad \tilde{y}_2]^T$ and $k_{p\lambda}$ and $k_{I\lambda}$ are the PI gains for the flux linkage estimation, respectively.

V. SIMULATION RESULTS

The block diagram of the proposed torque trajectory control is shown in Fig. 4. The estimated flux linkage is used not only for the calculation of the linearizing control inputs and output estimation, but also for the d-axis current command generation. The d-axis current command is generated with the estimated flux linkage as follows:

$$i_{ds}^* = \frac{\hat{\lambda}_m}{2(L_q - L_d)} - \sqrt{\frac{\hat{\lambda}_m^2}{4(L_q - L_d)^2} + i_{qs}^2}\ . \tag{21}$$

The calculated input voltages are applied to a motor using the space vector PWM technique. The nominal parameters used for the simulations are as follows:

$R_{s,nom}$=1.07[Ω], $L_{q,nom}$=12.0[mH]

$L_{d,nom}$=6.0[mH], and $\lambda_{m,nom}$=0.12[Wb].

Fig. 5 shows the simplified block diagram for the calculation of the control input voltages. Fig. 6 shows the torque and d-axis current responses at each sampling instants when the flux linkage is varied to 80% of its nominal value without the estimation algorithm. The torque command is given as 2 Nm and the sampling period is 0.1 msec. The d-axis current command is generated by the maximum torque control method using (21). Although the d-axis current response shows the specified output dynamic performance, the developed torque can not follow the torque command due to the flux linkage variation. Also, the proper orientation of the stator current vector for the

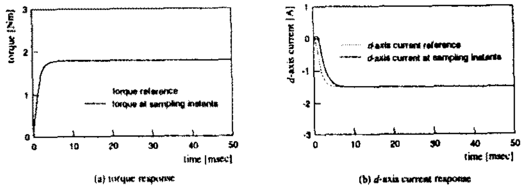

Fig. 6 Torque response under 20% flux linkage variation
(No Adaptation)

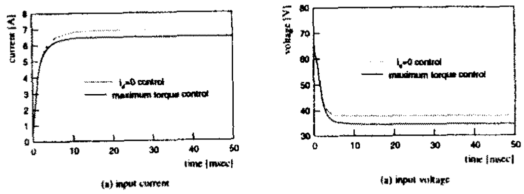

Fig. 8 Comparison of the input currents and voltages

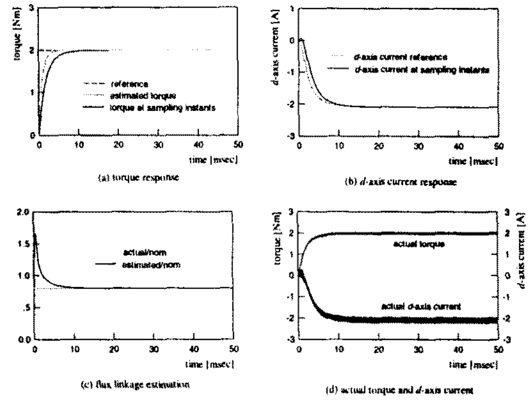

Fig. 7 Torque response under 20% flux linkage variation
(With Adaptation)

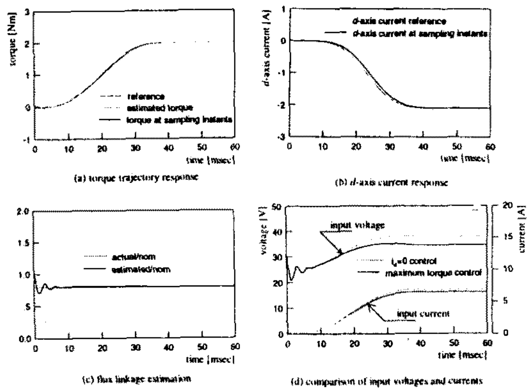

Fig. 9 Torque trajectory response under 20% flux linkage variation **(With Adaptation)**

maximum torque-per-current operation can not be obtained. Fig. 7 shows the torque and d-axis current responses of the proposed control scheme when the flux linkage is varied to 80% of its nominal value. Due to the exact flux linkage estimation, the torque command can be well tracked. Fig. 8 shows the magnitude of the input currents and voltages. In this figure, '$i_d = 0$ control' means the torque control with the flux linkage estimation algorithm when $i_{ds}^* = 0$. Although this control method gives a good torque tracking response, input voltage and current are larger than the maximum torque control method. Fig. 9 shows the torque trajectory response when the flux linkage is varied to 80% of its nominal value. The torque trajectory reference is given as

$$T_r^* = \frac{T_{rf}}{t_f} \cdot t - \frac{T_{rf} \cdot \sin(2\pi t/t_f)}{2\pi} \tag{22}$$

$$\dot{T}_r^* = \frac{T_{rf}}{t_f} - \frac{T_{rf} \cdot \cos(2\pi t/t_f)}{t_f} \tag{23}$$

As can be shown in these simulation results, by utilizing the proposed control scheme, a robust torque trajectory response and exact maximum torque control can be obtained. Also, the magnitude of the input currents and voltages are significantly reduced through the estimation of the flux linkage. Thus, the power capability of the inverter can be improved and it is more useful for the larger torque region.

VI. CONCLUSIONS

A torque trajectory control of an IPM synchronous motor using an adaptive input-output linearization technique is presented. To solve the difficulty for obtaining the torque feedback signal, the instantaneous torque signal is estimated from the estimated flux linkage and the measurements of currents. Using this estimated torque signal, the input-output linearization is performed. The linearized system gives the output torque error under the flux linkage variation. To give a good torque tracking in the presense of the flux linkage variation, an MRAS parameter estimation technique is employed. This estimated flux linkage is also used for the generation of the d-axis current command for the maximum torque-per-current control. This gives the performance improvement of the drive system such as the power factor and power capability. Also, the amplitude of the input currents and voltages are significantly reduced. As a result, a good torque trajectory tracking and the proper orientation of the stator current vector for the maximum torque control can be obtained.

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